This is the accepted version of a paper presented at *IEEE International Microwave Symposium, Hawaii*.

Citation for the original published paper:

Multitone Design for Third Order MIMO Volterra Kernels.
In:

N.B. When citing this work, cite the original published paper.

Permanent link to this version:
http://urn.kb.se/resolve?urn=urn:nbn:se:hig:diva-24581
Multitone Design for Third Order MIMO Volterra Kernels

Zain Ahmed Khan *,†, Efrain Zenteno‡, Peter Händel*, and Magnus Isaksson†
* Dept. Information Science and Engineering, KTH Royal Institute of Technology, Stockholm, Sweden
† Dept. Electronics and Telecommunications, Universidad Católica San Pablo, Arequipa, Peru
‡ Dept. Electronics, Mathematics, and Natural Sciences, University of Gävle, Gävle, Sweden
Email: zakh@kth.se

Abstract—This paper proposes a technique for designing multitone signals that can separate the third order multiple input multiple output (MIMO) Volterra kernels. Multitone signals fed to a MIMO Volterra system yield a spectrum that is a permutation of the sums of the input signal tones. This a priori knowledge is used to design multitone signals such that the output from the MIMO Volterra kernels does not overlap in the frequency domain, hence making it possible to separate these kernels from the output of the MIMO Volterra system. The proposed technique is applied to a $2 \times 2$ RF MIMO transmitter to determine its dominant hardware impairments. For input crosstalk, the proposed method reveals the dominant self and cross kernels whereas for output crosstalk, the proposed method reveals that only the self kernels are dominant.

I. INTRODUCTION

Measurement of Volterra kernels is an enduring problem in the research community due to the wide ranging applications of the Volterra theory such as mechanics, communications and biology, among others. Hence, several efforts have been made to devise suitable identification techniques for Volterra kernels but they suffer from stability and complexity issues [1].

Therefore, a method to overcome these drawbacks based on the separation of single input single output (SISO) nonlinear Volterra kernels was proposed using multitone signals in [2]. Excitation of Volterra systems by multitone signals yields a frequency grid which consists of all the permutations of sums of the input signal tones. This property is utilized in [2] to separate the second order Volterra kernels. Later, a sparse grid with odd multitone was presented in [3] to separate higher order nonlinear Volterra kernels. The sparsity of the input grid ensures that the Volterra kernels do not completely overlap with each other. A detailed study regarding the use of multitone signals for nonlinear identification is given in [4, 5].

In contrast to SISO systems, the multiple input signals of a MIMO system provide an extra degree of freedom to design excitations that fulfill certain requirements [6]. This feature is explored in this paper to separate the third order MIMO Volterra kernels using a similar technique as presented for SISO Volterra systems in [3]. Thus, the MIMO Volterra system is excited with $K$ multitone input signals such that the kernels fall in a non-overlapping frequency grid, as depicted in Fig. 1.

To this end, we derive the location of the output frequency tones of the MIMO Volterra system when excited with even and odd tones as defined in [3]. For a $2 \times 2$ MIMO Volterra system, this choice of the input grid separates the kernels into two distinct frequency grids, hence simplifying the analysis of third order MIMO Volterra kernels.

Furthermore, this paper also studies the effects of different input frequency grid excitations such that the contributions of the individual kernels fall in distinct frequency grids either partially or completely. This result is valuable in separating the individual third order MIMO Volterra kernels that can be identified directly or as band limited systems.

II. THEORY & BACKGROUND

This paper focuses on a $2 \times 2$ MIMO Volterra whose complex valued baseband output $y^{(i)}$ is given as,

$$y^{(i)} = g^{(i)}_1(u_1) + g^{(i)}_2(u_2) + g^{(i)}_3(u_1, u_2) + \cdots, \quad (1)$$

where $u_1$ and $u_2$ are the complex-valued baseband multitone input signals, $g^{(i)}_1(\cdot)$ and $g^{(i)}_2(\cdot)$ are the linear dynamics described by a finite impulse response (FIR) representation while $g^{(i)}_3(\cdot, \cdot)$ are the third order nonlinear dynamics described by polynomials of the product permutations of $u_1$ and $u_2$, the so called third order MIMO Volterra basis functions. The complex-valued parameters multiplying with these basis functions are called the third order MIMO Volterra kernels. Thus, for example, the kernels described by the basis functions $u_1 u_1 u_1^*$, $u_1 u_2 u_2^*$, and $u_2 u_2 u_1^*$, where $^*$ denotes the complex conjugate operator. Furthermore, it can be noted that $g^{(i)}_3(\cdot, \cdot)$ also includes dynamic functions covering basis such as $u_1(n) u_2(n-1) u_1^*(n-2)$ [7].

Fig. 1. $K$ multitone signals exciting a MIMO Volterra system.
From (1), it can be noted that the output of the linear dynamic functions in (1) falls in the same frequency grid as \( u_1 \) and \( u_2 \), respectively. Therefore, to separate the linear kernels, it is sufficient to excite the input signals with distinct tones. However, in order to separate the nonlinear contribution \( g^{(3)}_k(\cdot,\cdot,\cdot) \), this condition is insufficient because the output of the third order kernels fall in a frequency grid that is a permutation of the sums of the input signal frequency grids. For example, the output of \( u_2 u_1 u_1^* \) would fall in a frequency grid that is a permutation of the sums of the grids of \( u_2 \), \( u_1 \) and \( u_2^* \), i.e.,

\[
f_2(\cdot) + f_1(\cdot) - f_1(\cdot),
\]

where \( f_k(\cdot) \) denotes the \( k \)-th input multitone frequency grid.

Therefore, multitone signals result in frequency grids for the third order kernels that can be determined a priori. This a priori knowledge is used aiming to design input signals such that the third order kernels fall in a frequency grid that is a permutation of the sums of the grids of \( u_2 \), \( u_1 \) and \( u_2^* \), i.e., \( f_2(\cdot) + f_1(\cdot) - f_1(\cdot) \), where \( f_k(\cdot) \) denotes the \( k \)-th input multitone frequency grid.

III. PROPOSED FREQUENCY GRIDS

This paper proposes multitone input signals with a frequency grid of the form,

\[
f_k(z) = \Delta_k z + c_k,
\]

where \( c_k \) is the \( k \)-th input frequency offset and \( \Delta_k \) is the \( k \)-th input frequency spacing, i.e., if 6 tones are excited with \( c_k = 2 \), \( \Delta_k = 4 \), \( f_k = [-10 -6 -2 2 6 10] \). It can be noted that the frequency offset determines the center of the grid whereas the frequency spacing determines the sparsity of the grid. This paper proposes two such grids.

A. Even / Odd Grid

A frequency grid with even and odd tones is proposed such that \( c_1 = 1 \), \( c_2 = 0 \) and \( \Delta_k = 2 \). That is, if 6 tones are excited, then the input frequency grids are,

\[
\begin{align*}
  f_1 &= -5 -3 -1 1 3 5 \\ f_2 &= -6 -4 -2 0 2 4.
\end{align*}
\]

(3a) (3b)

Therefore, the contribution of the basis function \( u_1 u_1 u_1^* \) (and all its dynamic forms) will be \( f_1(\cdot) + f_1(\cdot) - f_1(\cdot) \), that is, a sum and difference of odd integers in (3) yielding,

\[
\begin{align*}
  f_{111} &= -15 -13 -11 \cdots -1 1 \cdots 11 13 15 \\ f_{222} &= -18 -16 -10 \cdots -2 0 \cdots 8 10 12.
\end{align*}
\]

(4a) (4b)

where, \( f_{111} \) is the frequency grid at the output of \( u_1 u_1 u_1^* \). Note that the grid expands three times the input bandwidth, as it is well known by Volterra theory. Furthermore, the output of this kernel overlaps the same grid as \( u_1 \). The basis \( u_2 u_2 u_2^* \) have a similar behavior considering \( u_2 \), as observed in (4b), where \( f_{222} \) is the frequency grid at the output of \( u_2 u_2 u_2^* \).

Furthermore, since the sum of two even and one odd integer is always an odd integer, \( u_2 u_1 u_1^* \) and \( u_2 u_2 u_2^* \) overlap \( u_1 \). Following a similar analysis, \( u_1 u_2 u_1^* \) and \( u_1 u_1 u_2^* \) would overlap the same grid as \( u_2 \). These results are summarized in Table I which displays all the basis functions of the third order MIMO Volterra kernels. It can be observed from Table I that the third order kernels are split equally into even and odd frequency grids. This result simplifies the analysis of MIMO Volterra systems and is extended to separate the individual third order MIMO Volterra kernels in the next section.

B. Sparse Even and Odd Grid

Section III-A proposes an even / odd frequency grid of input signal tones to separate the third order MIMO Volterra kernels into even and odd grids at the output. This section proposes a technique to separate the frequency overlapping contributions of the individual kernels within the even and odd frequency grids. To this end, an even / odd input frequency grid with different levels of sparsity is proposed. Thus for example, if 6 tones are excited with \( c_1 = 3 \), \( c_2 = 2 \), \( \Delta_1 = 4 \) and \( \Delta_2 = 8 \), the frequency grids are given as,

\[
\begin{align*}
  f_1 &= -9 -5 -1 3 7 11 \\ f_2 &= -22 -14 -6 2 10 18.
\end{align*}
\]

(5a) (5b)

as shown in Fig. 2(a). These frequency grids of the input signals yield partially or completely non-overlapping frequency grids at the output that can be used to separate the individual third order kernels. The kernels whose outputs fall in completely non-overlapping frequency grids are a consequence of the higher sparsity levels in the input frequency grids whereas the partially non-overlapping frequency grids at the kernel outputs are a consequence of the difference in sparsity levels between the input frequency grids. The frequency grids at the output of the third order kernels are described in Table II and a slice of the spectrum of these kernels is shown in Fig. 2(b).
TABLE II
FREQUENCY GRIDS AT THE OUTPUT OF THE THIRD ORDER MIMO VOLterra KERNELS FOR A SPARSE EVEN / ODD GRID.

<table>
<thead>
<tr>
<th>Basis functions</th>
<th>Frequency grid</th>
<th>Example for 6 excited tones</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1 u_1 u_1^*$</td>
<td>4z + 3</td>
<td>-29 -25 -21</td>
</tr>
<tr>
<td>$u_2 u_2 u_2^*$</td>
<td>8z + 2</td>
<td>-62 -54 -46</td>
</tr>
<tr>
<td>$u_1 u_2 u_1^*$</td>
<td>4z + 2</td>
<td>-42 -38 -34</td>
</tr>
<tr>
<td>$u_2 u_1 u_2^*$</td>
<td>4z + 3</td>
<td>-49 -45 -41</td>
</tr>
<tr>
<td>$u_1 u_1 u_2^*$</td>
<td>4z</td>
<td>-36 -32 -28</td>
</tr>
<tr>
<td>$u_2 u_2 u_1^*$</td>
<td>4z + 1</td>
<td>-55 -51 -47</td>
</tr>
</tbody>
</table>

It can be noted from Table II that for $u_1 u_1 u_2^*$, the sums of two odd integers with one even integer yield a non-overlapping even grid due to the sparsity in $u_2$. Similarly, $u_2 u_2 u_1^*$ lies in a frequency grid of non-overlapping odd tones. These results are illustrated in Fig. 2(b).

Furthermore, it can also be noted from Table II that for $u_1 u_2 u_1^*$, the sums of two odd and one even integer yield a partially non-overlapping even grid as shown in Fig. 2(b). The grid is partially non-overlapping because the sparsity in $u_1$ is less than $u_2$. This grid is useful in separating $u_1 u_2 u_1^*$ based on the study of band limited Volterra systems [8], using a comb filter to remove the overlapping tones.

It can be further noted from Table II that the frequency grid at the output of $u_2 u_2 u_1^*$ expands three times the bandwidth of $u_2$ and overlaps with all the tones in the grid of $u_2$ and $u_1 u_2 u_1^*$ as shown in Fig. 2(b). Therefore, it can be separated only in the spectral regrowth region as a band limited kernel [8] by first eliminating the previously separated $u_1 u_2 u_1^*$.

Finally, by switching the inputs to the MIMO Volterra, it can be noted that the frequency grids for $u_1 u_1 u_1^*$ and $u_2 u_1 u_2^*$ follow the same properties as $u_2 u_2 u_2^*$ and $u_2 u_1 u_2^*$. Therefore, these kernels can also be separated as described previously. Hence, instead of identifying all the third order kernels jointly, the separated kernels can be identified independently. This result is valuable in simplifying the analysis of MIMO Volterra systems by revealing the dominant basis functions.

Furthermore, since the MIMO Volterra kernels are identified using least squares estimation techniques [7], the identification complexity, given in terms of the number of arithmetic operations, grows as $O(N \times M)$ [9], where $N$ is the number of excited input tones and $M$ is total number of third order basis functions (static and dynamic). Separation of the individual third order kernels reduces the size of the regression matrix by a factor of 6. Therefore, the identification complexity is reduced to $6O(N \times M/6)$. Hence, in addition to simplified analysis, separation of the individual third order MIMO Volterra kernels yields low complexity identification processes as well. These results are summarized in Table III.

IV. EXPERIMENTAL INVESTIGATION

Experiments are performed to validate the proposed method for a $2 \times 2$ RF MIMO transmitter using a measurement setup similar to [10] as depicted in Fig. 3. The experiments are performed for two configurations of the RF MIMO transmitter: (i) only input crosstalk with directional couplers at the input of the amplification stage and (ii) only output crosstalk with directional couplers at the output of the amplification stage.

In these experiments, the MIMO transmitter is tested with the signal excitations described in Section III using equal amplitude multitone signals of the form,

$$u_k(n) = \sum_{z=0}^{N-1} e^{i2\pi f_k z + \phi_k(z)},$$  \hspace{1cm} (6)

where, $\phi_k$ is the $k$-th input phase. It must be noted that the phases of these signals can be chosen depending on the test requirements. For example, Schroeder phases [11] which minimize the peak to average power ratio (PAPR) or equal phases which create a pulse-like signal (large PAPR). For the experiments presented in this paper, random phases distributed uniformly over $[-\pi, \pi]$ were used with $N = 10000$, to generate communication-like signals. Finally, it must be noted that the sampling frequency was the same for all the input signals, $u_k(n)$.

Fig. 4 displays the output spectrum for 20 MHz multitone signals in an even / odd grid without sparsity for input crosstalk. It can be noted that the output spectrum is split equally into kernels with even and odd frequency grids at the output, as described in Table I. Furthermore, it can also be noted that the odd frequency kernels, $u_1 u_1 u_1^*$, $u_2 u_2 u_1^*$ and

![Fig. 3. Outline of the experimental setup used in measurements.](image-url)
Fig. 4. Spectral plot at the output of the $2 \times 2$ MIMO transmitter with input crosstalk. The transmitted is excited with 20 MHz multitone signals with an even / odd frequency grid without sparsity. The odd frequency grid includes $u_1 u_1 u_1^*$ and $u_2 u_2 u_2^*$, while the even frequency grid includes $u_2 u_2 u_2^* + u_1 u_2 u_1^* + u_1 u_1 u_2^* + \cdots$.

Therefore, the proposed method also reveals the dominant self kernels in the transmitter. Fig. 5 displays the spectrum at the output of the third order MIMO Volterra kernels excited by a sparse even / odd grid. It can be noted from Fig. 5(a) that for input crosstalk, the self kernels $u_1 u_1 u_1^*$ and $u_2 u_2 u_2^*$, and the cross kernels $u_1 u_2 u_1^*$ and $u_1 u_1 u_2^*$, are the dominant third order kernels. Hence, both the self and cross kernels are dominant hardware impairments in the case of input crosstalk, as presented in [12]. However, the proposed method also identifies the relevant cross kernels, that is, $u_1 u_2 u_1^*$ and $u_1 u_1 u_2^*$, which the results in [12], do not.

Furthermore, it can also be noted from Fig. 5(b) that for output crosstalk, self kernels are the dominant third order kernels while the contribution of each of the cross kernels is theoretically zero, upto the measurement noise. Hence, the self kernels are the dominant hardware impairments in the case of output crosstalk, as presented in [12].

V. CONCLUSION

This paper presents a technique for designing multitone input signals to separate the third order MIMO Volterra kernels. A sparse grid of even and odd tones is proposed such that the outputs of the third order kernels fall in a non-overlapping frequency grid, either partially or completely. These non-overlapping grids can then be used to separate the individual kernels, either directly or as band limited systems. This result simplifies the analysis of MIMO Volterra systems by revealing the dominant basis functions and yields simpler identification processes. The method is validated for a $2 \times 2$ RF MIMO transmitter for the cases of input and output crosstalk. For input crosstalk, the proposed methods indicates that the self kernels, $u_1 u_1 u_1^*$ and $u_2 u_2 u_2^*$ and the cross kernels, $u_1 u_2 u_1^*$ and $u_1 u_1 u_2^*$ are the dominant hardware impairments. However, for output crosstalk, the proposed method indicates that only the self kernels are dominant.

REFERENCES