METAMATERIALS
A field magnitude dependent and frequency independent model

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Dedicated to:

My Parents; Parviz and Roya
And to my sister Laleh

Also to the memory of my sister:
Zohreh
Abstract

In all attempts to analyze and realize Left-Handed materials, so far, most researchers have used the same idea of extracting only some or certain behaviors of Metamaterials from a set of unit cells gathered together in a designed order. Nevertheless meeting all criteria in order to consider a media as real double-negative material has never come true.

Starting with criticizing and arguing the validity of calling any set of unit cells as a medium of propagation, the work at hand will go further demonstrating analogies between a medium which could be assigned permittivity or permeability factors and the medium consisting a set of unit cells.

After presenting the critical analysis on previous studies in the field, here it is shown that it is impossible to build Metamaterials using any number of passive unit cells. A deep insight into the concept of phase and group velocities as well as Poynting’s vector will reveal weakness of the public perception of their relation with each other. Unlike the past and current trend in analyzing these two velocities in metamaterials, they will be proven to possess the same direction.

Moreover, in this work, a solid proof over violation of energy conservation in the intersection plane between a normal material and a Left Handed material is presented which requires us to believe and accept generation of energy at this plane. This view will consequently leave meaningless all attempts to build meta-materials by passive elements.

In present work a method is proposed at which a material with positive permittivity and permeability can behave like and yield all characteristics of Metamaterials only if the foregoing parameters, while remaining positive, can vary and be governed by the magnitude of the electromagnetic field. Independence of this method from frequency broadens the range of its application and also the interest it may attract.

*Keywords:* Metamaterials, Field Magnitude Dependent Model, Periodic Structures, FDTD
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1. Introduction

The word Metamaterial, literally, refers to any substance possessing characteristics beyond behavior of normal materials. However after Veselago’s paper [1], materials with negative permittivity and permeability became the first example of Metamaterials molded into minds. Since then the word metamaterial brings the Double Negative materials to mind. They are also called Left Handed materials because of direction of Poynting’s Vector. Consider Maxwell’s equations at (1) and (2).

\[
\begin{align*}
\nabla \times E &= -\frac{1}{c} \frac{\partial B}{\partial t} \\
\nabla \times H &= +\frac{1}{c} \frac{\partial D}{\partial t} \\
D &= \varepsilon E \\
B &= \mu H
\end{align*}
\]

(1)

(2)

Now if we consider a TEM wave propagating in \( \hat{K} \) then Maxwell’s equations reduce to (3).

\[
\begin{align*}
\nabla \times E &= \frac{\omega}{c} \mu H \\
\nabla \times H &= -\frac{\omega}{c} \varepsilon B
\end{align*}
\]

(3)

When \( \mu \) and \( \varepsilon \) are positive the direction \( E, H \) and \( K \) vectors are related to each other by the right hand rule. But when they are negative the vectors’ direction will be related to each other by the left hand rule.

In this chapter, first, an overview of previous studies on the subject is presented. Afterwards a general description of the work ahead of us will be given.

1.1. Background

Veselago’s paper published in 1968 introduced new materials to physics society at the time. Having negative values of both permittivity and permeability, meta-materials or double-negative materials were predicted to behave differently from their previously existing counterparts. Veselago’s action in assuming negative group velocity in Left-Handed material in [1] did not cause much opposition. However, recently, Caloz and Itoh in [2] on page 33 explained the phenomenon in different words. Unlike Veselago they assumed phase velocity to be negative and group velocity to be positive. “However, this appears more acceptable if one remembers that phase velocity simply corresponds to the propagation of a perturbation and not of energy” said Caloz and Itoh.

It will be shown that neither Caloz and Itoh’s approach to tackle problem of deciding direction of two velocities is true nor is that of Veselago and ultimately both interpretations are inconsistent with the philosophy and physics of equations forming electromagnetic propagation and therefore yields an incorrect insight into the propagation of energy and information. In my model causality will prevail while
energy propagation is interpreted to be in negative direction with a positive phase and group velocities. Causality is shown to be associated to group velocity through which information is transmitted while propagation of energy will take place at the direction extracted from Poynting’s vector. Poynting’s vector will be shown to be un-related to group velocity.

So it is understood that one of the pillars, upon which the building of current knowledge of metamaterial is erected, is different directions of phase and group velocities. Here a mathematical proof will expect all the velocities to possess the same direction which will lead to collapse of previous expectations.

Whatever has been done so far was based on two different ideas; first is to attempt to build a media at which permittivity and permeability are both decreasing as the frequency increases and finally we will be left with a material whose characteristics are demonstrating negative permittivity and permeability at certain frequencies. Collin in [3] has explained how, in a simple molecule, the growing time lag will cause the permittivity to decrease as frequency increases. Same approach could be taken for a rotating current loop in an alternating magnetic field. However, so far, any attack at the problem of realizing such a material with both negative permittivity and negative permeability through this approach has failed.

Nearly all realizations of a meta-material [8-14], summarized in [2] and [4] have concentrated on making a wave propagating backward or obtaining negative refraction index from a boundary plane between an LH and an RH material. It appears that totality of the concept of Double Negative Material is forgotten and forces are joined together insisting on having single or some limited characteristics of Metamaterials. Building a spatial matrix of unit cells whose sizes are comparable to wavelength and their spacing is at distances of considerable fraction of wavelength will be, in this work, shown to be more like designing an array antenna with pre-defined radiation pattern rather than realizing a media to which we can assign a different permittivity and permeability – negative, in this case – that could remain intact after applying any perturbation. This will be better explained later in this work especially considering applying a discontinuity of a size comparable to wavelength in the spatial matrix of unit cells. We will see that the work done so far does not qualify to be considered to possess negative permittivity and permeability. However the model I have presented here is consistent with all necessities of philosophy of the foregoing characteristics.

The same argument stands if one tries to challenge the validity of efforts to extract a negative index of refraction. Wavelength of a comparable size to the distance between elements and also some necessities on the angle of incident is strong enough to make us believe that any generated wave front producing negative index of refraction is rather an electromagnetic response of the whole system than any behavior connected to permittivity or permeability.

Electromagnetic structure of previous designs as well as their size which is comparable to wavelength has led to a solution completely dependent on frequency. This dependence on frequency is fundamental and is based upon the design itself. The model introduced here is independent from frequency. This characteristic of the model will help us to have a metamaterial medium in which the frequency response
is constant over a really broad range of frequency. Any possible limit on the bandwidth will have nothing to do with the model but the practical shortcomings of realization method of the proposed substance.

Another proof which reveals impossibility of building Metamaterials using passive elements will be given in this work. This will deal with conservation of energy between two media; one normal and the other metamaterial.

1.2. In this work
Adoption of a conceptual and physical approach will help us to obtain a deeper insight into the subject. A critical review of simple concepts which are frequently used in most of literature on Electrogmananetics will enable us to recognize the mistakes in some previous studies.

This will start with going through the definition of $\varepsilon_r$ and $\mu_r$. It will be discussed that under which conditions one is allowed to utilize the parameters $\varepsilon_r$ and $\mu_r$. Afterwards it will be shown that these conditions are not satisfied in previous works on metamaterial. Therefore what has been done so far will not fit into the category in which the idea of relative permittivity or permeability, whether positive or negative, can be used.

Explaining the philosophy of phase velocity and group velocity will be next step in illuminating our understanding of wave propagation in metamaterial. It will be discussed whether it is possible to have any of these velocities in backward direction. This will be concluded by a mathematical proof that completes the proof of having both velocities in the same direction. This will fortify and complete our belief in incompleteness of the previous studies on Metamaterials.

Another proof which illustrates how conservation of energy is violated at the plane between a normal material and a metamaterial will be presented and will force us to believe that there is a generation of energy at the intersection plane. This is another point clarifying impossibility of realization of Metamaterials utilizing a set unit cells which are passive components.

A very useful method to demonstrate all these behaviors is Finite Difference Time Domain-FDTD. Also through formulation it will be shown that it is impossible to have phase and group velocities of the opposite direction. We have a source which is varying with time by a sinusoidal function whose amplitude is a function of time. We need to calculate the field in the media and analyze the phase and group velocity’s direction. This is best supported by FDTD because of its capability of providing information on wave propagation in desired time steps.

A frequency-independent model will be presented which complies with all our, so far, reformed expectations from the behavior of Metamaterials. This model will be dependent on the magnitude of electromagnetic field; this dependence is exactly how the material is governed by the field’s magnitude to demonstrate the required behaviors.

A simulation of the above-mentioned model will be performed through FDTD method. It will be tried to investigate the behavior of this model at issues like phase and group velocities. Negative refraction will also be another characteristic which will be illustrated at this work.
2. Critical Approach

Before taking course on the main strategy of this chapter, I would like to emphasize on a question which we have never faced before! All experiments carried out by scientists during years and centuries leading to Maxwell’s equations have been performed in environments with positive permittivity and permeability. If we have a media whose permittivity and permeability are both negative then how do we know that the same rules and laws are still governing the dynamics of our environments and any possible propagation in it? How do we know that in a media of truly negative permittivity and permeability Coulomb’s law or Maxell’s equations will prevail? Here one might answer that in plasmas under certain circumstances we get negative relative permittivity and still Maxwell’s equations are completely successful in predicting and interpreting any propagation of waves. But as explained in [3] this happens only frequencies above the plasma frequency and changes will happen the relative permittivity. Imagine if even as low frequencies, even in DC, we had negative εr and μr then any same-sign charges would attract each other while opposite charges were repelling each other. This oddness would even grow bigger when any small perturbation on the DC current of a solenoid produced a Faraday voltage agreeing with its source. This would happen due to the following formula:

\[
emf = -\frac{d\Phi}{dt} = -A \cdot \frac{dB}{dt} = +A \cdot |\mu| \frac{dH}{dt}
\]

The perturbation will be increased by the Faraday’s voltage until goes to infinity. Maybe in environment which we have permeability and permittivity’s of intrinsic negative values, Maxwell’s equations would require some modification. So this might have made it clear that if once we are faced with a complete metamaterial any strange and weird behavior should be expected even violation of Maxwell’s equations.

We all have seen the conventional way of considering the point charge to be oscillating at a certain frequency and then choosing the solution which gives us the positive phase velocity to keep the causality of the radiation. Here I would rather take a different approach to the solution of a radiating point charge and conclude the argument with a different result on the direction of the velocity. However finally after presenting a proof on that the phase and group velocities should share the same direction and already proven the group velocity to be positive, we will choose the phase velocity to be positive. It will be shown that it is group velocity that is indicator of information transmission. Direction of group velocity has been considered as indicator of direction of energy propagation which will again be proven wrong.

2.1. Phase Velocity

Solving Maxwell’s equation in time domain generally is time consuming and tedious. Phasor representation of the source and equations help simplify the problem and ease the solution. However as mostly in every other case here also we have been inconspicuously doing a trade-off and therefore we
have lost some other points of values. Once we intend to utilize a phasor solution to the problem, we already accept to have a solution which is valid for one frequency at a time. A phasor vector is defined only by three parameters: magnitude, frequency and phase.

\[ E = E_0 e^{j\omega t + j\beta z} = |E_0| e^{j\omega t} e^{j\varphi + j\beta z} = |E_0| e^{j\omega t} e^{j\phi} \]

2.1.1. Steady state, information and phase velocity direction

Adoption of phasor vectors in approaching the problem of any wave equation means that once the magnitude, frequency and phase of the signal or solution are defined, we have a sinusoidal wave which has no information. This is true only in steady state at which the phasor vectors are used. Anything about our wave is already known by the three foregoing parameters. These parameters completely define our sinusoidal wave and no more information will be added to the information conveyed or received at this point.

This might seem a little inconvenient however once we remember that we are in steady state all perturbations have already been passed and we have entered steady state in which our signals main characteristics are steady, then it would seem wiser to accept this idea. Among the main three characteristics there is only phase which can take two different values at forward and backward propagation. However once it is changed from one to another (forward/backward) it will be constant. After deciding a negative phase velocity in steady state, all three parameters will remain intact which means on information is transmitted. Therefore having a negative phase velocity does not violate causality.

2.2. Group Velocity

Generally the idea of group velocity stems from having different frequency components around a central frequency. Dependence of wave number upon frequency in the medium will make different frequency contents travel at different velocities. The simplest form of this phenomenon could be explained when two tones are transmitted [6]. Imagine if at \( \omega_0 \) the wave number is \( \beta_0 \) for nearby frequencies we might get the following conditions stated in (4).

\[
\begin{align*}
\omega &= \omega_0 + \Delta \omega \rightarrow \beta = \beta_0 + \Delta \beta \\
\omega &= \omega_0 - \Delta \omega \rightarrow \beta = \beta_0 - \Delta \beta
\end{align*}
\]  

(4)

And if we assume our signal to be as:

\[ E(z, t) = E_0 \cos((\omega_0 + \Delta \omega)t - (\beta_0 + \Delta \beta)z) + E_0 \cos((\omega_0 - \Delta \omega)t - (\beta_0 - \Delta \beta)z) \]  

(5)

Then we will have:

\[ E(z, t) = 2E_0 \cos(t \Delta \omega - z\Delta \beta) \cdot \cos(\omega_0 t - \beta_0 z) \]  

(6)

Group velocity is defined as:
\[ t \Delta \omega - z \Delta \beta = \text{const.} \]

\[ \frac{dz}{dt} = \frac{\Delta \omega}{\Delta \beta} = \frac{1}{d\beta/d\omega} \]

The term \( \cos(t \Delta \omega - z \Delta \beta) \) is of course a low frequency signal and in a more general case this could be replaced by a baseband and low frequency signal like \( x(t) \) which is considered as our information source then the propagated signal will look like:

\[ E(z,t) = x(t) \cdot \cos(\omega_0 t - \beta_0 z) \]  

(7)

We can consider \( x(t) \) as a summation of its Fourier series or Integral of its Fourier transform which in turn is basically a summation. If the bandwidth of \( x(t) \) is low enough then all these Fourier components will be transmitted by the same velocity that is:

\[ u_g = \frac{1}{d\beta/d\omega} \]  

(8)

So one may truly interpret this as the velocity of information flow. Performing any analysis on group velocity it should be taken into account that we are dealing with information velocity. This velocity may not be chosen to be negative since it would be an obvious violation of causality. Therefore group velocity will always be positive. Any claim of getting negative group velocity [5] is therefore inaccurate and should be revised. This will be more explained later and any presence of waves travelling backward will be proven to be related to reflections and mainly to the pattern of unit cells and the spatial array they form.

2.3. Poynting Vector and direction of the velocities

If we review the extraction of Poynting’s vector, we will see that this has nothing to do neither with the solution of wave equations nor with the direction of phase or group velocity. Experiences from our everyday life make this difficult to accept. However there is no mathematical reason to associate the direction of Poynting’s vector to the direction of the two velocities. I can make it neither simpler nor more complex than this. Simply there is no reason to consider Poynting vector associated to the direction of phase or group velocities.

Therefore saying that group velocity should be positive does not mean that the flow is energy is also in forward direction. The direction of the flow of energy is related to Poynting’s Vector.

2.4. Phase and Group velocities in the same direction

Here it will be proven that phase and group velocities should be in the same direction. Otherwise at least one of the velocities will have to be derived from a constant function which is obviously impossible and will lead to loss of either information or the propagation itself.
Consider a simple wave equation like (9):

$$\frac{\partial^2 u}{\partial R^2} - \mu \epsilon \frac{\partial^2 u}{\partial t^2} = 0$$  \hspace{1cm} (9)

If you remember the formation of a signal containing information function and also the center frequency which was like:

$$E(x, t) = x(t) \cdot \cos(\omega_0 t - \beta_0 z)$$

Now we can even work with a more generalized function like (10):

$$u(R, t) = f_1(t - R \sqrt{\mu \epsilon}) \cdot f_2(t + R \sqrt{\mu \epsilon})$$  \hspace{1cm} (10)

In which $f_1(t - R \sqrt{\mu \epsilon})$ and $f_2(t + R \sqrt{\mu \epsilon})$ refer to phase and group velocities. If one is group velocity then the other is phase velocity.

Which one is phase or group velocity indicator is of no importance. Here we only want to show that having velocities at opposite direction is impossible. So the only necessary and important point is to have $t - R \sqrt{\mu \epsilon}$ at one of them and $t + R \sqrt{\mu \epsilon}$ at the other.

We start with:

$$\frac{\partial^2 u}{\partial R^2} = \frac{\partial u}{\partial R} \left( -\sqrt{\mu \epsilon} f_1'(t - R \sqrt{\mu \epsilon}) f_2(t + R \sqrt{\mu \epsilon}) + \sqrt{\mu \epsilon} f_1(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) \right)$$

$$= -\sqrt{\mu \epsilon} \left( -\sqrt{\mu \epsilon} f_1''(t - R \sqrt{\mu \epsilon}) f_2(t + R \sqrt{\mu \epsilon}) + \sqrt{\mu \epsilon} f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) \right)$$

$$+ \sqrt{\mu \epsilon} \left( -\sqrt{\mu \epsilon} f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) + \sqrt{\mu \epsilon} f_1(t - R \sqrt{\mu \epsilon}) f_2''(t + R \sqrt{\mu \epsilon}) \right)$$

Therefore:

$$\frac{\partial^2 u}{\partial R^2} = \mu \epsilon f_1''(t - R \sqrt{\mu \epsilon}) f_2(t + R \sqrt{\mu \epsilon}) - \mu \epsilon f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon})$$

$$- \mu \epsilon f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) + \mu \epsilon f_1(t - R \sqrt{\mu \epsilon}) f_2''(t + R \sqrt{\mu \epsilon})$$

And also:

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial u}{\partial t} \left( f_1'(t - R \sqrt{\mu \epsilon}) f_2(t + R \sqrt{\mu \epsilon}) + f_1(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) \right)$$

$$= \left( f_1''(t - R \sqrt{\mu \epsilon}) f_2(t + R \sqrt{\mu \epsilon}) + f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) \right)$$

$$+ \left( f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon}) + f_1(t - R \sqrt{\mu \epsilon}) f_2''(t + R \sqrt{\mu \epsilon}) \right)$$

Then we get:

$$\frac{\partial^2 u}{\partial R^2} - \mu \epsilon \frac{\partial^2 u}{\partial t^2} = -4 \mu \epsilon f_1'(t - R \sqrt{\mu \epsilon}) f_2'(t + R \sqrt{\mu \epsilon})$$
Hence we have:

\[-4\mu\varepsilon f_1'(t - R\sqrt{\mu\varepsilon})f_2'(t + R\sqrt{\mu\varepsilon}) = 0 \quad (11)\]

This means that either \(f_1(t - R\sqrt{\mu\varepsilon})\) or \(f_2(t + R\sqrt{\mu\varepsilon})\) should be constant values which obviously is not possible otherwise the information or the propagation itself would be lost or stopped.

So we have now proved that phase velocity and group velocity cannot have opposite directions. From this proof we know they both should be in either negative or positive direction. Since we have already shown that according to causality in information flow group velocity should be in positive direction therefore phase velocity is also positive.

This is not consistent with the outcome of previous studies in which the two velocities were assumed to be in opposite direction.

Our next step will be reviewing, criticizing and finally rejecting any attempt to build Metamaterials through any number of unit cells which will be shown to be more like and electromagnetic design rather than the design of a medium.

\[\text{2.4.1. Poynting’s Vector in Previous works}\]

Although it is claimed, in previous studies, that they have built some metamaterial – and of course in this work it has been and will be criticized and denied in many aspects – however they have forgotten to mention that the direction of Poynting’s Vector in all their models is exactly the same as that of phase or group velocity. This is observed from electromagnetic design in which everything is derived from Maxwell’s equations and all of the unit cells are placed in some normal material. Therefore any propagation from them will be Right-Handed.

\[\text{2.5. Unit cells: metamaterial or array antennae}\]

In [2] and [4] they have claimed to have built some Metamaterials. However they have only organized a set of unit cells put together aimed at generating certain behaviors of Metamaterials. In most cases the behavior tried to be demonstrated was a negative velocity of the wave like [5]. As stated previously the negative velocity does not exist in Metamaterials and this argument is enough to deny validity of previous works. However this is just one face of proving them wrong. Another face is to challenge totality and the idea behind their strategy. Once might find it easier to start with two dimensional structures built for realizing a plane with negative refraction index.

Negative refraction is realized by building a two-dimensional plane which consists of unit cells similar to antenna arrays. As discussed in [2] the required condition of this structure is that distance between the elements should be so that in the operating frequency the adjacent unit cells would get the signal with a phase difference greater than 180 degree which will cause the array to have a radiation pattern which in this configuration allows to say that the wave is refracted by a negative index. The simple idea I use in
this work to challenge the validity of this method is to bring up the fact that the wave travels a half wavelength between the elements. This means that our wave is propagating in free space also the whole wave is steered and its pattern is shaped an array of antennae. In this case we can study the propagation in a way similar to when the propagation is taking place at free space. Imagine inside a dielectric we have molecules and ions and still we can treat like free space with a compensating factor of relative permittivity. This is because the size of molecules is much smaller than the wavelength. Now if any perturbation of a size even comparable to wavelength is applied we still could consider the perturbations in a free space in which only a permittivity factor has changed.

At some planar array of unit cells they are getting a negative refraction; this is only at certain frequencies and second this is not because of metamaterial behavior. It is more like designing an antenna which has a defined radiation pattern.

2.5.1. Place of relative permittivity and permeability in previous works

In order to illustrate the point it is useful to start with a simple example about friction. Imagine you are pulling a big box on a gravel path. Here you can use the friction formula to model the forces applied on the box.

\[ F_{friction} = \mu_k mg \]  

(12)

In which \( \mu_k \) is the Kinetic Friction coefficient of the gravel path. Now if you change the gravel path and perform the experiment on some other part of the same path at which the density of the gravel or its type is a bit different we can still use the same formula but with some different coefficient \( \mu_k' \). If one tends to normalize the coefficient to the previous value then here we can say the relative Kinetic Friction coefficient is \( \mu_{kF} = \frac{\mu_k'}{\mu_k} \).

Now imagine that we are changing the size of the box and decreasing it dramatically until gets very small and half size as one of the small stones in the gravel path.
Here the box will move up and down between the small stones. Simply it has nothing to be compared to the previous stage at which the box was so big that saw gravels as just a path. There we could use the friction formula and even by slightly changing the type of gravel a relative coefficient could be applied. However here if we want to analyze the motion of the box we could not use the same friction formula and relative coefficient because simply now the scale has been changed so much that perhaps the motion should be considered and analyzed on each small stone separately. So here the idea of using relative friction coefficient – relative to the gravel path – collapses.

Now as mentioned earlier at some planar array of unit cells they are reporting obtaining a negative refraction; this is only at certain frequencies and second this is not because of metamaterial behavior. It is more like designing an antenna which has a defined radiation pattern. Also more importantly claim of obtaining negative permittivity and permeability will be challenged using the foregoing argument on (relative) Friction coefficient.

Technically, we cannot think of any collection of unit cells causing changes in permittivity and permeability as we see in dielectrics. Behavior of molecules and ions, regarding their much smaller size, contributes to clear change in permittivity of the media yielding new value of permittivity. Even if some discontinuity as the scale of wavelength is put into our media this new value will prevail and the same Maxell’s equations will be written to solve the problem; however we will apply some boundary conditions at the boundaries of the discontinuity which should also be done in free space. But if this discontinuity is applied to the spatial matrix of unit cells we will lose our configuration and changes in permittivity and permeability will no longer be valid; we cannot apply Maxwell’s equations in a perturbed spatial matrix of unit cells just by changing permittivity or permeability and considering the discontinuity. All we can do is to re-calculate the response or pattern of the new configuration.

Actually the idea of having and using relative permittivity or relative permeability is to benefit from using simple Maxell’s equations in free space except from applying the relative factors. If in every case of having discontinuity or change in path of propagation we had to go through the cumbersome calculation of vibrating molecules and current loops (orbiting electrons) together with the new discontinuity, we would not find it useful to have the idea of relative permittivity and permeability.
factors. This is the case happening with all attempts carried out so far to develop LH materials. Any change in the system would force us to re-calculate or re-simulate the whole structure. These kinds of negative phase velocity waves exist inside any microwave N-port networks.

The wavelength in these models is like the size of the box in the friction example. When it is comparable to the size of unit cells and the distance between them we no longer can use relative permittivity and permeability as before.

I would like to conclude this part by marking one last argument on the subject. The idea comes from imagining a point charge varying by time which will be propagating in every direction including $+z$ as well as $-z$.

2.5.2. Metamaterial or misunderstanding of coordinates
Final point on this issue could be considering the origin of our coordinate system to be at each unit cell at each time. Propagation from the cell will be in all directions and in that coordinate system all directions are considered positive. However some of directions happen to lie on the opposite to the direction of our main axis which shows the positive propagation in the whole system. So the waves propagating in positive direction in its own coordinate system will be added together by superposition theory and will contribute to a wave that appears to be travelling in negative direction in another coordinate system. Can we really call this a double negative material?

2.6. Violation of energy conservation
This will be the last and perhaps the most important argument on challenging and denying the previous works on Metamaterials. Here we prove that if there is wave traveling in medium 1 which is normal material and then hitting on a second material which is Left-handed then at the intersection plane of the two media we will see generation of energy. Refer to the second paragraph in “1. Introduction” to see that the direction Poynting’s vector in metamaterial is given by Left Hand Rule.

Imagine the structure illustrated at Figure 3.

![Figure 3. Boundary Surface between normal medium and metamaterial](image-url)
This is a simple plane wave propagation. Subscription “i” represents the incident wave and subscription “t” shows the transmitted wave and finally subscription “r” is for reflected wave.

Field components are tangential to the intersection plane. Therefore working with tangential boundary conditions will be sufficient.

\[ E_{1, \text{tangential}} = E_{2, \text{tangential}} \] (13)
\[ a_{n2} \times (H_1 - H_2) = J_s \] (14)

Because we assume the both environments to be lossless then there will be no current on the plane. And we will get:

\[ \begin{cases} E_{1, \text{tangential}} = E_{2, \text{tangential}} \\ H_{1, \text{tangential}} = H_{2, \text{tangential}} \end{cases} \] (15)

Or

\[ \begin{cases} E_i + E_r = E_t \\ H_i + H_r = H_t \end{cases} \] (16)

Which means:

\[ \begin{cases} E_i + E_r = E_t \\ H_i - H_r = -H_t \end{cases} \]

We get:

\[ \begin{cases} E_i + E_r = E_t \\ \frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = -\frac{E_t}{\eta_2} \end{cases} \] (17)

After solving this set of equations we will get:

\[ \begin{cases} E_r = \frac{\eta_1 + \eta_2}{\eta_2 - \eta_1} E_i \\ E_t = \frac{2\eta_2}{\eta_2 - \eta_1} E_i \end{cases} \] (18)

Here we obviously see that if \( \eta_1 \) and \( \eta_2 \) are selected very close to each other, the reflected and transmitted waves will have infinite values. This encourages us to calculate the power coming out of an imaginary closed boundary surface.

In Figure. 4 the structure is re-arranged to illustrate the new intention.
We have
\[ \oint \mathbf{P} \cdot ds = G \]  \hspace{1cm} (20)

In which:

\[ G = \text{total power leaving the surface contour} \]

So:
\[ \oint \mathbf{P} \cdot ds = \oint \mathbf{E} \times \mathbf{H} \cdot ds \]  \hspace{1cm} (21)

Therefore:
\[ \frac{\oint \mathbf{E} \times \mathbf{H} \cdot ds}{S} = -\frac{E_i E_i}{\eta_1} + \frac{E_T E_T}{\eta_1} - \frac{E_T E_T}{\eta_2} \]  \hspace{1cm} (22)

Which gives us:
\[
\frac{\oint E \times H \cdot ds}{S} = -\frac{E_i^2}{\eta_1} + \frac{(\eta_1^2 + \eta_2^2)E_i^2}{(\eta_2 - \eta_1)^2 \eta_1} - \frac{4\eta_2 E_i^2}{(\eta_2 - \eta_1)^2 \eta_1} = E_i^2 \cdot \frac{-(\eta_2 - \eta_1)^2 + (\eta_2 + \eta_1)^2 \eta_1 - 4\eta_2 \eta_1}{(\eta_2 - \eta_1)^2 \eta_1}
\]

\[
= E_i^2 \cdot \frac{\eta_1^2 + 2\eta_2 \eta_1 + \eta_1 \eta_2^2 - \eta_1^2 - \eta_2^2 - 2\eta_2 \eta_1}{(\eta_2 - \eta_1)^2 \eta_1}
\]

\[
= E_i^2 \cdot \frac{\eta_1 (\eta_1^2 + 2\eta_2 \eta_1 + \eta_2^2) - (\eta_1^2 + \eta_2^2 + 2\eta_2 \eta_1)}{(\eta_2 - \eta_1)^2 \eta_1}
\]

\[
= E_i^2 \cdot \frac{\eta_1 (\eta_1^2 + \eta_2^2) - (\eta_1 + \eta_2)^2}{(\eta_2 - \eta_1)^2 \eta_1} = E_i^2 \cdot \frac{(\eta_1 + \eta_2)^2(\eta_1 - 1)}{(\eta_2 - \eta_1)^2 \eta_1}
\]
That is:

$$G = \iint E \times H \cdot ds = S \cdot E_i^2 \cdot \frac{(\eta_1 + \eta_2)^2(\eta_1 - 1)}{(\eta_2 - \eta_1)^2 \eta_1}$$  \(23\)

This shows that for \(\eta_1 > 1\) which is almost always the case there will be a generation of energy at the boundary surface.

Generation of energy at the boundary surface is not something that could be achieved by using any number of passive elements. Therefore the idea of building Metamaterials by unit cells should be completely abandoned.

As last a comment on the previous work I would like to point the frequency band of the previous works. However this part is not necessary because we have already proven that everything with previous works is incorrect. A main shortcoming of the spatial arrays lies in their inability to provide a wave in which Poynting’s vector and phase velocity possess directions opposite to each other. I will try to show in the final version of my thesis report that even exhibiting a negative group velocity is impossible through the periodic structures. This proof includes a theoretical approach as well as simulation results. Any system consisting of causal components simply cannot exhibit non-causal behavior. Dr. Woodley and Dr. Mojahedi in [5] claim to build material with negative group velocity. As far as I understand from their paper they must have used different observing points for phase and group velocity and therefore have ended up in a result I prefer to challenge. Again having waves in a special direction radiated from an array of antennae with related pattern and associating it with negative velocity is like considering a reflected wave in a simple transmission line as meta-material.

Therefore if one is after creating on a medium which demonstrates all characteristics of Metamaterials at once, they should consider working on material aspects or characteristics of a substance. What is presented next satisfies all conditions to be called a medium of propagation and at the same time provides negative permittivity and permeability.
3. Field magnitude dependent model

Simply imagine a material possessing relative permittivity and relative permeability as illustrated in the following figure. Both relative parameters are positive. Such a material could be built by Solid State methods and using saturation and reversible break-down methods. Suppose we already bias the material with \( E_0 \) and \( H_0 \). And then we let the wave propagate. Of course the dimension of the biasing field and the propagating field should be consistent.

I anticipate that building such a material wouldn’t be too difficult, especially benefiting from today’s Nanotechnology. Imagine in a dielectric when an electric field is applied some dipoles turn until their Electric Dipole Moment Vector is parallel to the electric field. By increasing the intensity of the field more dipoles will follow the same procedure. However we should know that the number of dipoles is limited and of course probably not all dipoles can be re-directed. Therefore after a point by increasing the field less dipoles will follow with the direction of the field. This will make the slope of \( \varepsilon \) graph decrease. Imagine if by increasing the field intensity even more the some internal structure inside the substance collapses. If the substance is well make to agree with our intentions at this point we should lose some of already re-directed dipoles. Therefore increasing in the intensity of the filed not only will not increase the Polarization vector but will decrease it. This will make the slope of the \( \varepsilon \) graph be negative which means a negative relative permittivity for small signals around the point \( E = E_0 \). Of course the same argument stands for relative permeability and making it negative.

If the magnitude of the electromagnetic wave is very small compared to \( E_0 \) and \( H_0 \) then according to the superposition theory we will have:

\[
E_{total} = E_0 + e(t)
\]

\( (24) \)
And

\[ B_{total} = B_0 + b(t) \]  \hspace{1cm} (25)

And also

\[ \mu = \frac{\partial B_{total}}{\partial H_{total}} \]  \hspace{1cm} (26)

which will be equal to the slope of the related curve at the bias point that is a negative value.

And

\[ \epsilon = \frac{\partial E_{total}}{\partial P_{total}} \]  \hspace{1cm} (27)

which will be equal to the slope of the related curve at the bias point that is a negative value.

Simply by realizing this material we will have a double negative material which unlike previously developed periodic structures will have a real metamaterial-like behavior and will we expect it to have positive group velocity and negative phase velocity. It is independent of frequency and works at very low frequencies up to the highest possible frequencies which its microscopic structure allows. These behaviors will be investigated by FDTD.
4. Results and Discussions

The simplest case will be analyzing the wave propagation in an ideally double negative material. Here the source’s amplitude will be the varying by time in order to support an information source so that we could study the group velocity in addition to phase velocity. A simple TEM plane wave is considered. Of course our method is Finite Difference Time Domain – FDTD. For my simulation codes refer to the appendix.

![Diagram of wave propagation](image)

Figure 6. Propagation of TEM wave in an entirely Metamaterial medium

Figure 6 presents the result of a simulation from a single tone in an ideally metamaterial medium. The field values are shown at time points $T_0$, $T = T_0 + \Delta T$ and $T = T_0 + 2\Delta T$. As shown in the figure it is understood that the wave is propagating away from the source. Therefore we get a positive phase velocity.
In the same figure if at any point you take the vectors of $\mathbf{E}$ and $\mathbf{H}$, then you will find that the direction of Poynting’s vector as $\mathbf{E} \times \mathbf{H}$ at any point will be towards the source. This means that although the phase velocity is positive but energy is transmitting backwards. This completely fits our conclusion earlier.

Now the next step is to check the direction of the group velocity. Here according to our understanding from group velocity, mentioned earlier, the source is considered as

$$\frac{t^2}{100} \sin(\omega t)$$

The term $\frac{t^2}{100}$ is to allow us to understand the direction of the group velocity. The newer information is now associated to the larger field values. Figure 7 shows the result:

![Normalized Electric Field](image1.png)
![Normalized Magnetic Field Intensity](image2.png)

**Figure 7.** Wave propagation in metamaterial. Magnitude of the source is increasing by time

At the Figure 7 one can see that the point further from the source get have the lower values which means that older information. Later they will get bigger values that is new information. This means that the direction of the group velocity is also away from the source that is positive. This is in complete consistence with our previous discussion. However again it shows that previous works on metamaterial were wrong in assuming phase or group velocity to be negative.

But again at the last mentioned figure you will find that the direction of Poynting’s vector as $\mathbf{E} \times \mathbf{H}$ at any point will be directed towards the source. Like the previous example again this means that although
the group velocity is positive and away from the source but energy is transmitting towards the source. This proves the validity of our earlier discussions.

At both of previous examples the source was in the middle and the entire environment was metamaterial. Here the propagation of wave is in medium 1 which is normal material then it will enter a double negative material.

Figure 8 shows when the wave is just about to hit the boundary surface between a normal and metamaterial. See that the fields are normalized to 1. Figure 9 shows a bit later when get some reflection and transmission through the boundary surface as well. Here you see that the amplitudes of the waves transmitted and reflected are much bigger than the main incident wave which is 1. This shows that in the boundary surface there has been a generation of energy which was predicted before. This is also another strong consistence with the arguments of this work which are totally against all the previous studies on Metamaterials.

Figure 8. Wave right before entering metamaterial
Now simulating the Field Magnitude Dependent Model will reveal its capacity to be a good representative of Metamaterials.

At Figure 10 we see that phase velocity is positive while Poynting’s Vector shows the direction energy movement is backwards. This is matches our expectations from the model. The normalized (to the source) bias points as shown in Figure 5 are as follows:

\[
\begin{align*}
\{ & \varepsilon_0 E = 50; D = 50 \\
& \mu_0 H = 50; B = 50 \\
\end{align*}
\]

Again like what we did in Figure 10, here in Figure 11 it is shown that group velocity is positive while Poynting’s Vector show that energy flows backward. Again the model passes the test. For the group velocity here the same source is used as earlier for the theoretical investigation;

\[
\frac{t^2}{100} \sin(\omega t)
\]

The term \( \frac{t^2}{100} \) again says that newer information is now associated to the larger field values.
Figure 10, Propagation of TEM in a medium entirely made of the proposed model.

Figure 11, Group Velocity in the proposed model. Magnitude of the source is increasing by time.
Last two tests on the model were performed in an entirely metamaterial environment. Now the final test will be a wave first propagating in a normal medium then hitting on a metamaterial of the model proposed here. The reflected and transmitted waves are greater than the incident wave, as predicted earlier. Notice that the magnitude of the source is normalized to one. However the transmitted and reflected waves have magnitudes up to ten times. So the Field Magnitude Dependent Model prevails over the vital tests to be called a metamaterial. This is shown in Figure 12

Figure 12 Wave reflected from also entered into the biased metamaterial model, Magnitude of the source at No. 10 is 1.
5. Conclusions

At this work the validity of previous works on building Metamaterials using any collection of unit cells has been denied in different levels. First it has been shown that unit cell structures cannot be called a medium because the size of the cells and the difference between them are considerable portion of wavelength.

Before, phase and group velocities were assumed to be at opposite direction to each other. At this work we proved this idea wrong and showed that they are the same positive direction. However Poynting’s vector is of course towards the source which means flow of energy will be backwards.

Another important step in challenging and denying previous works was proving that in a boundary surface between and normal medium and a metamaterial there would be a generation of energy which is ultimately inconsistent with any passive design. Therefore incorrectness of previous designs was revealed.

A frequency independent model which is dependent on the magnitude of the field was presented. Theoretically it was shown to be working and producing negative permittivity and permeability. Afterward the result of a simulation on the model was an agreement with the idea. Phase and group velocities are positive. Poynting’s vector is obtained by Left-Hand rule.
6. References


