<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper presented at *89th ARFTG Microwave Measurement Symposium, Advanced Technologies for Communications, 9 June 2017, Honolulu, Hawaii.*

Citation for the original published paper:

Khan, Z A., Händel, P., Isaksson, M. (2017)

A Comparative Analysis of the Complexity/Accuracy Tradeoff in the Mitigation of RF MIMO Transmitter Impairments.

In: *Microwave Measurement Conference (ARFTG), 2017 89th ARFTG*, 8000827

<https://doi.org/10.1109/ARFTG.2017.8000827>

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:hig:diva-24580>

A Comparative Analysis of the Complexity/Accuracy Tradeoff in the Mitigation of RF MIMO Transmitter Impairments

Zain Ahmed Khan^{*†}, Peter Händel^{*}, and Magnus Isaksson[†]

^{*} Dept. Information Science and Engineering, KTH Royal Institute of Technology, Stockholm, Sweden
Email: zakh@kth.se

[†] Dept. Electronics, Mathematics, and Natural Sciences, University of Gävle, Gävle, Sweden

Abstract—This paper presents a comparative analysis of the complexity accuracy tradeoff in state-of-the-art RF MIMO transmitter mitigation models. The complexity and accuracy of the candidate models depends on the basis functions considered in these models. Therefore, a brief description of the mitigation models is presented accompanied by derivations of the model complexities in terms of the number of FLOPs. Consequently, the complexity accuracy tradeoff in the candidate models is evaluated for a 2×2 RF MIMO transmitter. Furthermore, the model complexities are analyzed for increasing nonlinear orders and number of antennas.

I. INTRODUCTION

RF MIMO transmitters are the contemporary technology in wireless communication for achieving high data rate capacities. However, an energy efficient operation of RF MIMO transmitters results in hardware impairments that are more severe than SISO transmitter impairments. This severity is caused by additional distortion effects such as crosstalk between the MIMO transmission paths [1].

Several mitigation models for SISO transmitter impairments have been proposed in literature [2]. Given an infinite order and complexity, these models can achieve a good error performance. However, at a given finite complexity, only part of the accuracy can be achieved [2]. Hence, it is necessary to analyze the complexity accuracy tradeoff in these models to get a complete picture of their effectiveness.

The complexity of SISO RF transmitter mitigation models is often determined based on the number of parameters which can be determined from the nonlinear order and memory. However, this representation may not always be an appropriate measure for complexity since mitigation models for RF transmitters are implemented on digital signal processing (DSP) hardware [3]. The computational resources of such digital platforms are predominantly allocated for performing additions, subtractions and multiplications for which the number of floating point of operations (FLOPs) is an accurate measure and not the number of parameters. Hence, a complexity accuracy tradeoff analysis for SISO transmitter mitigation models in terms of the number of FLOPs has been presented in [3].

Transitioning to RF MIMO transmitters, several mitigation models have also been proposed based on the MIMO Volterra series [4]. For example, the MIMO parallel Hammerstein (PH) model [5], the MIMO memory polynomial (MP) model [6] and the generalized (GMP) and extended generalized MIMO memory polynomial models (EGMP) [7]. These models incorporate higher complexity basis functions to account for the additional

crosstalk distortions in RF MIMO transmitters. Hence, a complexity accuracy tradeoff analysis is even more essential. The aforementioned works on MIMO mitigation models provide a necessary basis for analyzing the error performance of the candidate models, but the issue of complexity is not tackled.

Therefore, this paper derives the complexities of MIMO mitigation models in terms of FLOPs using a similar technique as presented for SISO transmitters in [3]. The resulting derivations are then used to analyze the requisite complexity accuracy tradeoff in RF MIMO transmitters.

II. THEORY & BACKGROUND

Following the methodology in [3], complexity of the candidate MIMO models is evaluated as the sum of the total number of FLOPs required in: (1) creation of the basis functions and (2) multiplication of the basis functions with their respective model coefficients referred to as the filtering process. Table. I summarizes the basis functions of the candidate models [5, 6, 7] up to the third nonlinear order for a 2×2 RF MIMO transmitter, where m_1 denotes the diagonal memory and m_2 denotes the off diagonal memory.

A. Complexity of MIMO PH

The filtering process for the MIMO PH model involves multiplication of the complex-valued coefficients with their respective complex-valued basis functions. From Table. I, it can be noted that the total number of p -th order basis functions in the PH model is $K \binom{p+1}{2} (M_1^p + 1)$, where M_1^p is the p -th order diagonal memory and K is the number of antennas. Since complex valued multiplications require 6 FLOPs and the complex-valued additions of the resultant basis functions require 2 FLOPs each [3], the filtering complexity of the PH model is,

$$C_f^{\text{PH}} = 8 \left[K \sum_{\substack{p=1 \\ p \text{ odd}}}^P \left(\frac{p+1}{2} \right) (M_1^p + 1) \right] - 2, \quad (1)$$

where P is the nonlinear order. The basis of the PH model include terms of the form $|u_k(n-m)|^2$. The $|\cdot|^2$ operator requires 6 FLOPs while the delay operator does not require any FLOPs [3]. Furthermore, creation of each of the p -th order basis of the form $u|u|^{\frac{p+1}{2}}$ requires $K \binom{p+1}{2}$ FLOPs. Thus, the

TABLE I
BASIS FUNCTIONS CONSIDERED BY THE CANDIDATE MODELS UP TO THE THIRD NONLINEAR ORDER.

PH	GMP	MP	EGMP
$u_1(n-m_1), u_2(n-m_1)$	$u_1(n-m_1), u_2(n-m_1)$	$u_1(n-m_1), u_2(n-m_1)$	$u_1(n-m_1), u_2(n-m_1)$
$u_1(n-m_1) u_1(n-m_1) ^2$	$u_1(n-m_1) u_1(n-m_1-m_2) ^2$	$u_1(n-m_1) u_1(n-m_1) ^2$	$u_1(n-m_1) u_1(n-m_1-m_2) ^2$
$u_2(n-m_1) u_2(n-m_1) ^2$	$u_2(n-m_1) u_2(n-m_1-m_2) ^2$	$u_2(n-m_1) u_2(n-m_1) ^2$	$u_2(n-m_1) u_2(n-m_1-m_2) ^2$
$u_1(n-m_1) u_2(n-m_1) ^2$	$u_1(n-m_1) u_2(n-m_1-m_2) ^2$	$u_1(n-m_1) u_2(n-m_1) ^2$	$u_1(n-m_1) u_2(n-m_1-m_2) ^2$
$u_2(n-m_1) u_1(n-m_1) ^2$	$u_2(n-m_1) u_1(n-m_1-m_2) ^2$	$u_2(n-m_1) u_1(n-m_1) ^2$	$u_2(n-m_1) u_1(n-m_1-m_2) ^2$
-	-	$u_1^2(n-m_1)u_2^*(n-m_1)$	$u_1(n-m_1)u_1(n-m_1-m_2)u_2^*(n-m_1-m_2)$
-	-	$u_2^2(n-m_1)u_1^*(n-m_1)$	$u_2(n-m_1)u_1(n-m_1-m_2)u_1^*(n-m_1-m_2)$

basis creation complexity of the PH model is,

$$C_b^{\text{PH}} = \left[K \sum_{\substack{p=3 \\ p \text{ odd}}}^P \left(\frac{(p+1)^2}{2} \right) (M_1^p + 1) \right] + 6. \quad (2)$$

B. Complexity of MIMO GMP

Since the MIMO GMP has the same structure as the MIMO PH model with additional off diagonal memory terms, its complexity can be described using (1) and (2) as,

$$C_f^{\text{GMP}} = 8 \left[K \sum_{\substack{p=1 \\ p \text{ odd}}}^P \left(\frac{p+1}{2} \right) (M_1^p + 1) (M_2^p + 1) \right] - 2, \quad (3)$$

$$C_b^{\text{GMP}} = \left[K \sum_{\substack{p=3 \\ p \text{ odd}}}^P \left(\frac{(p+1)^2}{2} \right) (M_1^p + 1) (M_2^p + 1) \right] + 6, \quad (4)$$

where, M_2^p is the p-th order off diagonal memory.

C. Complexity of MIMO MP

The filtering complexity of the MIMO MP model also depends on the number basis function and can be given as [6],

$$C_f^{\text{MP}} = 8 \left[K (M_1^1 + 1) + \sum_{\substack{p=3 \\ p \text{ odd}}}^P N_{b_p} (M_1^p + 1) \right] - 2 \quad (5a)$$

$$N_{b_p} = \left(K + \frac{p+1}{2} - 1 \right) \left(K + \frac{p+1}{2} - 2 \right), \quad (5b)$$

where N_{b_p} denotes the number of p-th order nonlinear basis functions. The complexity for creating the MIMO MP basis functions depends on the nature of the said basis. For example, the MP model considers not only the low complexity crosstalk basis of the form $u_{k_1}(n-m)|u_{k_2}(n-m)|^2$ but also the additional high complexity crosstalk basis of the form $u_{k_1}^2(n-m)u_{k_2}^*(n-m)$. Thus the complexity for creating

TABLE II
NUMBER OF TERMS N_g^p BELONGING TO THE COMPLEXITY REDUCED GROUP $\Omega_g(p)$ FOR A 2×2 MIMO PA

$\backslash g$	1	2	3	4	5	$\sum_g N_g^p$
p						
3	4	-	-	-	-	4
5	5	6	-	-	-	11
7	4	9	6	-	-	19
9	4	6	8	10	-	28
11	4	6	9	9	12	40

these basis functions is given as [6],

$$C_b^{\text{MP}} = \sum_{p=3}^P (M_1^p + 1) (S_p + \bar{S}_p + 6) \quad (6a)$$

$$S_p = \sum_{g=1}^{\frac{p-1}{2}} N_g^p C(\Omega_g(p)) \quad (6b)$$

$$\bar{S}_p = 6 \left(N_{b_p} - \sum_{g=1}^{\frac{p-1}{2}} N_g^p \right) (p-1), \quad (6c)$$

where S_p denotes the complexity of the low complexity crosstalk basis functions while $\bar{S}_p$ denotes the complexity of the additional high complexity crosstalk basis functions. $\Omega_g(p)$ describes the groups that contain terms of the form $u|u|^{2g}$ for the p-th nonlinear order and the corresponding number of terms N_g^p for a 2×2 RF MIMO transmitter in Table II. The resulting complexity $C(\Omega_g(p))$ of the group $\Omega_g(p)$ is given as,

$$C(\Omega_g(p)) = 6(p-2g-1) + (g-1) + 2. \quad (7)$$

D. Complexity of MIMO EGMP

Since the MIMO EGMP has the same structure as the MIMO MP model with additional off diagonal memory terms, its complexity can be described using (5) and (6) as,

$$C_f^{\text{EGMP}} = 8 \left[K (M_1^1 + 1) + \sum_{\substack{p=3 \\ p \text{ odd}}}^P N_{b_p} (M_1^p + 1) (M_2^p + 1) \right] - 2. \quad (8)$$

$$C_b^{\text{EGMP}} = \sum_{p=3}^P (M_1^p + 1) (M_2^p + 1) (S_p + \bar{S}_p + 6) \quad (9)$$

Fig. 1. Outline of the experimental setup used in measurements.

III. EXPERIMENTAL INVESTIGATION

Experiments are performed to compare the complexity accuracy tradeoff in the candidate models for a measurement setup mimicking a typical fourth generation (4G) 2×2 RF MIMO transmitter. The transmitter is excited with 64-QAM signals using two R&S SMBV100A vector signal generators (VSGs) with PAPR of 7.5 dB and RMS power level of -7 dBm. The transmitter consists of two ZVE-8G+ PAs placed between two coupling stages used for introducing crosstalk effects [8]. The ZVE-8G+ PAs have a gain of 30 dB each and a 1-dB compression point of 30 dBm. The outputs from the transmitter are measured using wideband down converters cascaded to a two-channel 14-bit resolution analog-to-digital-converter (ADC) operating at a sampling frequency of 400 MHz. The ADCs and the VSGs are then connected through a PC for control.

Figs. 2(a)-(b) plots the NMSE of the candidate models versus the number of FLOPs for 10 MHz and 20 MHz signals respectively. It can be noted that the PH model requires lowest complexity because it considers neither the off diagonal nor the high complexity crosstalk terms. The complexity of the GMP model is higher than the PH model because it considers off diagonal terms but lower than the MP model because it does not consider the high complexity crosstalk terms. Furthermore, it can be noted that the error performance of high complexity PH and GMP models deteriorates due to over-modeling.

It can be noted the MP model achieves good error performance with fewer FLOPs than the EGMP model for 10 MHz signals because the effect of the off diagonal terms is marginal. This is indicated by the tradeoff optimum in Fig. 2(a). Therefore, the MP model is suitable for this scenario since it does not involve redundant off diagonal terms which increase the complexity of the EGMP model. However, the EGMP model achieves better error performance with fewer FLOPs than the MP model for 20 MHz signals due to the increased effect of the off diagonal basis. This is indicated by the tradeoff optimum in Fig. 2(b). Here, the advantage of considering the off diagonal terms can be observed whose combination with a lower diagonal memory enables the EGMP model to achieve good error performance. The MP model achieves the same error performance but with higher diagonal

Fig. 2. NMSE versus the number of FLOPs for the candidate models with excitation signals having bandwidth of (a) 10 MHz and (b) 20 MHz.

memory, thus requiring more FLOPs.

Furthermore, it can be noted from Figs. 2(a)-(b) that the difference in error performance of the PH and GMP models compared to the MP and EGMP models is worse for 20 MHz signals because the effect of the high complexity crosstalk terms in the MP and EGMP models is compounded with increase in memory as the bandwidth increases. Comparing the results from Fig. 2(a) with Fig. 2(b), it can be noted that the error performance of all the models for the same number of FLOPs is worse for 20 MHz than 10 MHz. This is expected since the nonlinear dynamic distortions increases with an increase in bandwidth which require additional model complexity for mitigation.

For SISO transmitters, it is shown in [3] that the PH and GMP models achieve the best complexity accuracy tradeoff. This is valid for SISO transmitters where crosstalk is not present. However, for RF MIMO transmitters, it can be observed from Fig. 2 that both these models achieve the worst error performance because only the low complexity crosstalk terms are considered while the high complexity crosstalk terms are neglected (c.a Table. I). Thus, the mitigation of RF MIMO impairments requires high complexity MP and EGMP models to achieve good error performance.

The computational complexity of the considered models for different nonlinear orders is depicted in Fig. 3(a) for a low complexity memory scheme with $M_1^1 = 3$, $M_1^3 = 2$, $M_1^5 = 1$, $M_2^3 = 1$, $M_2^5 = 0$. It can be noted that the PH model requires lowest computational complexity because it does not consider either the higher complexity basis or the off diagonal memory basis. The complexity of the GMPNLC model is lower than the MP model since the effect of high complexity basis considered in the MP model is exponential compared to the linear effect of off diagonal basis considered in the GMPNLC model. The EGMPNLC model requires the highest complexity since it considers both the high complexity basis as well as the off diagonal memory terms. The complexity of the considered models for a high complexity memory scheme with $M_1^1 = 5$, $M_1^3 = 3$, $M_1^5 = 2$, $M_2^3 = 1$, $M_2^5 = 1$ is shown in Fig. 3(b). Similar trends are observed as Fig. 3a since the complexity

Fig. 3. Comparison of the complexity of the candidate models over different nonlinear orders with (a) low memory and (b) deep memory.

Fig. 4. Comparison of the complexity of the candidate models over different number of antennas with (a) low memory and (b) deep memory.

grows with the nonlinear order but linearly with memory.

Finally, the computational complexity of the considered models is analyzed with respect to the number of antennas for deployment in future higher order MIMO transmitters. Fig. 4(a) plots the complexity of the proposed models over different number of antennas for $P = 5$ and a low complexity memory scheme with $M_1^1 = 3$, $M_1^3 = 2$, $M_1^5 = 1$, $M_2^3 = 1$, $M_2^5 = 0$. It can be noted that the increase in complexity of the PH and GMPNLC models is significantly lower than the MP and EGMPNLC models. This shows that the effect of the high complexity basis functions is compounded as the number of antennas increases. Fig. 4(b) plots the complexity of the proposed models over different number of antennas for $P = 5$ and a high complexity memory scheme with $M_1^1 = 5$, $M_1^3 = 3$, $M_1^5 = 2$, $M_2^3 = 1$, $M_2^5 = 1$. It can be noted that the GMPNLC and EGMPNLC models require more FLOPs than the PH and MP models respectively, due to the increased effect of the off diagonal terms.

IV. CONCLUSION

This paper derives the complexities of four state-of-the-art MIMO mitigation models and analyzes their complexity accuracy tradeoff for a 2×2 RF MIMO transmitter. Com-

pared to SISO transmitters where the low complexity PH and GMP models achieve best error performance, the MIMO PH and GMP models achieve worst error performance because crosstalk is not completely accounted for in these models. Instead, RF MIMO transmitters require the high complexity MIMO MP and EGMP models to achieve good error performance for complete crosstalk mitigation. Furthermore, it is shown that the MP model achieves the best balance for low bandwidth signals due to the marginal the effect of off diagonal terms while the EGMP model achieves the best balance for higher bandwidth signals where the effect of off diagonal terms is more prominent. Finally, the complexity of the candidate models is analyzed with respect to nonlinear order and the number of antennas for low and deep memory schemes. It is shown that the exponential increase in complexity with nonlinear order and the number of antennas is the dominant effect compared to the linear increase in complexity with memory.

REFERENCES

- [1] S. A. Banani and R. G. Vaughan, "Compensating for non-linear amplifiers in MIMO communications systems," *IEEE Trans. Antennas Propag.*, vol. 60, no. 2, pp. 700–714, Feb 2012.
- [2] J. C. Pedro and S. A. Maas, "A comparative overview of microwave and wireless power-amplifier behavioral modeling approaches," *IEEE Trans. Microw. Theory Techn.*, vol. 53, no. 4, pp. 1150–1163, April 2005.
- [3] A. Tehrani, H. Cao, S. Afsardoost, T. Eriksson, M. Isaksson, and C. Fager, "A comparative analysis of the complexity/accuracy tradeoff in power amplifier behavioral models," *IEEE Trans. Microw. Theory Techn.*, vol. 58, no. 6, pp. 1510–1520, Jun. 2010.
- [4] A. K. Swain and S. A. Billings, "Generalized frequency response function matrix for MIMO non-linear systems," *Int. J. Control.*, vol. 74, no. 8, pp. 829–844, 2001.
- [5] D. Saffar, N. Boulejfen, F. Ghannouchi, A. Gharsallah, and M. Helaloui, "Behavioral modeling of MIMO nonlinear systems with multivariable polynomials," *IEEE Trans. Microw. Theory Techn.*, vol. 59, no. 11, pp. 2994–3003, Nov. 2011.
- [6] Z. A. Khan, E. Zenteno, P. Händel, and M. Isaksson, "Digital predistortion for joint mitigation of I/Q imbalance and MIMO power amplifier distortion," *IEEE Trans. Microw. Theory Techn.*, vol. 65, no. 1, pp. 322–333, Jan 2017.
- [7] S. Amin, P. Landin, P. Händel, and D. Rönnow, "Behavioral modeling and linearization of crosstalk and memory effects in RF MIMO transmitters," *IEEE Trans. Microw. Theory Techn.*, vol. 62, no. 4, pp. 810–823, Apr. 2014.
- [8] Y. Palaskas, A. Ravi, S. Pellerano, B. Carlton, M. Elmala, R. Bishop, G. Banerjee, R. Nicholls, S. Ling, N. Dinur, S. Taylor, and K. Soumyanath, "A 5-GHz 108-Mb/s 2×2 MIMO transceiver RFIC with fully integrated 20.5-dBm P_{1dB} power amplifiers in 90-nm CMOS," *IEEE J. Solid-State Circuits*, vol. 41, no. 12, pp. 2746–2756, Dec. 2006.