CONDITIONS AND CONSTRAINTS FOR TRANSFERRING JAPANESE STRUCTURED PROBLEM SOLVING TO SWEDISH MATHEMATICS CLASSROOMS

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CONDITIONS ET CONTRAINTES POUR LE TRANSFERT DE LA RÉSOLUTION DE PROBLÈMES STRUCTURÉE AU JAPON À LA CLASSE SUÉDOISE DE MATHEMÁTICAS

Résumé – Cette étude de cas examine dans quelle mesure une théorie et une pratique spécifiques de l'enseignement des mathématiques - la résolution de problèmes structurée japonaise telle que formulée par K. Souma - peuvent être transférées et appliquées dans un nouveau contexte (Suède). L'analyse est basée sur des outils de la théorie anthropologique du didactique. Il s'avère que l'enseignant suédois peut gérer les techniques didactiques proposées par Souma quand celles-ci sont soutenues par une technologie didactique qui est familière pour la communauté des enseignants suédois. Pour certaines techniques didactiques de l'approche de résolution de problèmes structurée, ceci n'est pas le cas ; ces techniques étaient en fait difficiles à utiliser pour les enseignants suédois. Concrètement, il s'agit de techniques liées au bansho (organisation du tableau); laisser les étudiants formuler le kadai (la tâche dérivée) d'une leçon; et le kikan-shido (suivi du travail des élèves pour planifier une discussion ultérieure). De plus, l'article fournit une analyse écologique des conditions et contraintes sous-jacentes qui ont provoqué ce phénomène - une divergence concernant les praxéologies didactiques des enseignants. Au Japon, les enseignants se concentrent sur le développement personnel des élèves dans les processus d'apprentissage réflexif et collectif, tandis que l’enseignant suédois se concentre davantage sur l’acquisition des connaissances par les élèves.

Mots clés: transfert de pratiques didactiques, théorie anthropologique du didactique, résolution structurée de problèmes, modèle de référence praxéologique, infrastructure paradidactique

CONDICIONES Y RESTRICCIONES PARA LA TRANSFERENCIA DE PROBLEMAS ESTRUCTURADOS JAPONESES PARA LA RESOLUCIÓN A AULA DE MATEMÁTICAS SUECO

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Resumen – Este estudio de caso(6,4),(993,994) investiga en qué medida una teoría y práctica específicas de la enseñanza de las matemáticas (la resolución de problemas estructurada japonesa, tal como lo formula K. Souma) se puede transferir y aplicar en un nuevo contexto (Suecia). El análisis se basa en herramientas de la teoría antropológica de la didáctica. Resulta que el profesor sueco puede gestionar esas técnicas didácticas, que están respaldadas por tecnología didáctica compartida dentro de la comunidad de profesores suecos. Existen técnicas didácticas comunes dentro del enfoque de resolución de problemas estructurados japoneses para los cuales este no es el caso, y que de hecho eran difíciles de manejar para el profesor sueco, a saber: técnicas relacionadas con bansho (organización de pizarra); dejar que los estudiantes formulen un kadai (tarea derivada) de la lección; y kikan-shido (monitoreo del trabajo de los estudiantes para planificar una discusión posterior de toda la clase). Además, el documento proporciona un análisis ecológico sobre las condiciones y limitaciones subyacentes que provocaron este fenómeno, una discrepancia con respecto a las praxeologías didácticas de los docentes. En Japón, los profesores se centran en el desarrollo personal de los estudiantes en los procesos de aprendizaje reflexivo y colectivo, mientras que el profesor sueco se centra más en el logro de los conocimientos de cada estudiante.

Palabras-claves: Transferencia de práctica pedagógica, la teoría antropológica de lo didáctico, resolución estructurada de problemas, modelo de referencia praxeológico, infraestructura paradidáctica

Abstract – This case study investigates to what extent a specific theory and a practice of mathematics teaching — the Japanese structured problem solving, as formulated by K. Souma can be transferred and applied in a new context (Sweden). The analysis is based on tools from the anthropological theory of the didactic. It turns out that the Swedish teacher can manage the didactic techniques, which are supported by a didactic theory shared within the community of Swedish teachers. However, there are common didactic techniques within the Japanese structured problem solving approach for which this is not the case, and which were indeed difficult to manage for the Swedish teacher. Among these were techniques related to bansho (blackboard organisation); letting the students formulate a kadai (derived task) of the lesson; and kikan-shido (monitoring students’ work to plan a subsequent whole-class discussion). Further, the paper provides an ecological analysis on the underlying conditions and constraints that brought about this phenomenon – a discrepancy concerning the didactic praxeologies of teachers. In Japan, teachers focus on the students’ personal development in the processes of the reflective and collective way of learning, while the Swedish teacher focuses more on individual student’s achievement of the knowledge.

Key words: transferring of teaching practice, anthropological theory of the didactic, structured problem solving, reference epistemological model, paradidactic infrastructure
1. INTRODUCTION

Since the 1990s, educational research has explored teaching approaches from East Asian countries such as Singapore, China, Taiwan and Japan, motivated in part by the performances of these countries in international surveys on students’ knowledge in mathematics, such as Trends in International Mathematics and Science Study (TIMSS) (Stigler & Hiebert, 1999). As it is suggested in Stigler and Hiebert's « The Teaching Gap » (ibid.), one may desire to try to import these teaching approaches that led to students’ high scores in TIMSS to less successful contexts. However, teaching is an entrenched practice, since it is based on a cultural script (ibid.) of each country. Teachers’ teaching practice are based on this script, which are founded on their own experiences through their time as students. This raises several issues: Is it possible for teachers to reproduce the methods, and will their students achieve as well as students in the country where the methods came from? What role does the individual teachers’ knowledge and routines play for the successful implementation? Then what kind of factors lie beyond the individual teachers’ control? In fact, we do not know much about transfer of teaching approaches from one country to another one, especially within longitudinal intervention projects that go beyond shorter episodes.

This paper reports on a case study concerning one Swedish teacher, who applied a variation of the Japanese problem solving oriented teaching approach provided by Kazuhiko Souma (the detail of this approach will be presented in the later section) as a design tool for her mathematics lessons throughout a full school year. The aim of the study is investigating exactly what parts of the practices of the Japanese approach were (or were not) successfully implemented, and to identify the institutional conditions and constraints that favoured and hindered certain techniques to be transferred. Note that the intention of the study is not to place the Japanese approach as « better » or « more efficient », and trying to « adopt » it to the Swedish classrooms but to study what kind of phenomena will appear regarding the transferring of a Japanese problem solving approach to the Swedish context.

2. RELATED STUDIES

Several international intervention studies have investigated « reconstructions » of teaching approaches of foreign origin (e.g. Ding,
Some of these studies focus on the Japanese mathematics teachers’ practice (e.g. Stigler & Hiebert, 2016), and some emphasise on the teachers shared theoretical assumptions related to the teaching approaches (Jacobs & Morita, 2002). The power of Japanese \textit{structured problem solving}, (hereafter, SPS) which supports students’ active participation to the lessons and their autonomous learning in mathematics is forcefully reported in « \textit{The Teaching Gap} » (Stigler & Hiebert, 1999) and is very often related to the practice of \textit{lesson study} (e.g., Fernandez & Yoshida, 2004; Miyakawa & Winslow, 2013; Takahashi, Lewis & Perry, 2013). In fact, the SPS (discussed in Section 4.1) plays a prominent role in the practice of Japanese lesson study (Fujii, 2014).

Mathematics teaching practices are shaped by a number of factors, which often are originated from conditions and constraints outside of the school. One of them is didactic theory (i.e. explicit and shared knowledge and principles related to the teaching of mathematics, cf. Bosch & Gascón, 2014), which differs among societies and among different communities of teachers. This could cause obstacles for applying a specific teaching approach based on a specific didactic theory, to a new context.

Despite the widespread international interests mentioned above, only few studies have investigated the transferability of Japanese teaching approaches to contexts outside Japan. In particular, two case studies especially focused on the implementation of the SPS. Groves, Doig, Vale and Widjaja (2016) investigated the possibilities and limitations for the implementation of the Japanese lesson study by applying the SPS in a small-scaled research project in Australia. They found that the Australian teachers valued the key-features of the SPS such as making detailed lesson plans with careful consideration of the mathematical goals of the lessons, the use of the initial problem which called on students’ previous knowledge, and which can lead student to multiple solutions. As a main obstacle to transfer, the authors mention the difficulty in finding suitable tasks to match the Australian curriculum, and teachers’ theoretical beliefs. Fujii (2014), in a case study involving two African countries, points out teachers’ misunderstandings of the SPS approach. He states that the African teachers’ misconception about the SPS as « just solving a task » is the converse of the Japanese teachers’ brief of SPS as « educational and mathematical values are taught through structured problem solving » (p. 79). In both cases, it seems that the differences of the teachers’ didactic theories has caused the difficulty of the transferring.
3. THEORETICAL FRAMEWORK AND RESEARCH QUESTIONS

This paper reports on research carried out within the anthropological theory of the didactic (ATD). Starting with the didactic transposition theory (Chevallard, 1985), ATD has been developed and offered several different tools to study various problems regarding the dissemination of knowledge in different forms of institutions. In this study, I will investigate didactic phenomena within various didactic systems (Bosch & Gascón, 2014), observed in Japan and Sweden. A didactic system is formed by learners (e.g. students), actors who help the learners (e.g. teachers) and so called didactic stake – a thing to be learned by the learners.

3.1 The theory of didactic transposition

Conflicts of such reported in Fujii’s study (2014) presented above naturally arise when specific didactic knowledge is transferred to different context. The didactic transposition theory generalises how certain knowledge created within one institution (e.g. scholarly knowledge created in academic institutions) is transposed through certain didactic work of certain groups of people in different educational systems (e.g. politicians and researchers making national curricula, school teachers planning their lessons from it) and takes distinct forms. By applying the notion of noosphere, Chevallard (1985) lets us recognise the social and educational long-standing influence that has formed certain knowledge as its current form. In this study, the notion of didactic transposition contributes to understand in what way the character of the Japanese teachers’ professional knowledge on the SPS (which is developed by the associations of Japanese researchers and teachers under the influence of the noosphere) is changed when it transposed to the Swedish teaching profession.

3.2 The theory of praxeology

The notion of praxeology contributes to model all kind of human activities, such as learning of mathematics. Since learning of mathematics rarely takes place without activities (such as listening an explanation by a teacher, solving a problem), a praxeology always consists of a practical unit (know-how) and a knowledge unit (know-why). The practical unit is called praxis, and the knowledge unit is called logos. Further, we can zoom in on each unit. The praxis is constituted by types of tasks $T$ and technique $\tau$, which gives a method
to solve the task. The logos is constituted by technology $\theta$, which is a discourse of the techniques, and a theory $\Theta$, which in turn justifies the technologies. The praxeologies that concern the realisation of mathematical tasks are called mathematical organisations. The teacher’s practices that aim to realise the mathematical organisations of the lessons are called didactic organisations. Student’s mathematical organisations and the teacher’s didactic organisations are affected by each other. The use of mathematical and didactic organisations is useful to investigate the structure of Souma’s design of the lessons, since the notions can technically describe how the didactic stake and the construction of Souma’s didactic organisation are interacted.

To identify which elements of Souma’s approach that the Swedish teacher realised and which elements were not realised, we need a tool to model the practice: a reference epistemological model (Bosch & Gascón, 2014). The reference model in Section 4 provides a theoretical model of the didactic techniques described in Souma’s books, and is then used to analyse the Swedish teacher’s didactic practice. In this reference model, the notion of didactic moments is employed to specify the didactic types of tasks in Souma’s praxeologies. ATD recognises six discernible moments in study processes, and these six moments models the process of creation or recreation of mathematical organisations (Chevallard & Bosch, 2020). The six moments are: 1 (FE) The moment of first encounter (or re-encounter) with a certain type of tasks $T$; 2 (EX) The moment of exploration of $T$ and elaborating techniques $\tau$ suitable for the $T$; 3 (TT) The moment of building the technological–theoretical block $[\theta/\Theta]$ in which alternative techniques are assessed and a technological discourse is taking place; 4 (TW) the moment of technical work to use and improve the praxeology produced, $[T/\tau/\theta/\Theta]$, particularly the techniques; 5 (IN) The moment of institutionalisation, where one is trying to identify and discern the elaborated praxeology; 6 (EV) The moment of evaluation aims to examine the value of the constructed praxeology. The didactic types of tasks are related to these six moments in the reference model.

3.3 Ecological analysis and the levels of didactic co-determinacy

The complexity of the didactic praxeology depends upon conditions of different kinds and origins. In ATD, such a set of conditions necessary for its existence of a specific praxeology in a given institutional setting, and constraints that impede its development is called the ecology of the given praxeology. I implement such ecological analysis
through applying the scale of levels of didactic co-determinacy (Chevallard, 2019). It models a hierarchy of origins of conditions and constraints from generic levels (humanity, civilization, society), and levels related to education (school and pedagogy), then to didactic system/condition, which are divided into five sub-levels (discipline, domain, sector, theme and subject). The co-determinacy model provides a clear illustration of how the generic levels influence the praxeologies at the lower levels in Japan and Sweden in different way. Identifying important conditions and constrains related to the higher levels in each country allows us to understand in what way the Japanese SPS approach can «live» in the Swedish didactic systems.

### 3.4 Paradidactic infrastructure
In ATD, the notion of infrastructure means the conditions indispensable to a certain activity. For instance, a textbook is one of such components if the given didactic system depends on it, and in that case, the textbook is an element of the didactic infrastructure. On the other hand, the didactic system is designed outside the classroom as teachers make lesson plans, search for appropriate tasks and revise the plans from lessons done in the past. The notion of paradidactic infrastructure (Winsløw, 2012) distinguishes teachers’ work within the didactic system and outside of the didactic system (paradidactic system). It describes «everything which conditions and constraints the paradidactic system in its different phases and in the interplay between the phases» (ibid., p. 293). A typical example in Japan is kyozaikenkyu (Miyakawa & Winsløw, 2019), which is the study and production of teaching materials. Other examples include open lessons and practice research (ibid.), and common educational conferences for math teachers and researchers. In this paper, I will examine the paradidactic infrastructure in each country to investigate the underlying base necessary to carry out the praxeologies of the SPS based lessons.

### 3.5 Research questions
We can now formulate the research questions of this paper:

1. What are the central didactic techniques in Souma’s version of the structured problem solving (SPS) approach, and what conditions are crucial for its realisation in Japan?
2. To what extent did the Swedish teacher in this case study realises Souma’s central didactic techniques described in RQ1? How can this be explained by a wider view based on
the levels of didactic co-determinacy, and on differences in
the paradidactic infrastructure?
To answer these questions, I firstly present a reference model that
serves to analyse the outcomes of the Swedish teacher’s attempt.
Secondary, I exhibit the conditions that has facilitated the
development and dissemination of the SPS in Japan. Then I compare it
with the conditions that has formed the Swedish teachers’ didactic
logos.

4. CONTEXT AND METHODOLOGY

4.1 The Swedish class and the teacher
From September 2010 to May 2011, a longitudinal study involving
observations of 40 lessons in one class in grade 7 with 15 students,
and from February to April 2011, seven lessons in one class in grade 8
with 17 students were implemented. The name of the teacher, who
conducted all lessons, was Eva (pseudonym). The school, operated by
a municipality, is partly adhering to the Montessori pedagogy. Five
months before the project began, I contacted a local mathematics
developer, who served to help mathematics teachers in the region. I
asked him distributing my request, where I sought a mathematics
teacher in lower secondary school, who would be willing to
collaborate with me on a weekly basis for a period of one year. Eva,
who signed up to be involved in the project was a qualified
mathematics teacher for lower secondary level and worked at the time
for three years, and had been serving at that school for two years.
According to the Montessori pedagogy, she lets her students freely
choose their working style (using manipulatives, working in-group, or
individually with the similar problems). She told me that the reason
she decided to join my project was that she was seeking a new method
to stimulate her students’ motivation for the learning of mathematics.
Besides observing Eva’s lessons twice a week, we usually had
meetings to plan the lessons for the coming week once every week.
All lessons were conducted in Swedish, and the analysis was done on
the transcripts translated from Swedish into English by the author.

4.2 Method of analysis

Analysis of Souma’s didactic praxeologies
To answer the first part of RQ1: What are the central didactic
techniques in Souma’s version of the structured problem solving
approach? I first studied the overall structure and flow of the approach as presented in two of Souma’s books: « Improvement of the lessons of mathematics with guessing » (Souma, 1995) and « The problem solving approach—the subject of mathematics » (Souma, 1997), since Souma’s pedagogical idea is well described in these two books. The analysis of the structure of Souma’s didactic praxeologies is used to construct the reference model to answer the first part of RQ1. This serves even as the reference for answering the first part of RQ2: To what extent did the Swedish teacher realise Souma’s central didactic techniques described in RQ1? To define the types of didactic tasks in the reference model, I implemented the notion of the didactic moments. To identify and illustrate the key elements of the model, I studied Souma’s books (Souma, 1995; 1997; 2000; 2017), and lesson plans, which were available at that time (Kunimune & Souma, 2009a; 2009b) and categorised the didactic techniques that corresponded to the six moments. The criteria for relating this literature to the techniques is detailed in Appendix 1. To compare the outcome of the Swedish lessons to the original lesson plan, the lesson plan « introduction to finding the general solution » (ibid., 2009a) for grade 7, which belongs to the domain of algebra for students to learn the use of variables is analysed.

The second half of RQ1: what conditions are crucial for its realisation of the central didactic techniques in Souma’s approach in Japan is strongly related to the second half of RQ2, the institutional conditions and constraints that explain the degree of the transferability of Souma’s version of the SPS into Sweden. To identify the conditions that advocate the existence of the praxis part of Souma’s approach, and to compare them to the Swedish counterpart, I studied the historical and cultural backgrounds that influenced the ideologies of the mathematics education in both countries.

Eva’s use of didactic techniques in 16 lessons
The main strategy to answer the first part of RQ2 was applying the reference model to study Eva’s lessons. I made a detailed analysis on her implementation of the lesson « introduction to finding the general solution », which was conducted in both grade 7 and 8. I observed the recorded videos and checked which of the techniques described in Section 5.2 (the summary of the didactic techniques from the Japanese lesson plan) she applied, did not applied, or attempted to applied. Then I studied her other 15 lessons in a similar manner. Based on the reference model in Table 1, I had categorised Eva’s use of the didactic techniques, observed in 16 lessons at grade 7 and 8 (see below). The
reason I chose those 16 lesson was that the lessons were implemented between January and March 2011 (including the lesson presented in sec. 5.3), and she had then tried using Souma’s approach for almost 6 months in all her lessons. Therefore, both Eva and the students become accustomed with this new approach, and a new didactic contract was established at this point. Several problems applied there (lessons 4, 6, 9, 10, 11, 13, 14, 15, 16) were adapted from lesson plans by (Kunimune & Souma, 2009a; 2009b) and from a problem collection (Souma, 2000). Other initial problems, mainly from the Geometry domain, were created by the author and Eva in a collaborative process. (lessons 1, 2, 3, 5, 7, 8, and 12). Since the Japanese curriculum in Geometry particularly for the lower secondary level has a strong coherent epistemological structure (e.g. firstly one learns about the parallel lines, then the properties of angles, and finally about the concept of the congruence of figures), we could not adopt any « part » of Souma’s lesson plans of Geometry. They did not fit to the Swedish curriculum, which does not emphasise such Euclidean structure. The topics of the lessons are shown below. In these lessons, Eva applied the initial problems from Souma’s problem collections (Souma, 2000), lesson plans from the lesson plan collections by Kunimune and Souma (2009a, 2009b), and problems that she has created by herself.


5. REFERENCE MODEL

5.1 Souma’s problem solving oriented lessons

Kazuhiko Souma is a professor of mathematics education in Hokkaido. He has written and edited a number of books regarding the practice of a variation of SPS approach, called Mondai kaietsu no jugyou (problem solving oriented lessons) (Souma, 1995; 1997; 2000; Kunimune & Souma, 2009a; 2009b) especially for the lower
secondary school. It is noteworthy that Souma focuses his work mainly on the lower secondary teaching practice, whereas many Japanese studies within the establishment of the SPS approach are addressed to primary levels (e.g. Tejima, 1985). In these books, he proposes lesson plans and collection of tasks, as well as his theoretical ideas. This was a main reason I chose Souma’s version of the SPS approach: there are sufficient materials to work in several subjects over an entire school year. Souma has established his method from long experience as a mathematics teacher in lower secondary school; his research is clearly practice-based. He gives numerous lectures and workshops every year for mathematics teachers all over Japan. As most other Japanese professors in mathematics education, he often participates in lesson study type activities. The flow of the lessons of Souma’s approach is based on the Japanese SPS approach, which is widely used by teachers in Japan (Shimizu, 1999). The standard flow of the lesson with SPS can generally be described by following didactical terms in Japanese, which are used by Japanese teachers on a daily basis. (Shimizu, 1999, pp. 109-111, by his English translations):

1. *Hatsumon*: to ask a key question that provokes students’ thinking at a particular point of the lesson.
2. *Kikan-shido*: teachers’ instruction at students’ desk. Scanning by the teacher of students’ individual problem solving process.
3. *Neriage*: whole-class discussion. A metaphor for the process of polishing students’ ideas and of developing an integrated mathematical idea through the whole-class discussions.
4. *Matome*: summing up. The teacher reviews what students have discussed in the whole-class discussion and summarizes what they have learned during the lesson.

In his book, Souma (2017, pp. 10-11) describes the flow of a lesson according to his approach from the students’ perspective as follows:

1. **Understanding the mondai (initial problem)**: students grasp the meaning of the initial problem shown on the blackboard, and try to work with it.
2. **Guessing**: students state a guess for the answer to the initial problem, or suggest one or more solution methods.
3. **Formulating the kadai (derived task) from the initial problem**: within the process of trying out their hypotheses,
students formulate the derived task (see the description below) that the teacher’s initial problem aimed at.

4. **Solving the kadai**: within the process of solving the derived task, students acquire sufficient knowledge and skills to solve tasks of this type.

5. **Solving the initial problem**: By using the knowledge they got during the process above, students finally solve the initial problem.

6. **Working with training tasks**: students work with tasks, which prove their learned techniques.

The teacher’s acts follow largely the SPS: showing the initial problem, observing students’ individual work, conduct a whole-class discussion. It differs from Shimizu’s description in that it initially lets the students state a guess or hypothesis and that it invites them to reformulate the initial problem and clarify the kadai (derived task). The flow described above is used commonly even at elementary level (e.g. Tejima, 1985).

Souma stresses two conditions on the initial problem that lead to successful lessons: 1. tasks should trigger students’ curiosity so that they immediately start tackling them, 2. tasks should enable students to learn new knowledge, techniques, concepts and ways of thinking (Kunimune & Souma, 2009b, p. 11). He calls the initial problem open-closed problems, in which the students’ answers, in the later stage after they make the guesses, should be unique (closed), but still give rise to a variation (openness) of the techniques. This is in contrast to « open-ended-problems » (Becker and Shimada, 1997) where the problems are designed to arouse different interpretation, and make students reason the precise conditions for finding several appropriate answers.

5.2 **The didactic moments detected in Souma’s lesson plan**

The main didactical terms within the SPS; *Hatsumon*, *Kikanshido*, *Neriage* and *Matome*, express certain processes of the didactic phases, where the teacher applies different didactic techniques. Most Japanese teachers recognise the signification of those terms, and it means that the terms themselves form part of their didactic technologies – the technologies which explain the use of didactic techniques by the teacher in order to realise those phases. In that way, several of the didactic techniques listed in the reference model are difficult to distinguish from the technologies which sustain the SPS.
The term *kadai* (a derived task) is also such an element of technology, which helps substantiating several techniques.

To understand the structure of Souma’s lesson, I present an example case from Souma’s lesson plan (Souma, 1995, pp. 102–105) using the notions of the didactic moments. The mathematical type of task T is finding the properties of integers, and prove that the difference of the squares of two consecutive integers is equal to the sum of the two numbers. The didactic phase *hatsumon*, which is corresponded to the first encounter (FE) provides a following initial problem: the teacher writes down the expressions
\[ 5^2 - 4^2 = 9, \quad 8^2 - 7^2 = 15, \quad 4^2 - 3^2 = 7 \text{ and } (-9)^2 - (-10)^2 = 19 \]
on the blackboard, and asks the class: “What can you say about these expressions?” The guessing technique strengthens (FE) with the initial problem t, and leads students into the exploratory moment (EX), where they start to comprehend the initial problem. The guessing technique motivates them to start to tackle the problem and to participate in the discourse. Students may come up with several initial guesses: “the differences equal the sum of the integers”, “the differences equals the first integer times two minus one”, “the last integer times two plus one”. After the class has verified each of these statements, they start to wonder if those statements always hold and why. They now want to prove them. Souma refers to this part of the lesson as the «clarification of the kadai» (ibid., p. 34) – students have formulated the *kadai* from their guesses. Thus, the *kadai* is derived from the initial problem to function handing over the responsibility of learning from the teacher to the students.

The formulated *kadai* can be approached using several possible mathematical techniques, and realises the moment of (TT) – building the technological–theoretical block: some students may use the formula for expanding the square of a sum, and others apply the rule of difference of two squares, etc. The techniques belong to *Kikan-shido* corresponds to the (EX) moment, where students produce new techniques to solve the *kadai*. After a few minutes of individual considerations, the students discuss in groups, and the teacher walks around to check which students come up with what kind of techniques, and let some of the students present their reasoning. In preparing the lesson plan, the teacher is supposed to consider all anticipatable techniques the students might come up with, so that she/he will be prepared to be able to evaluate the quality of the techniques and to give appropriate help and response. This moment usually is used to plan e.g., in which order the students’ various
techniques will be presented in the next phase *neriage* (the whole-
class discussion).

Techniques teacher usually use for *neriage* moment, which
corresponds to (TT), are following: asking who used which
techniques, and what kind of knowledge from previous lessons they
were using; asking the class which techniques they prefer and why;
asking the relevance and differences between the techniques.
Certainly, to enable any of the above, and to realise the (IN) and (EV)
moments of *institutionalisation* and *evaluation*, it is important that the
students have easy access to all techniques proposed. *Bansho* is
another didactic technology, and is one of the crucial elements to
achieve this moment. The notion expresses more than « board writing »
and includes teachers’ various act aiming to help students’ learning.
It constitutes an essential « instructional tool for organizing students’
(1997, pp. 74-75), watching the contents written on the blackboard
and copying the writings in their notebooks should help students
obtain a sense of:
1. necessity to solve the task
2. collective work of solving the problem with the classmates
3. understanding the process of the problem solving
4. contemplating on one problem

*Bansho* contributes developing a record of the whole lesson, since the
teacher usually does not erase any of the text written on the
blackboard. Thereby teacher’s techniques on *bansho* assist to
institutionalise the knowledge actually learned, and evaluate « the
praxeology produced and one’s relation to it » (Chevallard & Bosch,
2020, p. xxvi). Souma considers that the issues 1-4 above are all
essential components for the realisation of lessons within the SPS
approach. (ibid. p. 75).

### 5.3 Reference model for Souma’s didactic techniques

Here, I present the reference model of didactic techniques, which are
connected to the different didactic moments. The techniques were
identified in Souma’s books (Souma, 1995; 1997; 2000; 2017;
Kunimune & Souma, 2009a; 2009b). These techniques are based upon
the didactic technologies, which sustain the standard structure of the
SPS approach (see the previous section). In Section 7.1 I describe how
the didactic logos of the SPS is developed. Appendix 1 explains how I
identified the different techniques from Souma’s literatures.

<table>
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<tr>
<th>Type of didactic</th>
<th>Didactic technique</th>
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blackboard. Thereby teacher’s techniques on *bansho* assist to
institutionalise the knowledge actually learned, and evaluate « the
praxeology produced and one’s relation to it » (Chevallard & Bosch,
2020, p. xxvi). Souma considers that the issues 1-4 above are all
essential components for the realisation of lessons within the SPS
approach. (ibid. p. 75).

### 5.3 Reference model for Souma’s didactic techniques

Here, I present the reference model of didactic techniques, which are
connected to the different didactic moments. The techniques were
identified in Souma’s books (Souma, 1995; 1997; 2000; 2017;
Kunimune & Souma, 2009a; 2009b). These techniques are based upon
the didactic technologies, which sustain the standard structure of the
SPS approach (see the previous section). In Section 7.1 I describe how
the didactic logos of the SPS is developed. Appendix 1 explains how I
identified the different techniques from Souma’s literatures.
Transferring Japanese problem solving to Swedish classroom

| tasks | | |
|---|---|
| (FE) the moment of first encounter | 1. *Hatsumon*: Giving an initial problem for which every student is able to guess the answer by using old knowledge, or by simple guessing.  
2. Writing the initial problem on the blackboard.  
3. Using one among the four different question types of initial problems appropriately depend on the target knowledge and the levels of the students.  
4. Letting students guess in some way (according to the nature of task), and not reply immediately if guesses are correct or not.  
5. Writing the summarized students’ different guesses (showed during the technique 4 above) briefly on the blackboard.  
6. Comparing students’ different guesses recorded on the blackboard and let them consider which of the guesses may be correct. |
| (EX) the moment of exploration of T | 7. *Kikan-shido*#1: Letting the students work individually for a few minutes and monitor their work. |
| (TT) the moment of building technological & theoretical block | 8. Giving students a question, which helps them formulating the *kadai*, such as: «I wonder if it will be the same by any number. What do you think? If so, how can we prove it?» |
| (TT) | 9. Predicting the possible mathematical techniques in advance to be able to react students various solutions for giving them a quick response and comments regarding their techniques. |
| (TT) | 10. *Kikan-shido*#2: monitoring the students’ solutions and plan the *neriage* moment, e.g. the order of the presentation. |
| (TT) | 11. *Neriage*: Letting the students write and present their mathematical techniques on the blackboard and discuss their viabilities (or letting them raise the hands and present their solutions in any order). |
| (IN) the moment of | 12. Letting the students reflect the discussion. Comparing different techniques, and discussing the different aspects of the solutions. |
| | 13. *Matome*: Summing up the lesson by: |
Letting the students review the writings on the blackboard
b) Checking together the explanations in the textbook about the topic learned today

14. Letting the students copy everything on the blackboard at the same time as the teacher writes. When the lesson is over, the whole blackboard is copied in their notebook.
15. Never erasing what the teacher and students wrote on the blackboard for the students to be able to look at it and reflect the solution process they have discussed on the initial problem.
16. Letting students work with training tasks in the textbook by their own way using their learned techniques

Table 1. — The reference model of the didactic techniques of Souma’s version of the structured problem solving approach.

**6. ANALYSIS OF THE TRANSFERABILITY**

In this section, I firstly describe in what extent Souma’s approach were introduced to the Swedish teacher Eva. Then, a praxeological analysis of the Swedish lessons using the reference model (Table 1) is presented in two steps: first, the analysis on a particular lesson based on Souma’s lesson plan, second, the summary of her use of the didactic techniques applied in her 16 lessons.

**6.1 The details of the contents mediated to Eva**

At the first meeting with Eva, I described the structure of Souma’s approach, which I translated in Swedish according to Souma’s description precisely. I also summarized the following «script» for a lesson and showed to her:

1. Present an initial problem (usually on the blackboard), which represents a new concept or solution method for the students, in a way, which will incite them to guess answers, or make immediate observations
2. Let all students make such guesses, hypotheses, etc.
3. Discuss the viability of guesses and let students motivate these guesses. Derive a formulation of the *kadai* (derived task) to be worked on in the lesson
4. Let students work on the *kadai*, individually or in groups
5. Let students present their various solutions to the whole class, by writing their solutions, or orally, and discuss the differences among their solution techniques.
6. Turn to the textbook for an outline of the related theory.
7. Use the whiteboard as the record of the lesson so that the students understand the process of the lesson through looking at the board.

I translated Souma’s description regarding *kadai* in Swedish as it was presented in Section 4.2. And picked several initial problems from Souma’s problem collection (Souma, 2000), so that she could see the characteristics of his problems. I described the four different types of initial problem as follows (Kunimune and Souma, 2009b, p. 11):

I. Form of answer is closed: « How many cm is ~? », « What kind of triangle is this? »
II. The answer is among some given alternatives: « Which of these expressions are actually same? », « Which of these expressions are right /wrong? »
III. Having two options (yes or no) for the answer: « Is it correct that ~? » « Is it the same as ~? »
IV. An open answer: « What can you say about the following expressions? »

Regarding *Neriage* moment, I translated Souma’s text (1997) in which he described two types of methods for conducting whole-class discussions and two additional options. I summarized them as below:

A. Let students presenting different solution methods one by one
B. Let students write different solution methods simultaneously, at the blackboard
C. Plan the order of students’ presentation beforehand during *kikan-shido* (teacher’s observation of students’ solutions)
D. Have students raise their hands and let them present their solutions, without planning the presentation order.

Regarding *bansho* technology, I described the white board as a record of the whole lesson, so that students could reflect on the whole process that they went through. I asked Eva to try remaining the writings on the board as well as possible. I showed her the translated text presented in Section 4.2, namely, watching and copying the writings in their notebooks would help students obtain a sense of: 1. necessity to solving the problem; 2. collective work solving the problem with the classmates; 3. understanding the process of the problem solving; 4. a sense of contemplating on one problem.
In August, Eva and I started to fix the details of the lesson plans for the grade 7 class. We discussed about the goal of the lessons, the initial problems, the flow of the lessons, and the tasks for homework. She learned how to write up a lesson plan by reading the lesson plan described in Section 5.2 from the lesson plan collection book *Practical lesson plan collection for mathematical activities* (Kunimune & Souma, 2009a). Her lesson plan for every lesson involved three parts: 1. the goal, 2. the preparation and 3. The flow of the lesson. The last part included three columns, which together form a chronological script for the lesson:

- a) Teacher’s activity (what the teacher will do during the lesson),
- b) Students' activity (what the students will do during the lesson),
- c) Other things to consider or notice at particular points of the lesson.

In the first column (a), one describes e.g. the initial problem, questions to ask to the students during *kikan-shido* (teachers’ circulation during the individual work) and *neriage* (whole-class discussion), or in the process of *matome* (summing-up). In the second column (b), one tries to predict students’ guesses and solution techniques, and likely errors are included. In the third column (c), one notes didactical/pedagogical considerations, such as « make students write down sketches of the figure first » and « if nobody comes up with the solution 1-3, I will mention the subject of the previous lesson », etc.

To sum up, I mediated mainly praxis part of Souma’s praxeologies, especially didactic techniques: *hatsumon, kikan-shido, neriage, matome* and *bansho*. Also the character of the initial problem and derived task, the flow of a lesson. However, I did not convey the logos of his praxeologies fully. Surely, I showed her e.g. the translation of Souma’s description of the *bansho* technology presented above. Nevertheless, I did not stress the didactic theories (see Section 7.1 and 7.2) justifying these techniques. Sometimes I pointed out after the observed lessons that she was erasing the whiteboard, however I did not explain why she should plan the organisation of the writing well, so that the writings on the board could be remained. During the lessons, I basically devoted myself as an observer and did not speak to Eva nor the students. We did not regarded our relation as a trainer and learner but as colleagues of a project where we tried out a new teaching approach.
6.2. Analysis of a specific lesson plan

Here I describe a lesson plan « Introduction to finding the general solution » for grade 7, proposed by Kunimune and Souma (2009a, pp. 26–29) for the later analysis of Eva’s lesson with her grade 7 class.

The lesson plan begins with « The idea of this lesson », where the teacher describes his/her pedagogical and didactic considerations and aims (ibid., p.26). Here is an outline: In primary school, students had used some symbols like ¥ and ▲ for « numbers to find », as they made calculations with addition/subtraction and multiplication/division. They have also trained how to express a relationship between two quantities, and to interpret expressions. In this lesson, they will learn using variables as a basis of modelling, before starting to learn about solving linear equations. The focus of this first lesson is on letting the students realise the convenience of using formulae, such as in the formula for the area calculation of a triangle. Students will solve the task by using several methods such as induction, analogy and deduction. Then the initial problem is presented:

We will make a square by arranging stones as in the picture below. If one of the sides consists of five stones, how many stones are used in total? (See figure 1)

The teacher will let students reason by drawing his or her own figures. During the moment of whole-class discussion, students discuss their various way of thinking. The teacher now gives a second problem: to determine the total number of the stones of when there is 20 stones on each side. These first activities will lead to the kadai: to find a method to determine the total number of stones in a square with any number of stones on one side. When the students formulate such an expression, they will feel the inconvenience of using the phrase « number of stones on one side » repeatedly. In that way, students will appreciate the convenience of using a letter, rather than a phrase. During matome (summing up) moment, students will explain how the variable represents an unknown number within in some possible domain (here, positive integers). « The goal of the lesson » is then described as follows (ibid., p. 27):
• To understand the meaning of using variables
• To raise students’ ability to explain their reasoning clearly to their classmates, by using figures and mathematical expressions

The next part in the lesson plan is called « The mathematical activities in this lesson », where the teacher’s consideration on the students’ activities in this lesson are explained: Looking back one’s old knowledge about using formulae, recognising the convenience of using formulae, to explain one’s reasoning to the classmates by using figures and mathematical expressions, recognising the inconvenience of using the phrase « number of stones on one side » repeatedly, and thereby realising the convenience of using variables. After this section, presents the lesson plan the flow of the lesson.

The mathematical types of task in this lesson is to find a general expression (a formula) to calculate the total number of the objects, which is expressed as kadai in the lesson plan. The possible techniques are described as: \( \tau_1 \), \( (n \cdot 4) – 1 \cdot 4 \), \( \tau_2 \cdot (n – 1) \cdot 4 \), \( \tau_3 \cdot (n \cdot n) – (n – 2)^2 \) and \( \tau_4 \cdot (n – 2) \cdot 4 + 4 \), which are corresponding to four different ways to model the kadai. However, the lesson plan does not ask the class to verify that these four expressions are equivalent, since the students have not learned to simplify algebraic expression yet. This work is not included in the mathematical task of this lesson. Noticeable technologies are the following: \( \omega_1 \): partitioning of figures, \( \omega_2 \): inclusion-exclusion in counting, \( \omega_3 \): the distributive law and \( \omega_4 \): the rule for expanding a square (though the students have not learned yet).

According to the reference model (Table 1), this lesson plan realise all didactic moments but the moment of technical work. It does not use the technique of applying training tasks to the students.

6.3 Praxeological analysis of the lessons implemented in Sweden

In this section, the empirical data from the Swedish classroom observation and its analysis by the reference model are presented. Firstly the analysis on the lesson « Introduction to finding the general solution », then general analysis on her 16 lessons are presented.

Lesson « Introduction to finding the general solution » in grade 7

Eva begins the lesson by showing the initial problem about squares made by stones. Two students present their solutions and Eva writes them on the whiteboard:

Robert’s idea: \( 2 \cdot 5 + 2 \cdot 3 = 16 \)
Ronja’s idea: \( 4 \cdot 4 = 16 \)
Eva lets Robert explain what he means of the expression $2 \cdot 5 + 2 \cdot 3$:

Robert: I do not know… why I said so. Two times 5 like $2 + 2 + 2 + 2$, and further three times 2? Ah, I probably meant $2 \cdot 8$.

T (Eva): Two times eight? (To the class) Do you understand how he thinks? However, the question is that if it will be easier to reason by using $2 \cdot 8$. I can write it anyway (writes $2 \cdot 8$ on the whiteboard). Ronja, can you explain yours? (lets Ronja mark the figure)

Ronja: I thought like this way (drawing as in Fig. 3).

T: Is there another idea? Mary?

Mary: $15 + 1$.

T: Ok, we write it (writes $15 + 1$ on the board)

Figure 2. – Ronja’s explanation for the expression $4 \cdot 4 = 16$

Eva uses the didactic technique suggested by the Japanese original lesson plan: giving an initial problem, which is able to guess its solution for every student, and can lead to various mathematical techniques. The figure on the whiteboard gives the student an opportunity to find different solutions. Another technique she uses is, presenting students’ different guesses on the whiteboard including methods that are not correct, such as $8 \cdot 2$ and $15 + 1$. As it will appear below, her intention is to let the students realise that these expressions would not work in the case of other numbers than 16.

Eva now therefore lets the students apply these expressions to bigger numbers than 5 and find a certain pattern for the formula. She asks the class if they can express the total number of the stones with one side constructed by 100 stones. In Souma’s lesson plan, it is planned to give the students the task with 20 stones but Eva choses to continue with 100 stones to shorten the process.

Ronja: 4.

T: What are you saying?

Ronja: 4 times 99. (she applied her previous expression $4 \cdot 4$)

T: You think it will be 4 times 99. (To the class) What do you think? Which one of these expressions can be used to
calculate 100 stones on one side? How about this?
(Pointing $2 \cdot 5 + 2 \cdot 3 = 16$) What does this 5 mean?

Ronja: 5 is the number of the stones on one side.
T: (writes « The number of the stones » besides 5) OK, and how about this 3? Anyone?

Samir: 6 divided by 2.
T: But look this (the figure). 5 is these (marking the 5 stones on the both sides), and the rest?

Eva tries to give the student questions to make them conscious about the kadai (finding out the formula). However, only Ronja appears to see a pattern clearly. Thus, Eva now shows by herself how the expression of $2 \cdot 5 + 2 \cdot 3$ should be interpreted in the figure. Ronja, points out the expression of $2 \cdot 8$ and $15 + 1$ would not work in the case of 100. Most students still do not find any pattern in the case of 100 stones on one side. Consequently, Eva changes her strategy, and draws a figure of 3 stones on one side. She asks the class:

T: How can we write according to this expression (points to Robert’s idea $2 \cdot 5 + 2 \cdot 3$) if there are 3 stones on each side?

Samir: 2 times 3 plus…2 times 3 minus 2, it should be.
T: Ok, (writes $2 \cdot 3 + 2 \cdot 1$) And it is equal to?
Samir: 8.

Her didactic technique, giving the student a question here is not based on the didactic task (EX) the moment of exploration of T or (TT) the moment of building technological & theoretical block, since the mathematical technique to find the pattern is given by herself. Eva continues asking the class about the cases of 4, 6, 7, 8, 9 and 10 stones on one side for making the students recognise the pattern of the expression. She records numeric chart of the expressions presented by the students, and asks if they can see the pattern in the expressions:

- 3 stones $2 \cdot 3 + 2 \cdot 1$
- 4 stones $2 \cdot 4 + 2 \cdot 2$
- 5 stones $2 \cdot 5 + 2 \cdot 3$
  (etc., all the way up to 10 stones).

Several students answer that there are 2 stones between the front term and the rear term. Now Eva asks again the case of 100 stones on one side:
Tina: 2 times 100, anyway to begin with.
T: Ok. (writes $2 \cdot 100 +$) How can we express this part (points on the rear term)?
Kejo: 2 times 70. No, 2 times 80!
Someone: 2 times is agreed. (writes $2 \cdot 100 + 2 \cdot 70$) What is happening here? (points on 9 and 7 of $2 \cdot 9 + 2 \cdot 7$) 3 and 1, 4 and 2, 5 and 3, 6 and 4, 7 and 5, 8 and 6. Aisha?
Aisha: 2 in interval. [Someone talked at the same time and could not be heard in the class]
T: What Aisha said, Kejo?
Kejo: 2 in interval.
T: Then in that case, 2 times what?
Kejo: 80!
T: Is 2 in interval between 80 and 100?
Kejo: 98!

Kejo gives here the correct answer. However, the answer is strongly directed by Eva’s questioning. Then she asks about 1000 and one million stones on one side. Kejo, who now starts to see the pattern, replies «$2 \cdot 1,000,000 + 2 \cdot 999,998$». Eva now steers the direction of the lesson to come to the general solution using a variable $n$ on one side.

T: In that way, we can calculate whatever, any number. That is called a general solution in mathematics. A general solution is something, which we can apply to any of the cases. (writes $n$ on the whiteboard) How can we express this one (pointing on 4 of the expression $2 \cdot 6 + 2 \cdot 4$) using $n$? There are 2 in between to the $n$. How can we express it?
Someone: $n^2$?
T: $n^2$?
Ronja: No, $n - 2$.
T: (writes $n \ (n - 2)$) How can we write the whole expression? Compare to those expressions. $n$ is for those (points the number of the stones on the chart) and $(n - 2)$ for those (points the rear part of the terms). What is missing?
Someone: Plus.
T: Yes, (writes $n + (n - 2)$) and? What else?
Someone: 2 times and 2 times.
T: (writes $2 \cdot n + 2 \cdot (n - 2)$) Is that correct? Everyone understood?
Eva then gives the class a training task to calculate the value of \(2 \cdot n + 2(n - 2)\) when \(n = 20\) and \(n = 40\) to let the students agree that the formula is applicable for any number of stones on one side. At last, she lets the class apply Ronja’s initial expression \(4 \cdot 4 = 16\), by letting them make the numeric chart again and find the case of \(n\) stones on one side. Helped by Eva’s leading questions, the class eventually «finds» the formula \(4(n - 1)\). However there was no time left to compare this expression with the previous expression, \(2n + 2(n - 2)\).

Eva’s use of the didactic techniques in this lesson

During the lesson, Eva used several didactic techniques: all techniques FE (the moment of first encounter) 1–6 regarding hatsumon; TT (the moment of building technological & theoretical block) 9 predicting the possible mathematical techniques in advance; TT11 letting students present their various techniques on the board; TT12 comparing different techniques, and discussing the different aspects of the solutions were also observed.

On the other hand, she made substantial modifications on following techniques: EX (the moment of exploration of \(T\) 7 & TT10 regarding kikan-shido; TT8 giving students a question, which helps them formulating the kadai; IN (the moment of institutionalisation of the learned knowledge) 13, 14, 15; TW & EV 16 (the moment of technical work & the moment of evaluation). Technically, the reason that Eva did not use last technique 16 « letting students work with training tasks in the textbook by their own way using their learned techniques » were because of the time limitation. However, Eva’s non-use of the techniques regarding TT8 letting the students find kadai was caused of the modification of overall goal of the lesson. In the original lesson plan, the kadai was on « letting the students find the formula through making them realise the convenience of using letters ». The modification arose when Eva stated the question of the 100 stones on one side. At that stage, only few students could see the pattern in what were essentially ideas put forward by two students (Robert and Ronja). At this moment, the mathematical types of task T « finding the pattern from the figure » had been changed to «understanding and using informal induction from a numeric chart to make the pattern ». Subsequently, didactic techniques also were changed to the classic teacher initiated questions and students’ reply based on the didactical contract. First, she gave the students the task to express the total number of the stones with one side constructed by 100 stones, and then she suggested to use a variable \(n\) for the general case, while according to the original lesson plan, both the general
expression and the use of a letter should be suggested by the students. The fact that she did not use essential techniques concerning $kikan$-$shido$ also promoted the modification of the mathematical goal, since $kikan$-$shido$ could have given the students an opportunity to find the patterns.

Another techniques, which concern $bansho$, were not applied to realise the didactic task institutionalising (IN) 15 « to be able to look at it and reflect the solution process students have discussed on the initial problem ». What she wrote on the board was: the initial problem, the figure, the four ideas for the solution of the initial problem, Ronja’s figure, a comment on Robert’s idea, figures of the squares of the case of 3, 4, and 6 stones on one side, the numeric chart from 3 stones to 10 stones, the formula $2 \cdot n + 2 \cdot (n - 2)$, the control task $n = 20$ and $n = 40$, the numeric chart to find the pattern for making the formula corresponding to Ronja’s idea of $4 \cdot 4 = 16$, and the formula $4(n - 1)$. No comments were given to explain the intention of these (mainly symbolic) expressions. She did not pay attention to writing different issues in specific places of the board during the lesson. The fact that she erased the figures was actually necessary to get enough space for the final formulae $4(n - 1)$ and $2n + 2(n - 2)$ (see Figure 3), however, an attempt for the institutionalisation of the learned knowledge by the board was not observed.

![Figure 3. - The whiteboard at the end of the lesson](image)

*The didactic techniques actively applied in the 16 lessons*

The first three didactic techniques from Table 1 concern the initial problems. Eva worked diligently planning the flow of the lessons and constructing problems, especially when she would not (or would not directly) use Souma’s lesson plans. Regarding the technique 3 « using
one among the four different question types of the initial problems», Eva used all four types (see Section 5.1) in her lessons. The tasks applied in the lessons were mostly open-closed, where students should use their previous knowledge, and could result several solution methods (except lesson 6, 7 and 13, which were about the operations involving negative numbers). In the classroom, she started every lesson by stating the initial problems on the whiteboard.

Guessing techniques (FE) 4, 5, 6, were frequently used. In all lessons, Eva let the students guess the answers to initial problem, and recorded how many of them guessed what at the whiteboard. The following discussions often started based on these different guesses: «now we have two different answers. How can we find out which one is the correct?» Naturally, this technique is connected to the task design, where the task allows for several guesses. Regarding the technique 4 «let students guess in some way», Eva did not confirm the students’ answers immediately by replying, «Yes» or «It’s correct». She pretended she did not know the answer by saying «Hmm», «Ok». However, when student looked uncertain because of a request for explanation, it also happened that Eva confirmed an answer, saying something like «do not worry, your answer is correct. I am not asking you the explanation because your guess is wrong, but I want to know why you think in that way». Eva applied heritage techniques (TT) 11 and 12 in all lessons. The key to realise these techniques were often connected to the well-constructed initial problem. For example, few days after Eva conducted the lesson showed above, she repeated it with the same contents at the grade 8 class. There she changed the second problem to 20 stones as it was shown in the original lesson plan, and gave a few minutes to work in-group. She let four groups of them present their figures and patterns, and without using numeric chart, students eventually found out the formula.

Eva used technique (IN) 13b «checking the textbook to sum up the learned knowledge with the students. For the preparation, she often made stencils explaining different mathematical laws and rules, which the students have worked with during the lesson, and let them read it in the end of the lessons. This work had to be done since the textbook, which was used by the class, did not include such general explanations.

The didactic techniques not applied in the 16 lessons
In the lesson «introduction to finding the general solution», Eva did not try to make the students formulate the kadai. However, she sometimes tried, for instance in lesson 16 (with grade 8) which was
based on a lesson plan by (Kunimune & Souma, 2009a, pp. 30–33).
There, the initial problem is to compare the expressions $5m + 3m = 8m$, $9x + x = 10x$, $9a - a = 8a$, $5y - 8y = -3y$ and find out what is common to these four equations. The *kadai* is to find a rule to rewrite $ax+bx$ when $a$ and $b$ are given numbers and $x$ is some letter (thus, in essence, a version of the distributive law). At the beginning, Eva followed the Japanese lesson plan closely. However, when the class could not answer the initial problem, she wrote a new expression (which is her own, not from the Japanese lesson plan) $9a - 3x + 2b - 4x + 3a - 2$, on the whiteboard. She asked the class «Which of the terms are associated with $9a$?» to make the students notice that some terms include the same letters. Then the class understood they should have said «the equations that have the same letters». On the other hand, the Japanese lesson plan prescribed that the teacher would try letting students find some explanations of why $5m$ and $3m$ were able to be added: «If one add, say, 5 meters and 3 meters (of something), it will be 8 meters», «If one substitute $m$ for the constant 2, $10 + 6 = 16$, and 8 times 2 is also equal to 16», or «$5m$ is $m + m + m + m + m$ and $3m$ is $m + m + m$, thus $5m + 3m = 8m$» (Kunimune & Souma, 2009a, p. 33). Of course, these are potential students’ solution from the lesson plan. Nevertheless, it is a prominent idea in Souma’s approach to try to let students formulate general mathematical rules by their own, while this idea is not so common in the Swedish context.

While *kikan-shido*, which Eva tried to use as well as possible, she often gave hints and advice to the students. Compared to what was proposed in Souma’s original lesson plans, this phase often took longer time, more than 10 minutes. The most significant difference of Eva’s use of *kikan-shido* and the use proposed in Souma’s approach was that she hardly ever made use of this moment to plan the next moment, the *neriage*. Only in one lesson she effectively realised this planning function. Most of the time, she let the students raise their hands and present their solutions in the order they volunteered.

Finally, concerning *bansho*. It was quite obvious that realising the principle of «try remaining the writings on the board» was difficult, since the size of the whiteboard was much smaller than Japanese blackboards. The noteworthy phenomenon was however, she did not pay much attention of Souma’s interpretation concerning *bansho*, which serves for «collective work of solving the problem with the classmates» (see Section 5.1). She never used the whiteboard as a tool for the institutionalisation of the learned knowledge by e.g. letting the students look at the board together and reflect the process of the
lesson. But this is indispensable to carry out the matome moment according to Souma’s method (see (IN)13a in the reference model).

7. HISTORICAL AND CULTURAL CONTEXT IN JAPAN AND SWEDEN

What kind of educational and cultural factors made Eva apply some didactic techniques presented in Souma’s approach, while omitting? In this section, I provide a brief historical and cultural account of the mathematics education in Japan and Sweden for the preparation for proceeding ecological analyses in Section 8.

7.1 Japan

Mathematical thinking, mathematical activities and problem solving
During the 1950’s, the reform of mathematics education took a form as « emphasising students’ understanding of the mathematical concept » from the post-war-time’s « realising the social need by applying students everyday-life related tasks » (Souma, 1997; Nagasaki, 2007). The notion of mathematical thinking (sugakuteki na kangaekata in Japanese) was introduced to the Japanese national curriculum to demonstrate explicitly how student would discover and create mathematics (Nagasaki, 2007). There are many interpretations on this notion. Kikuchi (1969) defined it as « processes in which one finds mathematical facts through understanding of mathematical concepts, constructing mathematical problems and solving them » (p.25). He described the process of problem solving as (1) Configuration of kadai (the derived tasks) (2) Observation (to find the task) (3) Classifying the facts (4) Making the hypothesis (5) Examination of the methods (6) Proof (of the methods) (7) Application (of methods to another tasks) (ibid.). The notion of mathematical thinking continued to be developed by mathematics educators and mathematicians during the 1960’s, and the notion of problem solving eventually has established its position as a method to develop students’ mathematical thinking (Ito, 2010).

In the end of 1970’s, the notions of mathematical thinking became expanded to the notion of so-called mathematical activities (sugakuteki katsudou in Japanese). This was done by the work of the Japanese mathematics educators with the aim to implement the notion of mathematical thinking into concrete activities (Nagasaki, 2007). The problem solving was the most appropriate method to represent
those activities. For instance, Shimada (Becker & Shimada, 1997) advocated a problem solving approach called open-ended approach as an ideal method to achieve certain mathematical activities. This approach is based on open-ended problems (ibid.), which are conditional or incomplete problems that enable multiple answers and the search for different solution methods. Certain classroom activities that was developed within the frame of the open-ended approach, such as emphasising students’ explanation of their reasoning and comparing their various solutions were carried on to the current style of the SPS.

In the guidelines for the Japanese national curriculum of the lower secondary mathematics (MEXT, 2008), the mathematical activities is defined as « various activities related to mathematics where students engage willingly and purposefully » (MEXT 2008, translated by CRICED, 2010, p. 16). The guidelines mention that « mathematical activities are carried out as problem solving » (ibid., p. 51), and the series of the mathematical activities provides opportunities « to feel the joy of thinking and learning mathematics as well as the necessity for and the usefulness of mathematics » (p. 51). Even though these descriptions are about the problem solving in general, and are not declared as specifically the SPS, the descriptions have the same feature as e.g. Souma’s description of his approach. Shimizu (2011) claims that the major components of mathematical activities are united with the essential idea of the SPS. The SPS approach is established to go together with the development of the notion of the mathematical thinking and mathematical activities with the aim of substantiating the objectives of mathematics education in the teaching practices.

Consideration on the tradition of learning of mathematics

The guidelines underline not only the value of the new knowledge gained through the process of the mathematical activities but also the value of the obtained methods and perspectives that constitute the creation of the new knowledge (CRICED, 2010). In the description of the Framework of the contents of the lower secondary mathematics, the guidelines describe the Explaining and communicating as below:

During the process of mathematical activities, one must face himself by examining his own thinking and feelings. By expressing one’s own ideas and thoughts in his own words, he can reaffirm his own thought processes. (…) This process of internal dialogue can be further facilitated by the communication with others and increases the possibility of raising the quality of thinking. (ibid. p.29)
This account of the mathematical activities of students’ explanation and communication, has clear resonance that values the process of the learning of mathematics. Hirabayashi (2006) enlightens the traditional aspect of the learning of mathematics in Japan by describing the ritual of an indigenous tradition – gei. The notion gei generally means traditional arts such as tea ceremony, ikebana, noh, traditional dance and music. According to Hirabayashi, it is a kind of « self-culture or hobby ». It gives people « some good effects in living happily », or « a good reputation in their society » (ibid., p.55). Conventionally, one (a disciple, a learner) learns the substance of gei only by watching and following every movement of his master, and there is no shortcut to attain the skill of gei in any kinds. Hirabayashi explains the features of the process of gei-training as jutsu (technique) and do (path/way). He references comments of the master of the Japanese traditional martial arts judo to explain these two notions. Judo includes both the meaning of path and technique, however, the master states «Technique will be secondary to achieving an understanding of the way. To train men of good character for life, judo is the ideal way» (ibid., p.58). The point of Hirabayashi’s argument is corresponding the fundamental recognition in learning gei-do to the ideology of the Japanese mathematical education, where the aspect of « the way to develop students’ personality » (p.58) is prioritised in its curriculum and teaching.

Like Hirabayashi, Winslow and Emori (2006) describe the agent system of a teacher and students in the Japanese mathematics classroom as having iemoto characteristics, specifically in its forms of semiotic interaction. The term iemoto originally means a family line or the member that has inherited a certain Japanese traditional art. Here Winslow and Emori employ Hsu (1975)’s definition, which expresses a hierarchical arrangement of group of human beings following a common code of behaviour for a set of common objectives (p. 558). Their semiotic analysis on the data from a Japanese lower secondary mathematics classroom shows how the students learn using the codes of « good way of thinking », « good way of writing » and « good way of speaking » in their reflective discourse. Winslow and Emori state that these three codes are inseparable cornerstones of the mathematical way as a semiotic and discursive endeavour. The students are led (by the teacher) « to pay great attention to good way of writing as a manifestation of good way of thinking, with good way of speaking as a bridge between the two » (p. 563). The authors conclude that in the formation of the iemoto of mathematics classrooms, the assimilation and use of these codes are
part of the socialisation into adult society for the individual student in Japan.

7.2 Sweden

Individualisation and goal oriented teaching

In 1962, nine-year compulsory school system with the first national curriculum Lgr 62 was introduced. After this reform, the proportions of students in lower secondary school was increased from 30 % in 1950s to almost 100 % in 1960s (Carlgren, Klette, Myrdal, Schnack & Simola, 2006). Teaching the new kind of heterogeneous classes was a challenge, and Lgr 62 emphasised individualisation of teaching, which means treating student as separate individuals (ibid.). The state has introduced textbooks and other material with detailed instructions on how teachers should guide students in their individual learning process (Pettersson, 2013). Aspirations of individualisation took various forms in self-instructional teaching materials during the 1960s and 1970s. In 1980s, number of attempts of study projects regarding the teaching approaches that were tailored to individual student's ability were carried out (Vinterek, 2006). In the beginning of 1990s, due to the need for increased democratisation, efficiency and quality for the school operations, the political decision of the decentralisation of school governance was made (Pettersson, 2013). The municipalities got full responsibility for the school, and became able to design the organisation after the local needs and conditions. The previous curricula prescribed certain contents as syllabus for the student while the new curriculum Lpo 94 instead set objectives for the students, which were written in very general terms, and The goals to strive for in mathematics also were described by broad expressions (Ministry of education, 1993). These objectives did not give detailed prescriptions of what and how the teachers should teach but only what knowledge the students should acquire. By the enforcement of Lpo94, the previous norm referenced grading (relativa betyg) was replaced to criterion referenced grading (målrelaterad betyg). In this new grading system, teacher compares a students’ knowledge or skills against a predetermined learning goal. Since after students’ goal achievement would be followed up through national tests and the supervision of the school inspectors, teachers were urged to adapt their teaching to ensure students fulfilment of the goals of the national test (Jarl & Rönnerg, 2010), and not losing the sight of goals of the lessons for their students (Hemmi & Ryve, 2015).
Problem solving as mathematical competency

In 2011, a new curriculum Lgr 11 was enforced. Lgr 11 emphasised following student’s mathematical competencies: *problem solving, conceptual analysis, application of appropriate mathematical methods, reasoning and communication*. The notion of competencies had been discussed for a long period since 2000s not only within the subject of mathematics (e.g. Linde, 2004). The guidelines for the course plan in mathematics (Skolverket, 2017) described above mentioned five competencies as fundamental conditions to be developed through the mathematics education. In Lgr 11, problem solving is described as a tool to develop students’ ability to « formulate and solve problems using mathematics and also assess selected strategies and methods » (Skolverket, 2019, p. 55), and is connected to all other four competencies. The guidelines state that problem solving has a special position being applied to all other areas of knowledge (Skolverket, 2017). The descriptions of problem solving in the guidelines are focused mainly on the achievement of the target knowledge. The account such as « An ambition with the course plan is that students will develop problem solving strategies in a wider context. The strategies are a collective concept for (...) formulating and solve problems in different situations in everyday life and in different subject areas » (ibid. pp. 25-26) does not regards applying problem solving as a method for realising students’ e.g. personal development. On the other hand, issues such as *interaction with the classmates* to develop students’ mathematical idea are strongly recommended in the guidelines. Since 2000s, a number of Swedish research have emphasised the efficacy of communications within mathematics lessons, and thereby the mathematical tasks, which are able to achieve such mathematical discussions (e.g. Hansson 2010; Ryve, Nilsson & Pettersson, 2013).

8. DISCUSSION

8.1 The characteristic of Souma’s approach and its condition

Characteristics of the techniques

The central didactic techniques described in Souma’s approach are as follows: the first category that brings out the first encounter (FE) moment — presenting initial problem and letting students make some kind of hypothesis that leads them finding *kadai*. The second category that builds the technological–theoretical block (TT) moment —
conducting whole-class discussion based on the information of the students’ work gathered during kikan-shido. The third category that brings out the moments of institutionalisation (IN) and evaluation (EV) — institutionalising the learnt knowledge and letting students examine the value of the constructed praxeology by using bansho techniques. These techniques do not particularly differ from other versions of the Japanese SPS approach (e.g. Shimizu, 1999), and they are familiar for the majority of Japanese mathematics teachers.

As the title of Souma’s books The problem solving approach (1997) and Practical lesson plans for mathematical activities (Kunimune & Souma, 2009a; 2009b) express, the essential didactic logos of his approach is applying the SPS to embody an idea or philosophy of mathematical activities. The guidelines for the national curriculum stipulates:

Through mathematical activities, to help students deepen their understanding of fundamental concepts, principles and rules regarding numbers, quantities, geometrical figures and so forth, (...) to help students enjoy their mathematical activities and appreciate the value of mathematics and to foster their attitude toward to making use of the acquired mathematical understanding and ability for their thinking and judging (CRICED, 2010, p. 15).

Souma (2009b) considers that the implementation of the objectives related to mathematical activities should be realised through students’ autonomous thinking activity (pp. 10-11).

In general, mathematics teachers’ didactic concerns regarding their lessons tend to be centered on how to organize the teaching of a specific piece of target knowledge. In terms of the levels of didactic co-determinacy, teachers’ focus exclusively on the subject and (at most) the thematic levels (thematic confinement, see Barbé, Bosch, Espinaza & Gascón, 2005). However, since the didactic praxeologies described in Souma’s lesson plans are aligned with the objectives of school mathematics described in the guidelines for the Japanese national curriculum, they contain certain aims that are explicitly related to the higher levels (Kunimune & Souma, 2009a; 2009b). For example, the techniques for letting the students find the kadai – the derived task for the learning of the target knowledge – is based on the didactic theory which calls for validating students’ autonomously developed ideas within the collective discussion, as an important part of their path to the target knowledge. This didactic idea is explicitly related to the levels of: school (pursuing students’ collective efforts and experiences), pedagogy (student-centered learning, fostering students’ self-efficacy) and discipline (mathematical thinking, mathematical activities). Obviously, the posture of applying the idea
of mathematical activities in daily lessons is shared among the mathematics teachers in Japan. There exists a plethora of educational books, booklets, research articles on this matter, written by both researchers and teachers. They usually discuss and study about implementing of the mathematical activities in the teachers’ daily practice in the form of the SPS (e.g. JSME, 2001; Shimizu, 2011). Many of lesson plans presented in those books are related to theme and subject levels such as different possible ways for introducing specific mathematical techniques. However, as shown in Souma’s case, the lesson plans use to describe very precise objectives (fostering students’ attitude for using computer for the problem solving, raising students’ interest in studying figures observed in their daily life, etc.) to be achieved in the process of building mathematical praxeologies.

8.2 Ecology of the transferability of teaching knowledge

Actively applied techniques and substantially modified techniques

The didactic techniques that Eva actively applied and managed in her lessons relate to the technologies hatsumon and neriage. She constructed the initial problems that allow the students having different guesses, and various mathematical techniques. Most of the whole-class discussions indeed took the form of the whole-class discussions. The students actively participated in the discussions, as their different guesses made them interested in knowing the correct answers. Eva pretend not knowing the correct answer, and kept up the students’ curiosity by posing questions that encouraged them to explain their way of thinking.

The techniques that Eva modified substantially were related to the kadai, kikan-shido and bansho (including matome) technologies. She did not tried to let the students by themselves formulate the kadai based on the initial problems. Instead, she always told the target knowledge directly to the class. The whole class sharing and validation of solutions was hardly planned on the information collected during the kikan-shido. She always monitored students’ work, but most of the time, this led her to give the students hints and advice. Regarding the matome moment, she was very keen to produce handouts explaining mathematical laws and concepts, and let the students read it together in order to institutionalise the knowledge learned. However, she never tried conducting this moment through the bansho – letting the students think together by reflecting the contents written on the board and using it as the summary for the class. It was
transferring Japanese problem solving to Swedish classroom

used to display questions and answers but not for documenting the progress of students’ reasoning, from the initial problem to the kadai, and onto the mathematical theorem that justifies the hypotheses they have generated. To understand the conditions which has caused this phenomenon, we need to look at the higher conditions related to the level of society and school, which described in the section 7 – implicit conditions regard the traditional view of the learning and the teacher profession in Japan and Sweden.

Conditions and constraints of the transposition of the professional knowledge

Rohlen and Tendre (1996) identify some established patterns of teaching and learning, expressed in certain Japanese terms for ideal concepts that shape shared expectations. Among others, the terms mutuality, imitation, form and experience are mentioned as a manifestation of the cognitive differences between the Japanese and American teachers’ view on teaching. Following our description of gei-do training or formation within an iemoto, the students learn the body of knowledge by experiencing some form as a media of knowledge transmission, and they imitate that form by repeating its pattern. This is a mutual process, where all members are encouraged to participate, and what is crucial to the group is « the continued general advancement that all members can share » (p. 371). On the contrary, according to Rohlen and Tendre, American teachers tend to see imitation as something inferior to creativity. Also if they transmit the form, they do so verbally (ibid.), not by letting the student observe and imitate it. The idea of form as authority is unpopular among them, and they try to « ignore forms in the name of spontaneity, independence, and creativity » (p. 372). As the American teachers, the Swedish teachers prioritise verbal transmission of the knowledge. In that way, the Japanese didactic technologies such as conducting kadai is to a large extent obscure to them. As explained in section 7.2, Swedish teachers believe that the predetermined goal for the students should be explicitly stated by the teacher, not be some kind of tacit knowledge the students will find out by themselves.

The function of the bansho in the Japanese teachers’ professional practice is also extraneous to the Swedish teachers’ didactic knowledge. In Japan, bansho is common term used in didactic technology to explain and justify board-writing techniques. It is a source of inspiration and new perspectives for the solutions, creating an atmosphere of collective work and as a tool for the institutionalisation (Souma, 1997). This technology caters to needs
coming from the level of school, pedagogy and lower levels of the didactic system. On the other hand, the notion of bansho as a didactic technology does not exist in Sweden. The use of whiteboard mostly caters to needs coming from the subject level, e.g. the need to record the initial problem and students’ answers.

The other essential factor concerning the transferability of Souma’s didactic practice is students. The teachers’ practice in each country has shaped the didactic contract about how students are supposed to act in the mathematics classroom. Highly individualised teaching practice, comparatively equal relationship between teacher and students formed by the long-standing social democratic policy, and the knowledge centred teaching approach in Sweden, are constraints that impede changing their didactic contract for adapting this new approach. Both the kadai and bansho moments requires developing a collective-reflective attitude of the class, driven by an established iemoto formation. Costa and Faria’s study (2018) conducted in Asia and Oceania about students’ association between intelligence and academic achievements suggests that «collectivist societies might focus less in individual results and encourage students to value the learning process over academic achievement» (p. 13). On the contrary, the tendency in Europe is towards a more academically competitive society valuing individual results in the acquisition of knowledge (ibid.).

The reason that Eva modified the manner of kikan-shido is partly resulting from Swedish teachers’ preference for individualised teaching, and students’ inexperience with note-writing. The teacher’s scanning of students’ individual problem solving process is related to the belief that every student should achieve the target knowledge, and not for the preparation of the whole-class discussions. This also cause the difficulty for the students to adapt the manner of note-writing that valuing to describe one’s thinking process while reflecting the previously presented hypothesis during the hatsumon moment.

In the Swedish national curriculum, the notion of problem solving appears more as a tool for making students reach a target knowledge, and the aspect of the socialisation of the students, or their personal development are not especially emphasized. However, the tenet of the initiate problem and whole-class discussion that leads to various techniques and brings the development of students’ mathematical idea matched to the ideology of the students-centred learning and the achievement of the didactic stakes. This brought about that Eva actively applied the techniques regarding the hatsumon and neriage moments.
Paradidactic infrastructure in Japan

The dissemination of teachers’ professional knowledge, e.g. know-how to deal with the notion of mathematical activities in a form of the SPS, relies on the Japanese paradidactic infrastructure (Miyakawa & Winsløw, 2019). Teachers study collectively and share their experiences in the process of those kyozai-kenkyu as in the case presented above. Such knowledge they have obtained can be later developed by observing other didactic systems in different settings in, e.g. open lessons (Miyakawa & Winsløw, 2013). According to Chevallard, infrastructure refers to «the underlying base needed to develop any determined reality» (Chevallard, 2019, p. 84). The set of these teachers’ paradidactic systems are indispensable to carry out Souma’s and other SPS practice in Japan. In other words, the Japanese teachers’ professional practice in paradidactic systems (e.g. in lesson study, practice research, educational book writing) relies heavily on a paradidactic infrastructure.

In Section 7 and the previous section, I have described the higher conditions that affected the didactic ecologies of the Japanese mathematics classrooms. These conditions can be observed even in the teachers’ paradidactic practices, especially in the implementation of lesson study. Teachers observe a masterful expert teacher, and learn his didactic techniques from the observation (Elipane, 2012, p. 359). It can be quite natural that the Japanese noosphere and Japanese teachers also are affected by these traditional preferences of viewing the teacher as a strong authority who guides his disciples by letting them experience the form of teaching practice.

9. CONCLUSION

The central didactic techniques of Souma’s approach described in the reference model and Section 8.1 are based on didactic technologies, with central terms such as hatsumon, kikan-shido, neriage, matome and bansho, which make the SPS approach explicit. The SPS along with the notion of mathematical activities is widely accepted among Japanese teachers. Both notions are linked to the objectives of mathematics education by the longitudinal work of the Japanese noosphere to disseminate the substance of mathematics learning within the framework of Japanese didactic traditions. A discrepancy among the didactical theories that are shared by teachers in the two countries was one of the main constraints which brought about the difficulties to adapt Souma’s teaching practice for the Swedish
teacher. Even though, in both countries, the overall objective of school education is to raise good citizens, it plays out differently in the didactic theories which direct teachers’ practice in the classroom. The Japanese teacher traditionally emphasizes students’ personal development in the processes of achieving new knowledge, and devises a learning situation where students collectively develop their abilities of mathematical way of thinking. On the other hand, the Swedish teacher focuses on the students’ individual achievement of the knowledge itself, and considers the classroom as a venue where students develop their skills in mathematics learning in (essentially) their own ways.

Paradidactic practices, which include a kind of master-disciple like relations between the teachers, are essential to establish and validate the professional knowledge and community spirit of the teachers. The teachers’ didactical knowledge, related to SPS, will be further disseminated through the paradidactic infrastructure, while changing its form gradually. Without immersing oneself in these paradidactic practices, it seems difficult to comprehend the nature of this knowledge. Consequently, even if Eva could read books or watch films explaining and justifying the detailed practice of the SPS, she might not be able to reconstruct its practice. She would not value it in the same way as Japanese teachers do, with their immersion in the paradidactic practices that produced and sustain SPS.

The present study indicates why and how the transposition of didactic knowledge may meet with certain obstacles in a different teaching profession. The researchers’ responsibilities regarding the communication with the teachers about the praxeological equipment need to be taken seriously into account. If one provides a teacher with ideas for teaching practice without giving theoretical rationales, the resulting teaching may easily disappoint. Certainly, the empirical data of this study collected from one particular teacher’s practices is very limited, and other studies (e.g. Stigler & Hiebert, 1999) have indicated that the potential for success of method transfer is beyond what is determined by the individual teachers’ abilities or preferences, or by their students’ backgrounds. However, the analysis based on the reference epistemological model of the teachers’ didactic techniques, and the subsequent analysis based on the levels of co-determinacy, highlight which kind of particular conditions and constraints support and hinder for transplanting this didactic practice. Also, the historical and cultural evidence lend support to the conditions and constraints found in this paper are not simple conjectures but more general and fundamental. Of course, factors and the compatibility of curricula
matter a lot for the realisation of such a transplantation. Still, at a deeper level, one has to consider also the compatibility of the teaching professions’ shared technology and theory, which in turn is shaped by several factors at the level of society, school and pedagogy, in particular by the paradidactic infrastructure.

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### Appendix 1: Supplementary documentation of the reference model (corresponded to Table 1)

<table>
<thead>
<tr>
<th>DO τ</th>
<th>FE 1 &amp; 2</th>
<th>The source described in Souma’s text</th>
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<tbody>
<tr>
<td></td>
<td>1 &amp; 3</td>
<td>« a situation, where students have different guesses on a task, evoke the curiosity with the students and it bring out students’ positive attitudes for attending the lesson » (Souma, 1995, p 9). Also, see example i Section 4.2.</td>
</tr>
<tr>
<td></td>
<td>4, EX 5 &amp; 6</td>
<td>« The initial problem, which will be given in the beginning of the lesson, is a question, which gives the students an opportunity to start thinking. If the teacher gives a task such as “reason the solving methods to calculate…” or “prove it!” the students do not feel the aim and necessity thinking about the task. Thus we suggest the following 4 types of initial problems ». (Kunimune &amp; Souma, 2009b, p 11)</td>
</tr>
<tr>
<td></td>
<td>« Which of the sums of exterior-angles is the largest? The triangle’s, or the pentagon’s? » « Which of the sums of exterior-angles is the largest? » (Kunimune &amp; Souma, 2009b, p 54) The goals are:1. To understand the sum of exterior-angles of a polygon is 360°, 2. To demonstrate how to express the sums of exterior angles by using previous</td>
<td></td>
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</table>
knowedge. (ibid., p. 55)
« The students should come up with three alternatives: that the sum of the triangle’s exterior angles is largest, that the pentagon’s is largest or they are equal. The teacher should let the students briefly present the reasoning for each alternative and ask them », «How can we decide which of the guesses is the correct one? » (ibid., p. 54).

EX7 &8 (See the task example described in (FE)1& 2 above)
TT9 & 10 See Section 4.2

TT 11 I 13 See Section 4.2 description of neriage
« Students will have a goal of ‘what they think for what purpose’ through watching different things written on the blackboard; the initial problem, the kadai found by the students, and other classmates’ various way of thinking » (Souma, 1997, p. 74). Also, see Section 3.2 and the end of the lesson plan (process 6 & 7 in Table 1) in Section 6.1.
See the description of the bansho I Section 4.2

TW 16 I let the students create own tasks and let their classmates solve them (Souma, 2017, p.39) After we controlled the solving method used in today’s lesson, I showed a problem 2,1x = 0,5x – 3,2, and encouraged the students « Solve this task by any method you like » (Souma, 2017, p.51)