# GROUNDS AND CONSEQUENCES IN CONCEPTUAL SYSTEMS

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## *1. Introduction*

In the present paper, we will use a semi-formal framework of condition lattices to elucidate how two conceptual structures are coupled by an intermediary concept. The examples we will discuss are taken mainly from the legal area, regarding the coupling of descriptive and normative conditions in a normative system. In legal terminology, the descriptive conditions are usually called legal grounds, or conditioning facts, while the normative conditions are called legal consequences.

The rationale for calling 'owner', 'contract', 'possession', 'citizen' intermediary concepts is that the essential function of these concepts is thought to be that of serving as vehicles of inference between statements of legal grounds, on one hand, and legal consequences, on the other. In our paper we attempt to make this idea more precise by formulating it in terms of relations holding between two conceptual structures. In so doing, we distinguish between different kinds of relations, in particular those of joining, connection and coupling.

While the examples we will discuss are taken from legal contexts, the idea of intermediary concepts used for coupling conceptual structures has a range of application wider than that of legal or normative systems. A fanciful example of how an intermediate concept serves to couple two different empirical structures is given in the following quotation, taken from a little book, written at the beginning of the century by an author otherwise unknown:

> We do not often have occasion to speak, as of an individual whole, of the group of phenomena involved or connected in the transit of a negro over a rail-fence with a melon under his arm while the moon is just passing behind a cloud. But if this collocation of phenomena were of frequent occurrence, and if we did have occasion to speak of it often, and its happening were likely to affect the money market, we should have some name as a 'wousin' to denote it by. People would in time be disputing whether the existence of a wousin involved necessarily a rail-fence, and whether the term could be applied when a white man was similarly related to a stone wall.[1]

[1] Ingraham (1903). Quoted in Ogden & Richards (1960), p. 46 and in Kovesi (1967), p. 44.

In this example, "event *e* is a wousin" is introduced as a link between one conceptual structure concerning the transit of black or white men over rail-fences or walls and another conceptual structure concerning changes occurring in the money market.

In our paper we will not discuss earlier works dealing with the kind of problem we examine. A few remarks on earlier contributions, however, should be made at the outset. There is a well-known Scandinavian discussion of intermediary concepts in the law.[2] The debate among jurists, initiated in 1945 by Per Olof Ekelöf,[3] soon attracted the attention of professional philosophers. In his essay "Some problems in the logical analysis of legal science" from 1951, Anders Wedberg introduced three different methods for treating the concept of 'ownership' or 'property'.[4] Whereas Wedberg's first and second methods aim at a definition of ownership (namely in terms of conditioning facts and in terms of legal consequences respectively), his third method treats ownership as an undefined "vehicle of inference", used as a tool for inferring statements of legal consequences from statements of conditioning facts.[5] Other contributions to the philosophical debate were made by Sören Halldén, Sven Danielsson, and others.[6]

In the theory of language of Michael Dummett, there are features with some resemblance to the ideas mentioned above. According to Dummett, the meaning of an expression is determined, on one hand by the condition for correctly uttering it, and on the other hand by what the uttering of the expression commits the speaker to. Therefore, the meaning of a statement is identified in part by the conditions from which it can be inferred and in part by what can be inferred from the statement. In the case of utterances of sentences composed by the connectives "and", "or" etc., this is given by what are called introduction and elimination rules in Gentzen's system of natural deduction.[7]

Though the notions of grounds, consequences and intermediaries is a central theme in the present paper, there is another subject which is intertwined with it,

[2] The expression "intermediary concept" (Swedish *mellanbegrepp)* is often used by jurists. See, for example, Bengtsson (1963) regarding the legal notion 'possession'.

[3] Ekelöf (1945). For further references to the early debate among Scandinavian jurists, see Ross (1953) p. 226, note 5.

[4] Wedberg (1951).

[5] Wedberg's third method for treating the concept of ownership is close to Alf Ross's view of of ownership as a Tû-Tû term. See Ross (1951) or its English translation Ross (1956-57).

[6] Danielsson (1973), Halldén (1978). For contributions to the discussion by the authors of the present paper, see Lindahl (1985) and Odelstad (1989). Cf. as well Lindahl (1968) and (1996).

[7] Gentzen (1934). On Dummett's theory, cf. Prawitz (1981), p. 250, (1993), p. 154, and (1994), p. 134. As regards formal languages, cf. Prawitz (1977), pp. 8, 19 ff. and 23 ff.

namely the analysis of normative systems. As regards this second theme, the joint work of Carlos E. Alchourrón and Eugenio Bulygin is part of the background of our paper. The idea, to be presented here, of coupling a lattice of consequences to a lattice of grounds is clearly influenced by the method of correlating "cases" to "solutions", developed by these authors.[8]

The plan of our paper is as follows. We begin by introducing a semi-formal framework for conditions. Next, we present a legal mini-system, where a lattice of descriptive conditions (legal grounds) is coupled to a lattice of normative conditions (legal consequences). In particular we discuss how an intermediate concept can be used for coupling two lattices. After that, we widen the perspective, briefly discussing families of coupled lattices satisfying particular legal requirements. (This discussion is relevant for cases where the set of grounds is open.) Finally, in our conclusion, we suggest that the idea of intermediate concepts can be further developed for elucidating important concepts outside the legal area.

## *2. The framework of conditions*

One of our main notions is that of a condition. Conditions can be of different arity (unary, binary etc.). We use $A,B,C ,...,A_1,B_1,C_1,...,A_2,B_2,C_2,...$ for referring to conditions. Examples of (binary) conditions are: To be the father of, to be the guardian of, to administer the property of, having the obligation to administer the property of, having the right to compensation for, etc.

We often represent a condition by an expression $A(x_1,...x_\nu)$. In so doing we presuppose that if $A$ is a ν-ary condition, and $x_1,..,x_\nu$ are free variables which function as place-holders, then $A(x_1,...x_\nu)$ is a sentence-*form*. On the other hand, if $a_1,...,a_\nu$ are names of individuals or objects, then $A(a_1,...,a_\nu)$ is a *sentence* to the effect that $a_1,...,a_\nu$ fulfill condition $A$.

If $A$ and $B$ are ν-ary conditions, we form compound ν-ary conditions $A \sqcap B$ and $A \sqcup B$, where $(A \sqcap B)(x_1,...x_\nu)$ is defined by $A(x_1,...x_\nu)$ & $A(x_1,...x_\nu)$ and $(A \sqcup B)(x_1,..x_\nu)$ is defined by $A(x_1,...x_\nu)$ v $A(x_1,...x_\nu)$. The procedure of forming compounds can be iterated. Thus, for example $(A \sqcap B) \sqcup C$ is a condition. A condition $A$ is *simple* if it is not compound.

If $A$, $B$ are conditions, $A \doteq B$ expresses that $A$ and $B$ are extensionally equal. If $A,B$ are ν-ary conditions this means that for all $x_1,...,x_\nu$: $A(x_1,...,x_\nu)$ if and only if $B(x_1,...,x_\nu)$.

[8] Alchourrón & Bulygin (1971).

Two conditions $A$ and $B$ are said to be the same, or identical, which we denote $A \equiv B$, if they have the same meaning. $A \equiv B$ implies $A \doteq B$, but not conversely. Thus, $A \equiv B$ if and only if $A \doteq B$ holds in virtue of the meaning of $A$ and $B$.

The symbol $\sqsubseteq$ denotes a relation between conditions. $A \sqsubseteq B$ is read "$A$ implies $B$". For example, the condition to be the father of implies the condition to be a parent of. If $A$,$B$ are ν-are conditions, $A \sqsubseteq B$ means that for all $x_1,\ldots,x_\nu$ it holds that if $A(x_1,\ldots,x_\nu)$ then $B(x_1,\ldots,x_\nu)$. Note that $A \doteq B$ if and only if $A \sqsubseteq B$ and $B \sqsubseteq A$.

If $M$ is a set of ν-ary conditions, then $M^*$ is the set $M$ closed under the formation of compounds by the operations $\sqcap$ and $\sqcup$. $M^*$ is a lattice and we call it the lattice of conditions generated by $M$.[9]

## 3. *The example of guardianship and the coupling of lattices*

### *(i) 'To be a guardian of': Introduction of the example*

In the present section, the important notions of 'intermediary', 'connection' and 'coupling' will be defined. The basic point of the section is that intermediary concepts, or conditions, are appropriately introduced when there is a coupling between a lattice of grounds and a lattice of consequences rather than a plurality of connections. As an illustrative example we will use the rules of Swedish law concerning the condition of being a *guardian* (Swedish *förmyndare*, German *Vormund*) for someone.

If a person is under age, he/she must have a guardian. This guardian is either a parent or a specially appointed person. The guardian of a person administers his/her economic assets. Thus, according to the Swedish law the following holds:

(I) If
$y$ is under age ($P_1(y)$), and
$x$ is a parent of $y$ ($P_2(x,y)$), and
$x$ has $y$ under care ($P_3(x,y)$), and
$x$ is not under age ($P_4(x)$), and
$x$ is not under custody ($P_5(x)$), and
$x$ has not been removed from being guardian of $y$ ($P_6(x,y)$),

[9] Formally, $M^*$ is defined inductively as follows. (1) If $A \in M$ then $A \in M^*$. (2) If $A, B \in M^*$, then $A \sqcap B \in M^*$, $A \sqcup B \in M^*$. (3) The only members of $M^*$ are those resulting from a finite number of applications of (1) and (2). This means that $M^*$ is a partial ordering with least upperbound $A \sqcup B$ and greatest lowerbound $A \sqcap B$ for an pair $(A,B)$ of conditions. - To get fully adequate means of expression, we would certainly need Boolean algebras of conditions rather than only lattices, giving us the possibility of dealing simultaneously both with a condition and its negation. We do not think this widening of the framework is very problematic. However, since it is not needed for expressing our ideas about intermediaries, we will stick here to the simpler framework of partial orderings and lattices.

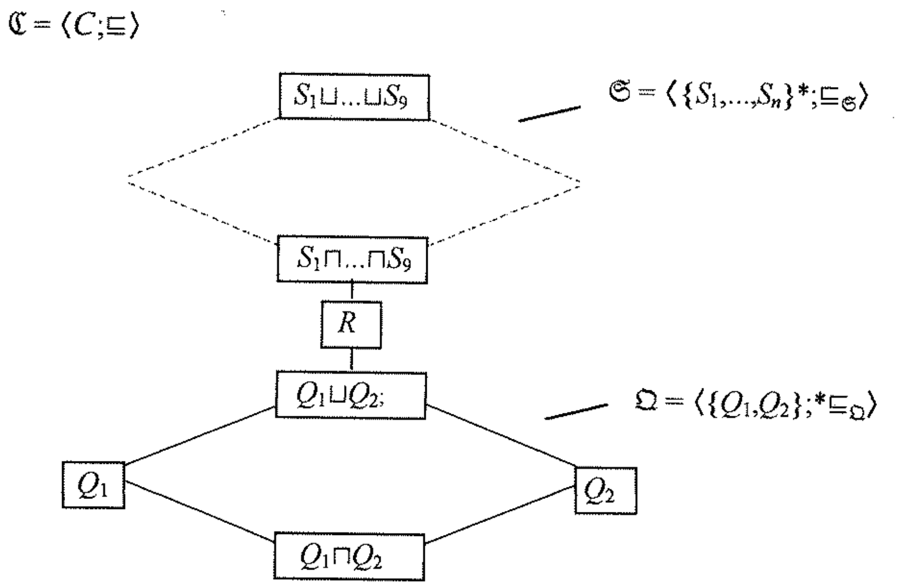
or, if
$y$ is under age ($P_1(y)$), and
$x$ has been appointed a guardian of $y$ ($P_7(x,y)$) and
$x$ has not been removed from being guardian of $y$ ($P_8(x,y)$),
then
$x$ is a guardian of $y$ ($R(x,y)$).

(II) If $x$ is a guardian of $y$ ($R(x,y)$),
then
$x$ shall administer the property of $y$ ($S_1(x,y)$), and
$x$ shall represent $y$ in any transaction which regards $y$'s property and where $x$ is not himself/herself a party ($S_2(x,y)$), and
$x$ shall represent $y$ in other matters when there is not, according to law, anyone else who shall represent $y$ ($S_3(x,y)$), and
$x$ shall always act so as to benefit $y$ in the best way ($S_4(x,y)$), and
$x$ shall see to it that money and securities belonging to $y$ are kept in such a way as not to be mixed up with assets otherwise taken care of by $x$ ($S_5(x,y)$), and
$x$ has a right to compensation for such expenses as are reasonably called for when $x$ acts as guardian for $y$ ($S_6(x,y)$), and
$x$ shall compensate $y$ for any injury that, intentionally or negligently, $x$ causes to $y$ ($S_7(x,y)$), and
if $y$ is sixteen years old or more, $x$ shall consult $y$ in any important matter which regards $y$ ($S_8(x,y)$), and
if $x$ is a guardian of $y$ in virtue of appointment, $x$ has a right to a reasonable remuneration for his/her charge as a guardian of $y$ ($S_9(x,y)$).

Define $Q_1$ as $P_1 \sqcap \ldots \sqcap P_6$. And define $Q_2$ as $P_1 \sqcap P_7 P_8$. Let $R$ stand for the condition 'To be a guardian of'. Then, according to the Swedish law, the following holds:

(1) $Q_1 \sqcup Q_2 \sqsubseteq R$.
(2) $R \sqsubseteq S_1 \sqcap \ldots \sqcap S_9$.

The following fragment of a picture of a lattice $\mathfrak{C} = \langle C, \sqsubseteq \rangle$, with $C = \{Q_1, Q_2, R, S_1, \ldots, S_9\}^*$ illustrates how the condition $R$ appears as a connecting link between the lattice $\mathfrak{Q} = \langle \{Q_1, Q_2\}^*; \sqsubseteq_{\mathfrak{Q}} \rangle$ and the lattice $\mathfrak{S} = \langle \{S_1, \ldots, S_9\}^*; \sqsubseteq_{\mathfrak{S}} \rangle$:




The statement that $\mathfrak{C} = \langle C;\sqsubseteq\rangle$ is a lattice means that, for any pair $(A,B)$ of conditions in $C$, the conditions $A\sqcap B$ and $A\sqcup B$ exist in $\mathfrak{C}$. (Since the lattice $\mathfrak{S}$ is not fully exhibited, some of the connecting lines are drawn in broken lines.) Moreover, for any pair $(A,B)$ of conditions in $C$ (or in $Q$, or in $S$), $A\sqsubseteq B$ means that $A\sqcap B \doteq A$.

As is seen from the picture, the two lattices $\mathfrak{Q}$ and $\mathfrak{S}$ are united by $Q_1\sqcup Q_2\sqsubseteq R\sqsubseteq S_1\sqcap\ldots\sqcap S_9$ where $Q_1\sqcup Q_2$ is the greatest element of the $\mathfrak{Q}$-lattice and $S_1\sqcap\ldots\sqcap S_9$ is the smallest element of the $\mathfrak{S}$-lattice. This is accomplished by enactments made by the legislator whose enactments correlate descriptive conditions with normative conditions. (A legislative enactment is to the effect that, for specific conditions $A,B$, it holds that $A\sqsubseteq B$.) Also, we see that $R$ appears as a link between, on one hand $Q_1\sqcup Q_2$, (the top of the $\mathfrak{Q}$-lattice), and $S_1\sqcap\ldots\sqcap S_9$ (the bottom of the $\mathfrak{S}$-lattice). It is appropriate to say that $R$ is *intermediary* between the two lattices.

*(ii) Formal development: joinings, connections, and couplings*

Suppose that $\mathfrak{Q} = \langle Q;\sqsubseteq_{\mathfrak{Q}}\rangle$ and $\mathfrak{S} = \langle S;\sqsubseteq_{\mathfrak{S}}\rangle$ are lattices of conditions where $Q=M^*$ and $S=N^*$ ($M$ and $N$ disjoint sets of simple conditions). We say that $\mathfrak{C} = \langle C;\sqsubseteq\rangle$ is a *joining of* $\mathfrak{S}$ *to* $\mathfrak{Q}$ if $\mathfrak{C}$ is a lattice such that the following three conditions are satisfied:

(i) $C = (M\cup N)^*$,

(ii) For all $Q_1,Q_2\in Q$, $Q_1\sqsubseteq_{\mathfrak{Q}}Q_2$ iff $Q_1\sqsubseteq Q_2$, and for all $S_1,S_2\in S$, $S_1\sqsubseteq_{\mathfrak{S}}S_2$ iff $S_1\sqsubseteq S_2$.

(iii) There are $Q \in \mathcal{Q}$ and $S \in \mathcal{S}$ such that $Q \sqsubseteq S$.

If $\mathfrak{C} = \langle C; \sqsubseteq \rangle$ is a joining of $\mathfrak{S}$ to $\mathfrak{Q}$ and there is $Q_1 \in \mathcal{Q}$, $S_1 \in \mathcal{S}$ such that $Q_1 \sqsubseteq S_1$ but there are no $Q_2 \in \mathcal{Q}$ and $S_2 \in \mathcal{S}$ such that $S_2 \sqsubseteq Q_2$, then we say that $\mathfrak{Q}$ is the *lower lattice* and $\mathfrak{S}$ is the *upper lattice* joined in $\mathfrak{C}$.

Suppose $\mathfrak{C} = \langle C, \sqsubseteq \rangle$ is a joining of the lattice $\mathfrak{S}$ to the lattice $\mathfrak{Q}$. We then say that $(Q_0,S_0)$ is a *connection* (in $\mathfrak{C}$) *from* $\mathfrak{Q}$ *to* $\mathfrak{S}$ if the following three conditions are satisfied:

(i) $Q_0 \in \mathcal{Q}$, $S_0 \in \mathcal{S}$ and $Q_0 \sqsubseteq S_0$.
(ii) If $Q \in \mathcal{Q}$ and $Q_0 \sqsubseteq Q \sqsubseteq S_0$ then $Q \doteq Q_0$.
(iii) If $S \in \mathcal{S}$ and $Q_0 \sqsubseteq S \sqsubseteq S_0$ then $S \doteq S_0$.

((ii)-(iii) are called the proximity principles.)

If for all $Q \in \mathcal{Q}$ and $S \in \mathcal{S}$ such that $Q \sqsubseteq S$ there is a connection $(Q_0,S_0)$ (in $\mathfrak{C}$) from $\mathfrak{Q}$ *to* $\mathfrak{S}$ such that $Q \sqsubseteq Q_0$ and $S_0 \sqsubseteq S$, we say that $\mathfrak{C}$ is *connected from* $\mathfrak{Q}$ *to* $\mathfrak{S}$.

We say further that $(Q_0,S_0)$ is a *coupling* (in $\mathfrak{C}$) of the lattice $\mathfrak{S}$ to the lattice $\mathfrak{Q}$ if $(Q_0,S_0)$ is a connection (in $\mathfrak{C}$) from $\mathfrak{Q}$ to $\mathfrak{S}$ and for all $Q \in \mathcal{Q}$ and $S \in \mathcal{S}$ the following holds: If $Q \sqsubseteq S$ then $Q \sqsubseteq Q_0$ and $S_0 \sqsubseteq S$. If $(Q_0,S_0)$ is a coupling such that $Q_0 \doteq S_0$, then we say that $(Q_0,S_0)$ is an *equivalence-coupling*. If there is a coupling (in $\mathfrak{C}$) from lattice $\mathfrak{Q}$ to $\mathfrak{S}$, then we say that $\mathfrak{C}$ *couples* $\mathfrak{S}$ *to* $\mathfrak{Q}$.

LEMMA 1: Suppose that $\mathfrak{C} = \langle C; \sqsubseteq \rangle$ is a joining of the upper lattice $\mathfrak{S}$ to the lower lattice $\mathfrak{Q}$, and that $\mathfrak{C}$ is finite. If $Q_1 \in \mathcal{Q}$, $S_1 \in \mathcal{S}$ and $Q_1 \sqsubseteq S_1$, then there is a connection $(Q_0,S_0)$ in $\mathfrak{C}$ from $\mathfrak{Q}$ to $\mathfrak{S}$ such that $Q_1 \sqsubseteq Q_0$ and $S_0 \sqsubseteq S_1$.
PROOF: Note that $Q_1 \in \{A \in \mathcal{Q} | A \sqsubseteq S_1\}$. Let $Q_0 = \mathrm{lub}\{A \in \mathcal{Q} | A \sqsubseteq S_1\}$. $S_1$ is an upper bound for $\{A \in \mathcal{Q} | A \sqsubseteq S_1\}$ and thus $Q_0 \sqsubseteq S_1$. Note that $S_1 \in \{A \in \mathcal{S} | Q_0 \sqsubseteq A\}$. Let $S_0 = \mathrm{glb}\{A \in \mathcal{S} \mid Q_0 \sqsubseteq A\}$. $Q_0$ is a lower bound for $\{A \in \mathcal{S} | Q_0 \sqsubseteq A\}$ and thus $Q_0 \sqsubseteq S_0$. Since $Q_0 \in \mathcal{Q}$ and $S_0 \in \mathcal{S}$, $(Q_0,S_0)$ satisfies condition (i) of a connection.

Suppose that $Q \in \mathcal{Q}$ and $Q_0 \sqsubseteq Q \sqsubseteq S_0$. Since $S_1 \in \{A \in \mathcal{S} | Q_0 \sqsubseteq A\}$ it follows that $S_0 \sqsubseteq S_1$. From $Q \sqsubseteq S_0$ and $S_0 \sqsubseteq S_1$ follows that $Q \sqsubseteq S_1$. Therefore $Q \in \{A \in \mathcal{Q} | A \sqsubseteq S_1\}$ and thus $Q \sqsubseteq Q_0$. Hence $Q \doteq Q_0$, which proves that $(Q_0,S_0)$ satisfies condition (ii) of a connection.

Suppose that $S \in \mathcal{S}$ and $Q_0 \sqsubseteq S \sqsubseteq S_0$. Thus $S \in \{A \in \mathcal{S} | Q_0 \sqsubseteq A\}$, which implies that $S_0 \sqsubseteq S$. From $S_0 \sqsubseteq S$ and $S \sqsubseteq S_0$ follows $S \doteq S_0$, which proves that $(Q_0,S_0)$ satisfies condition (iii) of a connection.

LEMMA 2: Suppose that $(Q_0,S_0)$ is a coupling in $\mathfrak{C}$ of $\mathfrak{S}$ to $\mathfrak{Q}$ and $(Q_1,S_1)$ a connection in $\mathfrak{C}$ from $\mathfrak{Q}$ to $\mathfrak{S}$. Then $(Q_0,S_0)=(Q_1,S_1)$.
PROOF: $Q_1 \sqsubseteq S_1$ and since $(Q_0,S_0)$ is a coupling it follows that $Q_1 \sqsubseteq Q_0$ and $S_0 \sqsubseteq S_1$. Since $Q_0 \sqsubseteq S_0$ we get $Q_1 \sqsubseteq Q_0 \sqsubseteq S_1$ and $Q_1 \sqsubseteq S_0 \sqsubseteq S_1$. $(Q_1,S_1)$ is a connection and

from condition (ii) and (iii) of a connection follows $Q_1 \hat{=} Q_0$ and $S_1 \hat{=} S_0$. Hence $(Q_0,S_0)=(Q_1,S_1)$.

COROLLARY 1: The coupling in $\mathfrak{C}$ of $\mathfrak{S}$ to $\mathfrak{Q}$ is unique.

COROLLARY 2: The coupling in $\mathfrak{C}$ of $\mathfrak{S}$ to $\mathfrak{Q}$ is the only connection in $\mathfrak{C}$ from $\mathfrak{Q}$ to $\mathfrak{S}$.

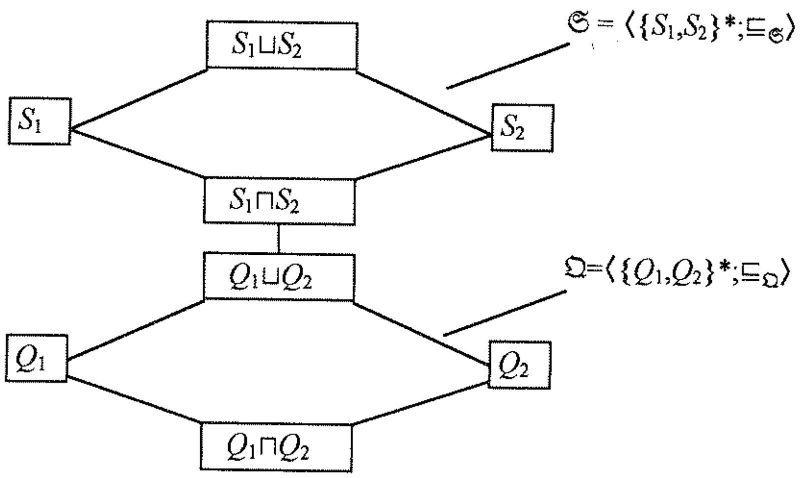
LEMMA 3: If $(Q_0,S_0)$ is the only connection between the lattices $\mathfrak{Q}$ and $\mathfrak{S}$ in the finite lattice $\mathfrak{C}$, then $(Q_0,S_0)$ is the coupling of $\mathfrak{Q}$ and $\mathfrak{S}$ in $\mathfrak{C}$.

PROOF: Suppose that $Q \in \mathcal{Q}$, $S \in \mathcal{S}$ and $Q \sqsubseteq S$. According to lemma 1 there is a connection $(Q_1,S_1)$ in $\mathfrak{C}$ from $\mathfrak{Q}$ to $\mathfrak{S}$ such that $Q \sqsubseteq Q_1$ and $S_1 \sqsubseteq S$. Since there is only one connection $(Q_0,S_0)$ in $\mathfrak{C}$ from $\mathfrak{Q}$ to $\mathfrak{S}$, $(Q_1,S_1)=(Q_0,S_0)$. Thus $Q \sqsubseteq Q_0$ and $S_0 \sqsubseteq S$. This shows that $(Q_0,S_0)$ is the coupling of $\mathfrak{S}$ to $\mathfrak{Q}$ in $\mathfrak{C}$.

*(iii) Illustrations of the notions of coupling and connection*

A few examples will help to elucidate the import of the defining requirements for connection and coupling, in particular Lemma 2. First, consider the lattice $\mathfrak{C} = \langle\{Q_1,Q_2,S_1,S_2\}^*;\sqsubseteq\rangle$ shown in part by the following picture:

$\mathfrak{C} = \langle\{Q_1,Q_2,S_1,S_2\}^*;\sqsubseteq\rangle$




There is only one connection from $\mathfrak{Q}$ to $\mathfrak{S}$, namely $\langle Q_1 \sqcup Q_2, S_1 \sqcap S_2\rangle$, and this is the (one and only) coupling of $\mathfrak{S}$ to $\mathfrak{Q}$.

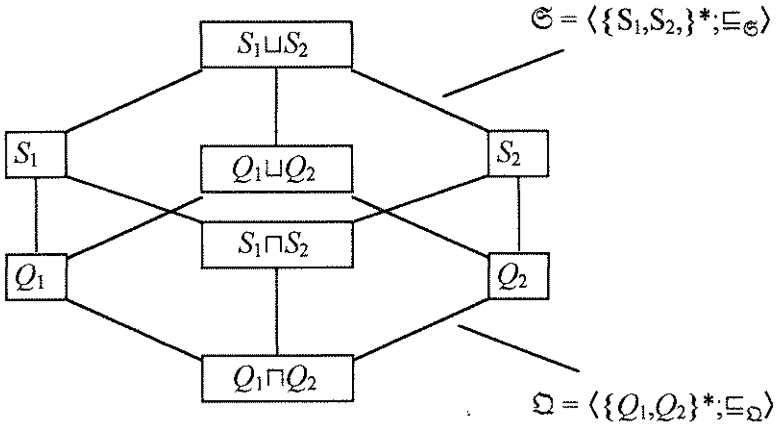
Examples of pairs that are not connections (and therefore, not couplings):

1. $\langle Q_1 \sqcap Q_2, S_1 \sqcap S_2\rangle$,
2. $\langle Q_1 \sqcup Q_2, S_1 \sqcup S_2\rangle$,
3. $\langle Q_1,S_1\rangle$.

In case 1, we have $Q_1 \sqcap Q_2 \sqsubseteq Q_1 \sqcup Q_2 \sqsubseteq S_1 \sqcap S_2$ but not $Q_1 \sqcap Q_2 \doteq Q_1 \sqcup Q_2$ (the requirement (ii) for a connection is not fulfilled). In case 2 we have $Q_1 \sqcup Q_2 \sqsubseteq S_1 \sqcap S_2 \sqsubseteq S_1 \sqcup S_2$ but not $S_1 \sqcap S_2 \doteq S_1 \sqcup S_2$ (requirement (iii) for a connection is not fulfilled). In case 3, we have $Q_1 \sqsubseteq Q_1 \sqcup Q_2 \sqsubseteq S_1$, but not $Q_1 \sqcup Q_2 \doteq Q_1$, and we have $Q_1 \sqsubseteq S_1 \sqcap S_2 \sqsubseteq S_1$ but not $S_1 \sqcap S_2 \doteq S_1$. (Neither requirement (ii) nor requirement (iii) for a connection is fulfilled.)

Next, consider the following lattice:

$\mathfrak{C} = \langle \{\boldsymbol{Q_1, Q_2, S_1, S_2}\}^*; \sqsubseteq \rangle$




In this case, there is no coupling of $\mathfrak{S}$ to $\mathfrak{Q}$. There are four connections, namely:

1. $\langle Q_1 \sqcap Q_2, S_1 \sqcap S_2 \rangle$,
2. $\langle Q_1, S_1 \rangle$,
3. $\langle Q_2, S_2 \rangle$,
4. $\langle Q_1 \sqcup Q_2, S_1 \sqcup S_2 \rangle$.

Neither of them, however, is a coupling. In case 1, for example, we have $Q_1 \sqsubseteq S_1$ but not $Q_1 \sqsubseteq Q_1 \sqcap Q_2$. In case 2 we have $Q_1 \sqcup Q_2 \sqsubseteq S_1 \sqcup S_2$ but not $Q_1 \sqcup Q_2 \sqsubseteq Q_1$. In case 3, once more we have $Q_1 \sqcup Q_2 \sqsubseteq S_1 \sqcup S_2$ but we do not have $Q_1 \sqcup Q_2 \sqsubseteq Q_2$. In case 4 we have $Q_1 \sqsubseteq S_1$ but not $S_1 \sqcup S_2 \sqsubseteq S_1$.

*(iv) 'Guardian' as intermediary*

We now introduce the important notion of an intermediary. Suppose that $\mathfrak{C}$ couples $\mathfrak{S}$ to $\mathfrak{Q}$ and $\mathfrak{Q}$ is the lower lattice and $\mathfrak{S}$ is the upper lattice joined in $\mathfrak{C}$. We extend $\mathfrak{C}$ with a condition $R$ representing the coupling $(Q_0,S_0)$ of $\mathfrak{S}$ to $\mathfrak{Q}$ in the sense that $R$ lies between $Q_0$ and $S_0$ (i.e. $Q_0 \sqsubseteq R \sqsubseteq S_0$) and the element in $\mathfrak{Q}$ which is nearest to $R$ is $Q_0$ and the element in $\mathfrak{S}$ which is nearest to $R$ is $S_0$. (If $(Q_0,S_0)$ is an equivalence-coupling then $Q_0 \doteq R \doteq S_0$.) More formally, let $\mathfrak{C} = \langle C, \sqsubseteq \rangle$, with $C = (M \cup N)^*$, be a lattice. Then, the extension of $\mathfrak{C}$ with $R$ is a lattice $\mathfrak{C}' = \langle C', \sqsubseteq' \rangle$, with $C' = (M \cup N \cup \{R\})^*$, $R \notin (M \cup N)^*$, which is as unrestricted as possible for $\mathfrak{C}'$ being a lattice, satisfying $\sqsubseteq'/C = \sqsubseteq$ and $Q_0 \sqsubseteq' R \sqsubseteq' S_0$. (This can be more carefully stated using the notion of a free lattice.) Note that the intermediary $R$ is a condition regulated by the requirement that $Q_0 \sqsubseteq' R \sqsubseteq' S_0$, which in a sense gives the meaning of $R$. A condition $R$ introduced in this way will be called an *intermediary* (in $\mathfrak{C}'$) between the lattices $\mathfrak{Q}$ and $\mathfrak{S}$.

In the example of guardianship, let $Q_0$ be $Q_1 \sqcup Q_2$ and let $S_0$ be $S_1 \sqcap ... \sqcap S_9$. Then, according to the definitions, the pair $(Q_0,S_0)$ is a coupling of $\mathfrak{S}$ to $\mathfrak{Q}$ in $\mathfrak{C}$.. Also, condition $R$ ("to be a guardian of") is intermediary in $\mathfrak{C}$ extended with $R$. (See the picture given above.) It should be emphasized, however, that what is shown is that $R$ is intermediary in relation to $\mathfrak{C}$ (extended with $R$) and the coupling of $(Q_0,S_0)$ in $\mathfrak{C}$. This is important, since a pair $(Q_0,S_0)$ which is a coupling in relation to one lattice $\mathfrak{C}$ can be a mere connection in relation to another lattice $\mathfrak{C}'$. This fact can be illustrated by enlarging our example of guardianship so as to take into account that, beside the condition of being a guardian, there is another legal condition called being an *administrator for* (Swedish *förvaltare*, German *Verwalter*).

If a person is not under age but lacks the physical and/or mental ability to administer his/her affairs, an administrator can be appointed. The legal consequences of $x$'s being an administrator for $y$ depend on what is prescribed when the appointment is made but can be largely similar to the consequences of $x$'s being a guardian of $y$.

With regard to the condition of "being an administrator for", a lattice $\mathfrak{C}'$ can be constructed as a coupling of an upper lattice $\mathfrak{S}'$ to a lower lattice $\mathfrak{Q}'$, in a way analogous to the way the lattice $\mathfrak{C}$ for guardianship was constructed as coupling lattice $\mathfrak{S}$ to lattice $\mathfrak{Q}$. In this lattice, the condition $R'$ of "being an administrator for" is the intermediary. However, it is possible as well to construct a larger lattice $\mathfrak{C}''$ where the lower lattice $\mathfrak{Q}''$ encompasses the grounds for guardianship as well as the grounds for administratorship, and where the upper lattice $\mathfrak{S}''$ encompasses the consequences of each of these positions. In this larger lattice $\mathfrak{C}''$, neither the condition of being a guardian nor the condition of being an administrator is an intermediary. This shows that a condition which is an intermediary in one lattice need not be so in another (larger) one. Thus, the example illustrates that the notion of an intermediary concept should be made relative to the coupling in a particular lattice. Therefore, it can be misleading to speak of "intermediary concepts" in general, as a special class of concepts.

*(v) The meaning of intermediary concepts*

In a rational reconstruction of Swedish law there are different ways of explaining the meaning of "guardian" ("$x$ is a guardian of $y$"). The most important of them are as follows. Firstly, a descriptively oriented definition can be aimed at, i.e. a definition in terms of conditioning facts. Suppose that $R \doteq Q_1 \sqcup Q_2$ , and that, moreover, $R$ is defined by $R =_{def.} Q_1 \sqcup Q_2$. Then, $R$ is a descriptive condition and $R \sqsubseteq S_1 \sqcap \ldots \sqcap S_9$ is a norm. Secondly, a normatively oriented definition can be aimed at, i.e., a definition in terms of legal consequences. Suppose that $R \doteq S_1 \sqcap \ldots \sqcap S_9$ and that $R$ is defined by $R =_{def.} S_1 \sqcap \ldots \sqcap S_9$ . Then, $R$ is a normative condition (expressed in terms of duties and rights) and $Q_1 \sqcup Q_2 \sqsubseteq R$ is a norm.[10] A third possibility, however, is an implicit definition, namely a statement that the meaning of $R$ is given by the conjunction of $Q_1 \sqcup Q_2 \sqsubseteq R$ and $R \sqsubseteq S_1 \sqcap \ldots \sqcap S_9$.[11] (It is presupposed that for each $Q_i$ and $S_j$ the meaning of $Q_i$ and $S_j$ is given independently of $R$.) The last kind of definition is supported by the fact that a concept such as 'guardian' is not understood by someone who does not know both conceptual structures, the one relating to grounds and the one relating to consequences. Consider the two inferences:

| | | | |
|---|---|---|---|
| I. | $Q_i\,(a,b)$ | II. | $R(a,b)$ |
| | $Q_i \sqsubseteq R$ | | $R \sqsubseteq S_j$ |
| | -------- | | ------- |
| | $R(a,b)$ | | $S_j\,(a,b)$ |

Inferences of these two kinds are purposeful only in combination. On one hand, the first premise in inference II is verified by an inference of the kind I. On the other hand, an inference of the kind I serves no purpose if it is not followed up by an inference of the kind II.

[10] These two alternatives correspond to Wedberg's first and second method for treating the expression "O is the property of P at t". See Wedberg (1951), pp. 266 ff., and cf. Lindahl (1996).

[11] This third alternative resembles Wedberg's third method for introducing "O is the property of P at t" insofar as it emphasizes the use of $R$ as a vehicle of inference. However, it differs from Wedberg's third method by not implying that "X is the guardian of Y" is a meaningless expression. As regards "O is the property of P at t" Wedberg states that his third method implies "regarding this basic expression as a »meaningless» linguistic vehicle of inference", see Wedberg (1951), p. 273.

Combining the second premises of the two inferences, i.e., combining $Q_i \sqsubseteq R$ and $R \sqsubseteq S_j$, the norm $Q_i \sqsubseteq S_j$ can be inferred. Therefore, if the premises of both inferences are maintained, this norm is implicitly maintained as well. This shows that when an intermediary concept such as "guardian" is purposefully used in inferences, a norm is implicitly maintained. Thus the use of $R$ presupposes the norm $Q_1 \sqcup Q_2 \sqsubseteq S_1 \sqcap \ldots \sqcap S_9$ (or part of it) as its background.[12]

*4. Sets of couplings satisfying a constraint*

*i) "A relationship similar to being married": Introduction of the example*

In some cases it is not possible to specify the coupling of $\mathfrak{S}$ to $\mathfrak{Q}$ completely. In this section we will suggest that it is possible to characterize it as a member of a set of "feasible" couplings. We will illustrate these cases by an example concerning what, in Swedish law, is called *having a relationship similar to being married* (Swedish *äktenskapsliknande förhållande*).

If two persons are not married, nevertheless they can have a relationship similar to being married. From such a condition particular legal consequences follow. First, if the relationship is dissolved, property acquired by one of the parties for use in common shall be partitioned between the parties according to rules similar to those applied when a marriage is dissolved. Secondly, if the relationship of the parties is dissolved, their dwelling can be allotted to that party who needs it most.

The law does not specify what is meant by a "relationship similar to being married". However, there is a number of criteria, in particular the following conditions:

1. cohabiting,
2. housekeeping in common,
3. having children in common
4. having sexual intercourse,
5. having confirmed the relation by a contract,
6. living in emotional community
7. being faithful,
8. giving mutual support,
9. sharing economic assets and debts,
10. having no legal impediments to marriage,
11. having no similar relationship to another person.

[12] Cf Hare (1989), p. 141.

If all of these criteria are satisfied by persons $a$ and $b$, their relationship is "similar to being married". Conversely, if none of them is satisfied, their relationship is not "similar to being married". However, the law does not tell what is the result if some of the conditions are satisfied while others are not. This means that, in a sense, the set of grounds for having a relationship similar to being married is "open", and it is not possible to specify the grounds completely. Therefore, in this case, neither is it possible to specify completely a coupling of the upper lattice $\mathfrak{S}$ of legal consequences to a lower lattice $\mathfrak{Q}$ of grounds.

*(ii) The closeness of a coupling*

In the case of relationship similar to being married, it may be possible to characterize a coupling $(Q_0,S_0)$ as a member of a set of "feasible" couplings. When doing this, a useful tool is the notion *closeness of a coupling*. This notion can be explained as follows.

Suppose that $\mathfrak{C}=\langle C,\sqsubseteq\rangle$ couples $\mathfrak{S}$ to $\mathfrak{Q}$ with $(Q_0,S_0)$ as the coupling and that $\mathfrak{C}'=\langle C,\sqsubseteq'\rangle$ couples $\mathfrak{S}$ to $\mathfrak{Q}$ with $(Q'_0,S'_0)$ as the coupling. We say that the coupling $(Q_0,S_0)$ (in $\mathfrak{C}$) is *at least as close as* the coupling $(Q'_0,S'_0)$ (in $\mathfrak{C}'$) if $Q'_0\sqsubseteq_{\mathfrak{Q}}Q_0$ and $S_0\sqsubseteq_{\mathfrak{S}}S'_0$. We say further that $(Q_0,S_0)$ is *closer than* $(Q'_0,S'_0)$ if $Q_0,S_0)$ is at leas(t as close as $(Q'_0,S'_0)$ and it holds that $Q'_0\sqsubset_{\mathfrak{Q}}Q_0$ or $S_0\sqsubset_{\mathfrak{S}}S'_0$.

LEMMA 4: Suppose that $\mathfrak{C}=\langle C,\sqsubseteq\rangle$ couples $\mathfrak{S}$ to $\mathfrak{Q}$ with $(Q_0,S_0)$ as the coupling and $\mathfrak{C}'=\langle C,\sqsubseteq'\rangle$ couples $\mathfrak{S}$ to $\mathfrak{Q}$ with $(Q'_0,S'_0)$ as the coupling. Suppose further that the coupling $(Q_0,S_0)$ is at least as close as the coupling $(Q'_0,S'_0)$. Then the following holds: For all $Q\in\mathcal{Q}$ and for all $S\in\mathcal{S}$, $Q\sqsubseteq' S$ implies $Q\sqsubseteq S$.

PROOF: Suppose that $Q\in\mathcal{Q}$, $S\in\mathcal{S}$, and that $Q\sqsubseteq' S$. Since $(Q'_0,S'_0)$ is the coupling in $\mathfrak{C}'$ it follows that $Q\sqsubseteq' Q'_0$ and $S'_0\sqsubseteq' S$. Hence, $Q\sqsubseteq_{\mathfrak{Q}}Q'_0$ and $S'_0\sqsubseteq_{\mathfrak{S}}S$. Since $(Q_0,S_0)$ is at least as close as the coupling $(Q'_0,S'_0)$ it holds that $Q'_0\sqsubseteq_{\mathfrak{Q}}Q_0$ and $S_0\sqsubseteq_{\mathfrak{S}}S'_0$. From this together with $Q\sqsubseteq_{\mathfrak{Q}}Q'_0$ and $S'_0\sqsubseteq_{\mathfrak{S}}S$ follows that $Q\sqsubseteq_{\mathfrak{Q}}Q_0$ and $S_0\sqsubseteq_{\mathfrak{S}}S$, which implies $Q\sqsubseteq Q_0$ and $S_0\sqsubseteq S$. Since $(Q_0,S_0)$ is the coupling in $\mathfrak{C}$ it holds that $Q_0\sqsubseteq S_0$. Therefore, $Q\sqsubseteq S$.

*(iii) Couplings satisfying a constraint*

The legislator may require that the coupling of $\mathfrak{S}$ to $\mathfrak{Q}$ must be at least as close as a particular coupling, call it $(Q^{**},S^{**})$. All couplings not fulfilling this requirement are regarded as contradicting the law, while the status of those couplings fulfilling the requirement is left undecided.

Let $K$ be the class of couplings that are at least as close as $(Q^{**},S^{**})$. For any coupling $C$ in class $K$, we can use the same linguistic expression "$R$" to designate the intermediary obtained for $C$ by extending $\mathfrak{Q}$. If so, however, the expression

"$R$" does not any longer represent a particular coupling $(Q_0,S_0)$ but every member of the class $K$ of couplings which are at least as close as $(Q^{**},S^{**})$. Thus, $R$ can be used for extending different lattices without violating the rules determining $R$.

Let us simplify the situation and consider an intermediary $R$ for which two requirements $D_1$ and $D_2$ are specified by the law. Let $Q_1$ be the condition that both $D_1$ and $D_2$ are fulfilled, $Q_2$ the condition that $D_1$ but not $D_2$ is fulfilled, and $Q_3$ the condition that $D_2$ is fulfilled but not $D_1$. Thus, we have a lattice $\mathfrak{Q}=\langle \mathcal{Q},\sqsubseteq_{\mathfrak{Q}}\rangle$ where $\mathcal{Q}=\{Q_1,Q_2,Q_3\}$. We suppose that a lattice $\mathfrak{S}=\langle S,\sqsubseteq_{\mathfrak{S}}\rangle$ of normative conditions will be coupled to $\mathfrak{Q}$. Furthermore, let us suppose that $S$ is a normative condition in $S$, and that the legal consequences of $R$ are well specified in the law by the sentence $R \sqsubseteq S$. On the other hand, we suppose that the legal grounds for $R$ are not as clearly stated. In particular, let us suppose that it is uncontroversial that $Q_1 \sqsubseteq R$ but that there exist different opinions among experts as to whether $Q_2 \sqsubseteq R$ and whether $Q_3 \sqsubseteq R$.

Thus, there is a dispute about how to couple $\mathfrak{S}$ to $\mathfrak{Q}$ and we can distinguish between four different theories:

$T_1$ : the coupling is $(Q_1,S)$
$T_2$ : the coupling is $(Q_1 \sqcup Q_2, S)$
$T_3$ : the coupling is $(Q_1 \sqcup Q_3, S)$
$T_4$ : the coupling is $(Q_1 \sqcup Q_2 \sqcup Q_3, S)$

Note that the coupling in $T_4$ is closer than the coupling in $T_3$ and in $T_2$, and both the last mentioned are closer than the coupling in $T_1$.

As is seen, there will be a class of four possible couplings not contradicting the law and we denote them $C_1$, $C_2$, $C_3$ and $C_4$ respectively. In all four systems generated by these couplings, the expression $R$ can be used to represent the couplings. In a sense, the minimal meaning content of $R$ is determined by the least close coupling, that is $C_1$. Therefore, $T_1$ has a special status among the four theories by restricting the grounds for $R$ to what follows from a minimal meaning content. $T_2$, $T_3$ and $T_4$ are less restrictive by adding further grounds for $R$.

In the example of relationship similar to being married, the situation is much more complicated than in the simplified case just described. (With eleven criteria, we get $2^{11}$ possible combinations.) Nevertheless, the general features of a theory for this case is of the same character. In practice, there will exist a consensus on various combinations of criteria as constituting grounds for the relationship in view. Therefore, the number of plausible theories is considerably reduced.

*5. Conclusion*

We end this presentation by suggesting, though very briefly, that the idea of intermediary concepts as connecting two different conceptual structures seems to be of much wider interest than we have said so far.

The typical situation in which an intermediary occurs is in connection with two phenomena described by two different conceptual structures. This feature is perspicuous within the law, where usually there is one descriptive structure and one normative structure. But a difference between two conceptual structures can be present even if both structures are descriptive, or empirical, as was suggested in the wousin example, presented at the beginning of our paper. The conceptual structure for observations about black or white men climbing over rail-fences or walls at full moon is different from the conceptual structure for changes in the money market (for example, changes such as a rising of the rate of interest).

In many contexts outside the law, the different structures to which two phenomena belong are specified only vaguely. In such contexts the characterization of grounds and consequences can seldom be made quite specific. Two examples may be suggested. The first concerns probability. With regard to a statement that the probability of the event $A$ equals $r$, there is, on one hand, a conceptual structure concerning frequencies and symmetries and, on the other hand, a conceptual structure concerning how one ought to choose between different games.[13] The second example concerns illnesses, for example rheumatism. One hand we have a structure dealing with the symptoms and etiology of rheumatism, on the other hand there is a structure dealing with how rheumatism should be treated. These examples might reinforce our suggestion that an intermediary concept is not properly understood if not both of the different conceptual structures are taken into account.

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[13] Probability as a concept with grounds and consequences is discussed in Odelstad (1989).

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