Designing of High Reflectance Distributed Bragg reflectors (DBRs), mirrors using AlGaInN material system in the UV wavelength range

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Abstract

Gallium Nitride (GaN) and its alloys with aluminum (Al$_x$Ga$_{1-x}$N) and indium (In$_y$Ga$_{1-y}$N & Al$_z$In$_{1-z}$N) have gained lots of attention in optoelectronics research community in the last decade. These semiconductors relativity have a wide bandgap and easy to make n-type layers by Si doping and p-type layers by Mg doping. Consequently nitride alloys have proved to be very attractive candidate for the fabrication of Distributed Bragg reflectors (DBRs) for VCSEL including other optoelectronic devices.

Distributed Bragg reflectors have prime importance in the performance of VCSEL. DBRs not only provide highest possible reflectivity, usually 99.9% is required, but also conduct electricity, confine current, to the gain region. That’s why accurate modeling and simulation of DBRs have significant importance for the future fabrication of VCSEL.

Matrix transmission method is employed to model and simulate the nitride based DBRs. As refractive index and absorption coefficient is key parameters to design optoelectronic devices and necessary to model the DBRs are modeled. Refractive index is modeled in this thesis work by Adachi’s model and Sellmeier model.

In first part of thesis, matrix transmission based DBR model is implemented on Al$_x$Ga$_{1-x}$N based DBRs. Refractive index is modeled by Adachi’s model and Sellmeier model. As Adachi’s model is widely considered most accurate model so it is used to calculate the refractive index over the entire range mole fraction of Al (Aluminum). Absorption is also modeled. Reflectivity spectrum is plotted by AlGaN/AlGaN, AlGaN/GaN, AlN/GaN and AlGaN/AlN based DBRs. Phase response and transmissivity also are plotted. The bandwidth is an important parameter in nitride alloys. So how bandwidth is influenced by number of parameters like reflectivity, number of periods and Bragg wavelength are analyzed. The bandwidth with and without dispersion is plotted. Two design curves are plotted that are very useful to practical realization of DBRs. These curves give us minimum operating wavelength and minimum Al content in 2nd layers over the number of periods.

In second part of thesis, In$_y$Ga$_{1-y}$N alloy is investigated and try to find hurdles to calculate accurate optical parameters e.g. energy bandgap, refractive index and absorption coefficient. Stress free InGaN/AlGaN DBRs are also modeled and their reflection spectrum is plotted.

In third and last part of thesis, Al$_z$Ga$_{1-z}$N alloy is investigated and discussed obstacles to calculate its accurate optical parameters. AlGaN/GaN lattice matched DBRs are also discussed with details and reflection spectrum is plotted.

In conclusion, we demonstrated which nitride alloy is most promising for designing and fabrication of DBRs in this date and which one could be in future.
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<td>AlGaN</td>
<td>Aluminum Gallium Nitride</td>
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<tr>
<td>AlInN</td>
<td>Aluminum Indium Nitride</td>
</tr>
<tr>
<td>AIN</td>
<td>Aluminum Nitride</td>
</tr>
<tr>
<td>BJT</td>
<td>Buried Tunnel Junction</td>
</tr>
<tr>
<td>DBRs</td>
<td>Distributed Bragg Reflectors</td>
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<tr>
<td>EELs</td>
<td>Edge Emitting Lasers</td>
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<tr>
<td>FWHM</td>
<td>Full width at half maximum</td>
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<tr>
<td>GaN</td>
<td>Gallium Nitride</td>
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<tr>
<td>HEMT</td>
<td>High Electron Mobility Transistor</td>
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<tr>
<td>InN</td>
<td>Indium Nitride</td>
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<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
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<tr>
<td>MBE</td>
<td>Molecular Beam Epitaxial</td>
</tr>
<tr>
<td>MOVPE</td>
<td>Metal-organic Vapor Phase Epitaxial</td>
</tr>
<tr>
<td>MOVCD</td>
<td>Metal-organic Chemical-vapor Decomposition</td>
</tr>
<tr>
<td>QW</td>
<td>Quantum wall</td>
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<tr>
<td>RT</td>
<td>Room temperature</td>
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<tr>
<td>UV</td>
<td>Ultra violet</td>
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<tr>
<td>VCSELs</td>
<td>Vertical Cavity Surface Emitting Lasers</td>
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1 Introduction

1.1 III-nitrides: State of art

The III-V nitride semiconductor material systems that we have used in this project are aluminum gallium nitride (AlGaN), indium gallium nitride (InGaN) and aluminum gallium nitride (AlInN). So we would like to present a brief overview of fundamentals of these nitride materials [1].

All these nitride materials have direct bandgap energy, aluminum gallium nitride (Al$_x$Ga$_{1-x}$N) alloy has bandgap energy ranging from 3.42eV for gallium nitride (GaN), to 6.28eV for aluminum nitride (AlN), indium gallium nitride (In$_y$Ga$_{1-y}$N) alloy has bandgap energy ranging from 1.99eV for indium nitride (InN) to 3.42eV for gallium nitride (GaN) and aluminum indium nitride (Al$_z$In$_{1-z}$N) alloy has bandgap energy ranging from 1.99eV for indium nitride (InN) to 6.28eV for aluminum nitride (AlN). So in this consequence, they are choice of materials for the visible light emitter applications (Light emitting diodes or lasers) or photodetectors. They are also very suitable candidate for fabrication of high power and high frequency electrical devices because of its robustness nature [1].

Until now several devices has been demonstrated and some of them are playing a vital role in industrial’s applications. In 2004, gallium nitride (GaN) material of worth $3.2 billion was dominated by light emitting diode (LED) sale. InGaN-based LEDs are fabricated with operating range from UV (340nm) to the amber (600nm). White light emitting diodes (LEDs) are produced using a phosphorus cap on UV LEDs as shown in fig. 1. These light emitting diodes (LEDs) could be used in traffic signals, outdoor displays, sign boards and automotive lighting. Other applications of light emitting diodes (LEDs) are possible in data communication field, in medical field, in bio photonics field, and in sensing tech etc... The current LED market is running by several enterprises including Nichia, Lumiled, Cree, Osram, or Toyoda Gosei and we expect in coming years GaN market including LEDs and non LEDs commercial devices will grow faster and it could be hit to worth of $7.2 billion in 2009 [1].
With the development of non-LEDs commercial applications i.e. blue-violet LEDs for the next generation HD-DVD recorder and players, High Electron Mobility Transistor (HEMTs) is used in power amplifiers in base stations for mobile networks, and power switches is used in high power supplies and high power rectifiers, etc… In addition, the development of GaN based UV LEDs could be attractive for applications such as water purification and UV curing [1].

Despite, advancement in fabrication technologies and in research on nitride materials, we are still facing lots of hindrance like, technical limitations, Poor material quality and device life time [2]. Compared to GaAs/AlAs system, nitride semiconductors suffer from large lattice-matching and absence of suitable substrate which determines the growth of high quality thick layers. The built-in-strain in nitride vertical structure leads to the formation for cracks and dislocations that directly affect the performance of devices. Consequently, fabrication of nitride based devices is limited to a certain thickness level, called “critical thickness”.

Figure 1: InGaN based blue, white, red and green LEDs [1]
Because of these issues, the designing of structures such as distributed Bragg reflectors (DBRs) and vertical cavity surface emitting lasers (VCSELs) require much attention [1].

1.2 VCSELs-Review

The vertical-cavity surface emitting laser (VCSEL) is a type of semiconductor laser device emits laser perpendicular to the chip surface contrary to its counterpart edge emitting semiconductor lasers (EELs) [3].

The vertical-cavity surface emitting laser (VCSEL) is a relatively new semiconductor device among the other optoelectronics devices. The first vertical- emitting laser (VCSEL) was design and fabricated in 1979 by K. Iga [3] [4] at Tokyo Institute of Technology, Japan [5]. Soda et al. [2] had also suggested this revolutionary approach at the same year [2]. They proposed in order to realize the low threshold current VCSEL, VCSELs must have (1) extremely small active region (2) high optical gain and (3) high reflectivity mirrors at both ends of active region. Despite many technical difficulties and materials limitations, they demonstrated first electrically pump InGaAsP/InP VCSEL successfully at 77K in 1979 [5]. In beginning, devices were made with metallic mirrors and they had higher threshold current (900mA) at liquid nitrogen temperature (77k) under pulsed conditions. The improvements of VCSELs in the GaInAsP/InP and AlGaAs/GaAs material systems had been slow in next decade. The most of advancements in VCSELs are associated with the advances in epitaxial growth techniques of Molecular-Beam Epitaxial (MBE) and Metal-Organic Vapor Phase Epitaxial (MOVPE). These techniques amazingly improved the characteristics of the mirrors and they lead the introduction of the DBRs, providing highest possible reflectivity and lowest absorption that enable the VCSELs to operate at lower threshold current on room temperature (RT). The first RT pulse operated VCSEL with GaAs active region was demonstrated in 1984. Early surface emitting laser devices had higher threshold current compared to conventional stripe lasers due to poor reflectivity and short gain path that was main hindrance for room temperature (RT) CW operation until 1988 but now after some progress has enabled to reduce the RT CW threshold current from 32 to $10^{-3}$mA. Even VCSELs with Al oxide confinement have lower threshold current and high frequency modulation [3].

The local area networks which are used for short distance data communication have opened the door for development of 850nm VCSELs. While for larger distance communication 1.3/1.55μm operating wavelength VCSELs with larger bandwidth is also attractive in telecommunication field. InP based quantum well (QW) is one of possible solution for 1.3/1.55μm operating wavelength but InP is not suitable material system for VCSEL applications because InP based Distributed Bragg Reflectors (DBRs) suffer from a limited
refractive index contrast and poor thermal conductivity as well as conduction band offset with InGaAsP quantum walls (QWs) leads to a poor high temperature performance [3].

High performance long wavelength VCSEL is fabricated with InP-based quantum wall (QW) and AlGaAs/GaAs DBRs using wafer bonding or using buried tunnel junction and flip-chip bond that is a solution for efficient heat dissipation but because its fabrication procedure is complex so it is not suitable for large scale production [3].

Long-wavelength VCSELs are ideally fabricated on GaAs substrate because of low cost and availability of well-established technology for GaAs substrate. 1.3μm VCSELs based on GaInNAs material system fabricated on GaAs substrate was first proposed but its main drawback is difficult growth of GaInNAs. Other 1.3μm InAs/InGaAs quantum dot VCSELs fabricated on GaAs also reported having best performance [3].

Additionally, 1.3μm wavelength VCSELs with AlGaAsSb/AlAsSb based DBRs [6], 1.5μm and 1.55μm wavelength VCSELs with GaAsSb/AlAsSb based DBRs with lattice matched InP [7], [8]. 2-3μm wavelength VCSELs with GaSb/GaInAsSb based DBRs and with a GaInAsSb/AlGaAsSb multi quantum wells active region and an InAsSb/GaSb tunnel junction have been reported [9], [10].

1.3 VCSEL Structure

Typically, VCSEL structure can be divided into three parts: the upper Distributed Bragg Reflectors (DBRs), the bottom Distributed Bragg Reflectors (DBRs) and central quantum-well active region as shown in fig. 2. DBRs those consist of repeated pairs of quarter-wave thick layers having high and low refractive index are called quarter-wave stack or Bragg stack. DBRs are also sometimes referred as a bilayer stack or binary stack [11]. By combining the repeated pairs of high and low refractive index layers will result in a maximum reflectance greater than 99.9%, depending on number of pairs and refractive index contrast and some other process parameters [12]. Multiple interference effects at each layer of DRBs are also shown in fig. 2 lead to reflectivity more than 99.9% that is essential for VCSEL operation to compensate the short active region and to obtain lowest threshold current [44]. Many complex VCSEL structures also have been demonstrated by many research groups [44], [45], [20] having application specific advantages.
Distributed Bragg reflectors are fabricated by either monolithically grown semiconductor or by other dielectric materials, such as hybrid DBR mirrors. The reason to use dielectric DBRs is to achieve a large refractive index contrast that provides highest reflectance using minimum number of periods which is a primary requirement of DBRs [12] and one of unique attraction to use dielectric mirror is the use of a buried tunnel junction (BTJ) provides both electrical and optical confinement [13]. We don't prefer to use pure metal mirrors because of its higher absorptive nature [1]. Monolithic semiconductor DBRs have a limited refractive index contrast result in a large number of periods for higher reflectance and allow current to be conducted to active region. Nevertheless, we would like to use semiconductor based DBRs with large refractive index contrast and be transparent for the laser light [12].
1.4 Epitaxial Growth of VCSEL

Epitaxy is a technology related to the growth of single-crystal semiconductor layers on a single-crystal semiconductor substrate. Epitaxy word is taken by Greeks words epi (meaning “on”) and taxis (meaning “arrangement”). Epitaxy can be divided into two categories on the basis on material using for epitaxial layers and for substrate. If epitaxial layer and the substrate are of same material then it is called homoepitaxial and if epitaxial layer and substrate are of different materials, chemically and often crystallographically different, is called heteroepitaxy [14].

By epitaxy, entire VCSEL structure including mirrors and active region and conduction layers is grown. Once VCSEL epitaxial structure is ready, it is quite easy to fabricate working VCSEL as compared to its counterpart conventional edge-emitting laser (EEL) [14].

There are two dissimilar epitaxial techniques Molecular Beam Epitaxy (MBE) and Metal-organic Vapor-Phase Epitaxy (MOVPE) [14]. In MBE technique, the epitaxial layer is grown by the attraction of one or several molecular or atomic beams that occur on a surface of a heated crystalline substrate [15]. Whereas, in Metal-organic Vapor-Phase Epitaxy (MOVPE) technique layers are grown from chemical semiconductor compounds transported to a crystalline heated substrate in a vapor stream [14].

Initially, VCSELs are grown by Molecular Beam Epitaxy (MBE) because of its superior precision and control compared to other common compound semiconductor-epitaxy techniques, such as liquid-phase epitaxy and hybrid vapor-phase epitaxy. MBE was considered most successful technique till at the end of the 80’s. Since that MOVPE is proved to be better technique for producing laser diodes and light-emitting diodes. The main reasons to prefer MOVPE over MBE is advancements of engineering and components of MOVPE technology and a comprehensive monitoring and control system for growth. In beginning, VCSEL used to suffer from high series resistances at each interface of different layers in DBR stack but in MOVPE, the alloys composition is continuously graded at interfaces of layers that considerably reduce the series resistance that improve the VCSEL performance [14].

With further technical improvements finally the first state-of-art VCSEL grown by MOVPE in 1993. The VCSEL operating at 980nm wavelength was a truly breakthrough in VCSEL history grown by MOVPE. This VCSEL used to operate at lowest voltage and with highest power conversion efficiency reported at that date [14].

Nowadays, VCSELs operating range from 500nm to 1100nm and beyond is grown by MOVPE holding many performance benchmarks. Now it should not be doubt that currently MOVPE technique is a first choice [14].
1.5 VCSEL Fabrication

Since the invention of VCSEL, several VCSEL structures have been proposed and investigated. All these structures have some pros and cons but optimum device geometry is chosen depending on material system and the purpose and application of device. However for the realization of VCSEL, we must ensure the current confinement i.e. the electrical pump area must be defined suitably and moreover the optical confinement must be provided. We take all these issues into account to fabricate the VCSEL [16]. Here we will discuss three methods briefly, namely Etched air-post VCSELs, Ion-Implanted VCSELs and Selectivity Oxidized VCSELs.

1.5.1 Etched air-post VCSELs

The air-post VCSEL structure affects the several electrical and optical properties. The first monolithic VCSEL were etched air-post [12]. The fabrication of VCSEL by air-post has many advantages. First, we can define a pillar with small diameter that allows the fabrication of high densely ultra low threshold current VCSELs. For instance millions of VCSEL with diameter as small as 1.5μm with submilli ampere threshold current have been reported. Secondly, due to index guiding presence in air-post VCSELs (Because of air/semiconductor interface surrounding the laser cavity), VCSEL with diameter <5μm emits single mode laser while VCSEL with diameter ≥10μm emits several transverse optical mode typically separated by 0.1nm. Anisotropic geometry which can be implemented easily in air-post VCSEL induced index guiding enable the efficient control of polarization [17]. VCSEL can be fabricated in its simplest form by a mesa structure. The etched pillar or post not only provides wave guiding but also provides efficient current confinement. Both top and bottom geometry is possible depending on substrate’s material, if substrate is transparent then a bottom emitting design is possible [12], bottom VCSEL’s geometry and top VCSEL’s geometry are shown in fig. 3 and fig. 4 respectively.

Selection of top and bottom emission VCSEL’s geometry is basically applications specific. The top emission geometry may be suitable when devices are desirable to use in arrays form to be mounted in conventional headers that can be used in Very Large Scale Integration (VLSI) chips and the bottom emitting geometry can be used for solder bump mounting of arrays on the driving circuitry as well as soldering upside-down is useful for heat sinking without thinning the substrate [12].

There are two drawbacks in bottom emitting geometry, first the requirement of windowed low electrode for light and second the reflection from an uncoated substrate-air interface introduces instability [12].

We also take into account the doping of upper and bottom DBRs for the emission of laser light. We most of the time prefer to take out light through the N-doped mirror by a window or a grid for the out-coupling of the laser beam because it commonly affect the homogeneity of injected current (Electrons) due to their high mobility and we prefer to conduct current to
active region through P-doped mirror because the current can be injected homogeneously through the P-doped mirror to active region. However if we take out light through P-doped mirror then in most cases it leads to inhomogeneous current injection due to much low mobility of the holes [16].

In summary, etched air-post VCSEL have many advantages such as, ease of fabrication, strong index guiding, efficient control on polarization and low threshold current. The drawbacks of etched air-post VCSEL are large non-radiative recombination, highest diffraction and scattering losses and inefficient thermal dissipation leading to high thermal impedance [12], [16], [17].

![Figure 3: Bottom geometry of VCSEL](image-url)
1.5.2 Ion-planted VCSELs

In this method, ion-implantation is used to obtain electrical confinement. Commonly, implementation of ions is performed into the top mirror to spread the material around the laser cavity non-conductive and thus by selectively insulating the region, current is injected efficiently to active region. Usually, positive ions such as O\(^+\), N\(^+\), F\(^+\) and H\(^+\) can be used for ion-implantation while the mass and depth of ion implantation determine the required implantation energy [12].

The fig. 5 shows the Ion-implanted VCSEL topology [12]. Despite ion implantation damages the crystal, ion implanted VCSEL has demonstrated good reliability. However possibility to damage the active region by implantation of ion and undefined ion implantation boundary due to ion’s scattering arise some precautions like type of ions used and high precision to define apertures and in other draw backs include lack of index guiding and lack of control on polarization [12].

**Figure 4:** Top geometry mesa of VCSEL [18]
1.5.3 Selectively Oxidized VCSELs
A recent and most efficient fabrication technology until now is selectively oxidized VCSEL that provides excellent transverse optical and electrical confinement by employing selective oxidation of buried AlGaAs layers. The electrically insulating low refractive index layers define the cavity region and proved effective confinement of electrons and holes inside the laser cavity. Oxidized AlGaAs layers have been employed with dielectric Distributed Bragg Reflectors (DBRs) and selectively oxidized DBRs have been used with monolithic semiconductor DBRs as shown in fig. 6 [12].

Selectively oxidized VCSELs gave many advantages over air-post and ion-implanted structures including best index guiding, good heat dissipation etc. Selectively oxidized VCSELs operating range from 604nm to 1550nm have been demonstrated with promising reliability [12].

As low threshold current, low threshold voltage and efficient heat dissipation is primary focus in the development of VCSELs due to their same size of active region. The selectively oxidized VCSEL has been reported with low threshold current <10μA and low threshold voltage <1.33V that is lower than using same wafer for ion-implanted VCSEL and air-post VCSEL [12].

---

**Figure 5:** Ion-Implanted VCSEL [18]
Since until now, selectively oxidized VCSEL have superior performance among above discussed fabrication methods and it has promising feature for commercial applications. But still need to work more on reliability issues [12].

\[ \text{Figure 6: Selectively Oxidized VCSEL [18]} \]

**1.2 VCSEL’s applications**

The first VCSEL commercial applications was a need of promising optical data links which required a stable laser output, low threshold current, circular output beam for efficient coupling to optical fibre and low-cost sources [12]. Consequently, nowadays, 850nm VCSEL is completely dominated in market with high bit-rate for short distance data communication [19]. In addition, compact disc, printing heads, optical sensors, optical displays, projection systems and optical scanner are also some VCSEL’s applications [12].
The extensive research is going on to design high output power VCSEL to replace high cost EEL for long distance data communication. InP-based 1.55μm and GaAs-based 1.3μm is already reported [19]. But still longer wavelengths (2-10μm) VCSEL are required for sensor applications. While short wavelengths (e.g. <550nm) VCSELs are being used for display applications. Integration of VCSELs, particularly two-dimensional VCSEL arrays shown in fig. 7 with microelectronics will open new doors for VCSEL applications and will move forward towards new technologies like advance optoelectronics systems including optical transmitter, optical receiver, optical amplifier and optical signal processors [12].

![2-D VCSEL Array](image)

**Figure 7:** Two dimensional VCSEL array [47]

### 1.3 VCSEL versus EEL

Vertical Cavity Surface Emitting Laser (VCSEL) has many fascinating advantages over Edge Emitting Laser (EEL) that are briefly discussed following one by one. But first see fig. 8 that is showing an excellent comparative view of VCSEL and EEL.
1) Beam Characteristics & Shape

The emitted beam characteristics of a VCSEL are very much associated with the characteristic of DBRs. The beam characteristics of VCSEL can be controlled by the number of parameters like number of DBRs, thickness of DBRs, type of material used to fabricate the DBRs and other process parameters. This kind of fine tuning is difficult to find in EELs [12].

A typical low power conventional edge emitting laser (EEL) consists of a p-n junction and laser light emits from active region, the typical size of area of emitting laser is 1×5μm that is narrow and long because p-n junction where thickness is almost zero. Whereas, the shape of emitting area of VCSEL can be the form of doughnut or ring leads to efficient coupling to multimode’s optical fibre. The typical size of emitting circular region of VCSEL is about 5 to 25μm in diameter that too short in contrast to emitting region of EEL. Due to small diameter of emitting region, emitting beam has less divergence and don’t need to do something complicated measures for collimation of light, just simply a lens can provide excellent collimation [12] see fig. 9.

Figure 8: Schematic diagram of VCSEL and EEL [46]
2) **Threshold current**

A typical EELs required 30mA threshold current for telecom applications while VCSELs require typically 1 to 2mA threshold current that leads to low operating voltages, simple driving circuitry and reduced RF interference. These all features push forward to use VCSELs as arrays and the development of high speed optical buses and interconnects [12].

3) **Size**

The size of typical conventional EEL is about micrometers in length while its counterpart VCSEL has fabricated very small, the typical size of VCSLE is microns which is determined by the dimension of its gain region. The die for a VCSEL is slightly larger than the emitting beam [12].

4) **Manufacturing**

We don’t have any method to test EELs laser diodes before the diced up and cleaved the edges. This is why, it is an expansive and time consuming process and we can determine only on final stage that fabricated EELs are working well or not. But on the other hand, entire
VCSEL’s structure can be tested step by step and we are able to evaluate beam characteristic, threshold current, reliability and other process parameters during the fabrication of entire VCSEL [12].

5) Simplified mounting and packaging

VCSEL’s mounting and packaging are not so complicated even the equipments used for mounting and packaging of VCSEL are same that are used for the final assembly of devices like integrated circuits (ICs). DBRs that are most important part of VCSELs are fabricated vertical on substrate and then over it active lasing semiconductor is fabricated [12].

On the other hand, VCSEL’s packaging is also not so expensive. We can use simple inexpensive plastic packaging or we can combine it with other optical components or chip-on-board assembly result in reduced cost [12].

Despite these advantages of VCSEL over EEL, a complete replacement of EELs with VCSELS seem very difficult in near future like in today’s Blu-ray and HD DVD players etc. That’s because, first we need to use VCSEL in form of an array to deliver high power and major changes are also required in technology like Reading/Writing functions. But still VCSEL future is quite bright and it could enjoy success in a variety of applications for instance Xerox has used infrared VCSEL arrays in newly built high quality 2400dpi printers [20].

2 Distributed Bragg Reflectors

2.1 Introduction

The distributed Bragg reflectors have great importance to design vertical cavity surface emitting laser (VCSEL) because of its dual purposes. They not only provide highest reflectivity but also transport the carriers from metal contacts to gain region. If the highest
reflectance is only requirement from mirrors then fabrication would be straightforward but because the DBR carry current to the gain region, requiring that it be a low-resistance electrical conductor [11]. Because of dual tasking requirements of DBRs, they play an important role in the performance of VCSEL and its accurate modeling has great importance.

2.2 Bragg reflector theory

In this section, we will analyze the reflection of transverse electromagnetic (TEM) waves at normal incidence on planner reflector stacks. We generally only consider normal incidence of angle in VCSEL because the lasing mode propagates in the vertical direction so other angles of incidence are not interested for us. The transverse modes in finite dimension devices are never be transverse electromagnetic (TEM). The lateral dimension in electrical pumped devices having diameter greater than 5µm is many times the wavelength in the material (~3µm), that’s why the transverse component of the wave vector will be considerably small for low order modes. Therefore, the transverse electromagnetic (TEM) approximation is valid [4].

2.3 Methods for calculating the reflection coefficient

There are several methods available to calculate the reflection coefficient of a multilayer stack but two famous methods used to calculate reflection coefficient are coupled mode method and transmission method. The coupled mode method is based on coupled mode theory and it is first introduced by Kögelnik & Shanks. Other transmission method is a versatile method being a discrete method therefore, it is easy to implement on arbitrary finite structures. The transmission method first introduced by McLeod [4].

In this thesis, we employ transmission method so before going to the formulation for matrix method it is necessary to understand the terminology used for the reflectance and transmittance at an interface of two different layers.
2.3.1 Notation for multilayer calculation

In this thesis, those notations are used for the calculation of multilayer reflectivity are taken from Heavens. The terms $E^+_m$ and $H^+_m$ represent the electric and magnetic vectors of the Electromagnetic (EM) wave propagating in positive direction respectively while $E^-_m$ and $H^-_m$ represent the propagation of Electromagnetic (EM) wave in negative direction. The resultant electric and magnetic tangential fields are represented by $E_m$ and $H_m$ and they can be expressed as: [12]

\[ E_m = E^+_m + E^-_m \]  \hspace{1cm} (2.1)
\[ H_m = \mu_m (E^+_m + E^-_m) \]  \hspace{1cm} (2.2)

Optical admittance for s and p polarization can be written as:

\[ \mu_m = \frac{n_m Y}{\cos \theta_m} , \text{ for the p-polarization,} \]  \hspace{1cm} (2.3)
\[ \mu_m = n_m Y \cos \theta_m , \text{ for the s-polarization,} \]  \hspace{1cm} (2.4)

Where, $\theta_m$ is the angle of incidence in the $m$th layer and $Y$ is characteristic optical impedance. The angle of incidence can be related to refractive index by Snell’s law:

\[ n_i \sin \theta_i = n_m \sin \theta_m \]  \hspace{1cm} (2.5)
\[ n_m = \sin \theta_m = n_s \sin \theta_s \]  \hspace{1cm} (2.6)

The phase change of electromagnetic radiation passing through the $m$th layer is calculated by:

\[ \delta_m = \frac{2\Pi n_m d_m \cos \theta_m}{\lambda} \]  \hspace{1cm} (2.7)

Where $\lambda$ is wavelength of radiation, $d_m$ is the thickness of the $m$th layer that is expressed by:
\[ d_m = \frac{\lambda_B}{4n_m} \]  

(2.8)

Where, \( \lambda_B \) is the Bragg or design wavelength which is also sometimes called central wavelength and \( n_m \) is refractive index of mth layer [12]. Thickness \( d_m \) is sometimes called optical thickness because thickness is determined by the wavelength that is present inside the each layer [11].

2.3.2 Reflectance and Transmittance at the interface

The reflectance \( R \) and Transmittance \( T \) are defined as the ratio of reflected and transmitted waves respectively to the incident wave. Because the amplitudes of reflected and transmitted wave are complex quantities so their arguments give us phase response of reflected and transmitted wave [12].

Here we would like to calculate the reflectance and transmittance at a single boundary between two homogeneous media [12].

Poynting’s vector is represented by \( S \) and it defined as the instantaneous power flow per unit area. Poynting’s vector is measured by W/m\(^2\) and it is defined as: [12]

\[ S = (E \times H) \]  

(2.9)

Where, \( E \) and \( H \) represent the electric and magnetic fields and measured in V/m and A/m respectively. If we have complex values of \( E \) and \( H \) then we must use real parts. In practice, it is difficult to find instruments that can measure time-averaged quantity \( \bar{S} \). For real \( n \): [12]

\[ \bar{S} = \left( \frac{1}{2} \text{Re} \left( EH^* \right) \right) \hat{r} \]  

(2.10)

Where \( H \) and \( E \) are amplitude vectors, \( \hat{r} \) is unit vector in the direction of electromagnetic wave and asterisk shows the complex conjugate. For a medium of refractive index \( n \), \( n(\hat{r} \times E) = H \), so equation (2.9) and equation (2.10) can be written as [12]

\[ S = n |E|^2 \hat{r} \]  

(2.11)
The reflected or transmitted light can be determined by applying the boundary conditions to the Maxwell’s equation [12].

The boundary conditions require that tangential components of both electrical and magnetic vectors must be continuous across the bounds. The fig. 10 represents the whole situation. A plane is shown on the boundary defined by \( z = 0 \) between the medium of incidence having refractive index \( n_0 \) and other medium having refractive index \( n_1 \). The amplitude of the electric vectors of the incident wave are represented by \( E_{0p}^+ \), \( E_{0s}^+ \), while \( E_{0p}^- \), \( E_{0s}^- \) are for reflected waves and \( E_{tp}^+ \), \( E_{ts}^+ \) are for transmitted waves. The subscripts \( p \) and \( s \) denote the \( p \) polarization and \( s \) polarization [12].

The phase factors with these waves are given below [12]:

**Incident wave:**
\[
\exp \left\{ i \left[ wt - \frac{2\pi n_0}{\lambda} \left( x \sin \theta_0 + z \cos \theta_0 \right) \right] \right\}
\]
Reflected wave: \[ \exp \{i [wt - (2\pi n_0/\lambda) (x \sin \theta_0 - z \cos \theta_0)] \} \]

Transmitted wave: \[ \exp \{i [wt - (2\pi n_1/\lambda) (x \sin \theta_0 + z \cos \theta_1)] \} \]

By applying the boundary condition \( z = 0 \), we obtain [12]:

\[
E_{0x} = (E_{0p}^+ + E_{0s}^-) \cos \theta_0 = E_{1s}^+ = E_{1p}^+ \cos \theta_1 \quad (2.13)
\]

\[
E_{0p} = E_{0s}^+ + E_{0s}^- = E_{1y} = E_{1s}^+ \quad (2.14)
\]

\[
H_{0x} = n_0 (-E_{0s}^+ + E_{0s}^-) \cos \theta_0 = H_{1x} = -n_1 E_{1s}^+ \cos \theta_1 \quad (2.15)
\]

\[
H_{0p} = n_0 (E_{0p}^+ - E_{0p}^-) = H_{1y} = n_1 E_{1p}^+ \quad (2.16)
\]

By solving the equations from (2.13) to (2.16) we obtain the ratios of the transmitted and reflected amplitudes to incident amplitudes for each component of polarization that give us the expressions for the Fresnel coefficients of reflection and transmission [12]:

\[
r_{1p} = \frac{E_{0p}^-}{E_{0p}^+} = \frac{n_0 \cos \theta_1 - n_1 \cos \theta_0}{n_0 \cos \theta_1 - n_1 \cos \theta_0} \quad (2.17)
\]

\[
r_{1s} = \frac{E_{0s}^-}{E_{0s}^+} = \frac{n_0 \cos \theta_0 - n_1 \cos \theta_1}{n_0 \cos \theta_0 - n_1 \cos \theta_1} \quad (2.18)
\]

\[
t_{1p} = \frac{E_{1p}^+}{E_{0p}^+} = \frac{2n_0 \cos \theta_0}{n_0 \cos \theta_1 - n_1 \cos \theta_0} \quad (2.19)
\]

\[
t_{1s} = \frac{E_{1s}^+}{E_{0s}^+} = \frac{2n_0 \cos \theta_0}{n_0 \cos \theta_0 - n_1 \cos \theta_1} \quad (2.20)
\]

The reflection coefficients or reflectance using equation 2.11 are [12]:

\[
R_p = \left( \frac{E_{0p}^-}{E_{0p}^+} \right)^2 = \left( r_{1p} \right)^2 \quad (2.21)
\]
\[ R_s = \left( \frac{E_{0p}^-}{E_{0p}^+} \right)^2 = (r_{1s})^2 \]  
(2.22)

and the transmission coefficients or transmittances are [12]:

\[ T_s = \frac{n_1 \cos \theta_1 (E_{1s}^+)^2}{n_0 \cos \theta_0 (E_{0s}^+)^2} = \frac{n_1 \cos \theta_1}{n_0 \cos \theta_0} t_{1s}^2 \]  
(2.23)

\[ T_p = \frac{n_1 \cos \theta_1 (E_{1p}^+)^2}{n_1 \cos \theta_1 (E_{1p}^+)^2} = \frac{n_1 \cos \theta_1}{n_0 \cos \theta_0} t_{1p}^2 \]  
(2.24)

For \( \theta = 90^0 \)

\[ R_s = R_p = \left| \frac{n_0 - n_1}{n_0 + n_1} \right|^2 \]  
(2.25)

\[ T_s = T_p = \frac{4n_0 n_1}{(n_0 + n_1)^2} \]  
(2.26)

The following expression is valid on the condition that \( n_0 \) and \( n_1 \) are real [12].

\[ R_p + T_p = 1 \]  
(2.27)

\[ R_s + T_s = 1 \]  
(2.28)

### 2.3.3 Transmission Matrix Formulation for Multilayer

The matrix method is easiest and versatile method for calculating the spectral coefficients of the layered media was first demonstrated by F. Abeles [12].

The fig. 2 shows a basic structure of multilayer coating consisting of a finite alternative homogeneous layer of refractive index \( n_1 \) and \( n_2 \).
In order to tangential components of E and H be continuous across any boundary to the wave’s equations travelling at the interface between the \((m-1)\)th and \(m\)th layers is necessary to apply the appropriate boundary conditions are given below [12]:

\[
E_{m-1}^+ = \frac{1}{t_m} \left[ E_m^+ \exp(i\delta_m) + r_m E_m^- \exp(-i\delta_m) \right] \\
E_{m-1}^- = \frac{1}{t_m} \left[ r_m E_m^+ \exp(i\delta_m) + E_m^- \exp(-i\delta_m) \right]
\] (2.29) (2.30)

Equation (2.29) and (2.30) look complex to follow so we use easiest approach by considering the electric field (E) and magnetic field (H) at either side of layer using Maxwell’s equation, a transmission matrix can be found relating to these fields are given below [4].

For the modeling of DBRs, if \(M_T\) represents the product of characteristic matrices of the mirrors and \(M_{i=1,2}\) represent the individual layers in a stack then we can write \(M_T\) as [4]:

\[
M_T = (M_1 \times M_2 \times M_1 \times M_2)^{p/2}
\] (2.31)

Where \(p\) is number of periods

\[
\begin{bmatrix}
E_{p-1} \\
H_{p-1}
\end{bmatrix} = M
\begin{bmatrix}
E_p \\
H_p
\end{bmatrix}
\] (2.32)

Thus for an \(N\)-layer filter:

\[
\begin{bmatrix}
E_0 \\
H_0
\end{bmatrix} = \prod_{m=1}^{N} M_m
\begin{bmatrix}
E_N \\
H_N
\end{bmatrix}
\] (2.33)

Where \(M_m\) is the characteristic matrix and is given by:
\[ M_m = \begin{bmatrix} \cos(k, d_r + j\alpha, d_r) j \sin(k, d_r + j\alpha, d_r) \\ jn_r \sin(k, d_r + j\alpha, d_r) \cos(k, d_r + j\alpha, d_r) \end{bmatrix} \]  
\quad \text{(2.34)}

and
\[ \begin{bmatrix} E_N \\ H_N \end{bmatrix} = \begin{bmatrix} 1 \\ n_s \end{bmatrix} E_s^+ \quad \text{(2.35)} \]

The above equation can also be written as:
\[ E_N \begin{bmatrix} 1 \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ n_s \end{bmatrix} E_s^+ \quad \text{(2.36)} \]

Where \( Y \) is optical admittance and it can be expressed in the form of ratio:
\[ Y = \left( \frac{H_0}{E_0} \right) \quad \text{(2.37)} \]

Substituting equations 2.33, 2.35 and 2.37 into equation 2.32 that gives an expression for calculating coefficient \( B \) and \( C \) are given below:
\[ \begin{bmatrix} B \\ C \end{bmatrix} = \prod_{m=1}^N \begin{bmatrix} \cos(\delta_r + jmad_r) j \sin(\delta_r + jmad_r) / n_r \\ jn_r \sin(\delta_r + jmad_r) \cos(\delta_r + jmad_r) / n_r \end{bmatrix} \begin{bmatrix} 1 \\ n_r \end{bmatrix} \quad \text{(2.38)} \]

Where \( \delta_r \), change of electromagnetic wave and is written by
\[ \delta_r = 2m_r \frac{d_r}{\lambda} \quad \text{(2.39)} \]

\( d_r \) is optical thickness and is written by
\[ d_r = \frac{\lambda_B}{4n_r} \]  

(2.40)

and \( \alpha \) is absorption coefficient that is explained in detailed in section 4.1.2.

Finally, the reflectance, transmittance, and phase changes respectively can be calculated by given expressions using coefficient \( B \) and \( C \) [12]:

\[ R = \left| \frac{n_0 \times B - C}{n_0 \times B + C} \right|^2 \]  

(2.41)

\[ T = \frac{4 \times n_s - n_0}{\left| n_0 \times B + C \right|^2} \]  

(2.42)

\[ \psi = \arg \left( \frac{n_0 \times B - C}{n_0 \times B + C} \right) \]  

(2.43)

### 2.3.4 Maximum reflectivity from a Bragg stack

The repeated quarter-wave layers having high and low refractive index are called quarter-wave stack or Bragg stack. It is also sometimes referred as a bilayer stack or binary stack. The reflectivity influenced by number of parameters listed below [11]

1. Number of periods
2. Refractive index contrast b/w layers
3. Bragg wavelength
4. Refractive index of substrate

These parameters need to be optimized to ensure the minimum number of periods \( p \) and minimum lattice mismatching \( (a_0(x_1) - a_0(x_2))/a_0(x_1) \). The reflectivity \( (R) \) at resonant wavelength of a stack with \( p \)th number of non-absorbing pairs of DBR can be calculated by [4]:

\[ \ldots \]
\[
\sqrt{R} = \frac{1 - \frac{n_2}{n_0} \left( \frac{n_1}{n_2} \right)^{2p}}{1 + \frac{n_2}{n_0} \left( \frac{n_1}{n_2} \right)^{2p}}
\]  

(2.44)

Where, \( p \) is number of periods, \( n_0 \) is refractive index of air (for outside the VCSEL) and refractive index of layer (for inside the VCSEL), \( n_s \) is refractive index of substrate (Sapphire (\( \text{Al}_2\text{O}_3 \)) = 1.78) and \( n_1 \) and \( n_2 \) are the refractive index of layer 1 and layer 2.

3 Different arrangements of layers

After developing the DBR’s model for the calculation of reflectivity, the study of different arrangements of periodic quarter wavelength thick layers having high and low refractive index and their interface with air and substrate is very important. Finally, we will try to find an optimal arrangement of layers for bottom DBRs to obtain maximum reflectivity.

Case 1:

In this case we analyze the air interface with layer of refractive index \( n_1 = 2.5145 \) and its alternative arrangement with layer of refractive index \( n_2 = 2.4614 \) with 20th number of periods.

![Diagram of layer arrangement](image)

**Figure 11:** Alternative arrangement of layers \( n_1 \) and \( n_2 \) with air interface with layer \( n_1 \)

The reflectivity shown in fig. 12 for outside the VCSEL is obtained with arrangement of layers \( n_1 \) and \( n_2 \) shown in fig. 11.
Case-2:

In this case, we have air interface with layer of refractive index $n_1=2.5145$ and its alternative arrangement with layer of refractive index $n_2=2.4614$ while sapphire is used as substrate with 20th number of periods.

Figure 12: Reflectivity versus wavelength with air-interface
The reflectivity shown in fig. 14 for outside the VCSEL is obtained with arrangement of layers $n_1$ and $n_2$ shown in fig. 13.

**Figure 13:** Alternative arrangement of layers $n_1$ and $n_2$ with air interface with $n_1$ and with substrate (Sapphire) interface with $n_2$ layer

**Figure 14:** Reflectivity versus wavelength with air and sapphire interface
Case-3:

In this case, we will discuss further four cases. In these cases we will place layer \( n_1 = 2.5145 \) and layer \( n_2 = 2.4614 \) on one side of alternative reflectors and air and sapphire on other side of reflectors.

Case-3.1

Here we will have \( n_1 \) layer on one side of reflectors and on the other side of reflectors we will have air interface. Reflectors contain 20\(^{th}\) number of periods.

![Figure 15: Alternative arrangements of layers \( n_1 \) and \( n_2 \) with air interface with \( n_1 \) layer](image)

The reflectivity shown in fig. 16 for outside the VCSEL is obtained with arrangement of layers \( n_1 \) and \( n_2 \) shown in fig. 15.
Figure 16: Reflectivity versus wavelength with air and sapphire interface

Case-3.2

Here we will have \( n_1 \) layer on one side of reflectors and on the other side of reflectors we will have air interface. Reflectors contain 20\(^{th}\) number of periods.

![Diagram showing layers](image)

Figure 17: Alternative arrangement of layers \( n_1 \) and \( n_2 \) with air interface with \( n_2 \) layer
The reflectivity shown in fig. 18 for outside the VCSEL is obtained with arrangement of layers $n_1$ and $n_2$ shown in fig. 17.

**Figure 18:** Reflectivity versus wavelength with air and sapphire interface

**Case-3.3**

Here we will have $n_1$ layer on one side of reflectors and on the other side of reflectors we will have sapphire interface. Reflectors consist of 20$^{th}$ number of periods.

**Figure 19:** Alternative arrangement of layers $n_1$ and $n_2$ with substrate (Sapphire) interface with $n_1$ layer
The reflectivity shown in fig. 20 for inside the VCSEL is obtained with arrangement of layers $n_1$ and $n_2$ shown in fig. 19.

![Reflectivity versus wavelength with air and sapphire interface](image)

**Figure 20:** Reflectivity versus wavelength with air and sapphire interface

**Case-3.4**

Here we will have $n_2$ layer on one side of reflectors and on the other side of reflectors we will have sapphire interface. Reflectors consist of $20^{th}$ number of periods.
Figure 21: Alternative arrangement of layers $n_1$ and $n_2$ with substrate (Sapphire) interface with $n_2$ layer.

The reflectivity shown in fig. 22 for inside the VCSEL is obtained with arrangement of layers $n_1$ and $n_2$ shown in fig. 21.

Figure 22: Reflectivity versus wavelength with $n_2$ and sapphire interface.
We conclude that our model based on matrix method is working well for all kind of interfaces and we can use it to find internal and external reflectivity. We also found reflectivity calculated by formula given in section 2.3.4 and obtained with matrix method has good agreement. We also conclude that layers arrangement in case-3.3 is best for reflection inside the VCSEL and in case.3.2 is best for reflection outside the VCSEL.

4 Implementation of Model on Nitride based DBRs

4.1 Al$_x$Ga$_{1-x}$N alloy Based DBRs

The realization of the VCSEL requires the pair of high reflectivity mirrors, usually in the form of distributed Bragg Reflectors (DBRs) on both sides of the active region to form the laser cavity [21]. The main requirements that we want from DBRs are high reflectivity and spectral broadness of the stop band that is main obstacle to fabricate nitride based DBRs, the reason of these shortcomings is small refractive index contrast that can be achieved within the entire AlGaN alloy composition [22].

There are many papers already published, for the demonstration of AlGaN based DBRs. Joan M. Redwing et al. [23] demonstrated a VCSEL’s structure consists of a 10μm GaN active layer sandwiched between 30 periods Al$_{0.40}$Ga$_{0.60}$N-Al$_{0.12}$Ga$_{0.88}$N (397Å/372Å) Bragg reflector stacks [23]. As, GaN-based materials have obtained lots of attention because of large band gap and promising potential for the optoelectronics devices. Over 50% Al content, produced AlGaN/GaN DBRs are free from cracks but in this situation the whole structure relaxes to an average in-plane lattice parameter so the GaN/GaInN multiple quantum well grown on top of AlGaN/GaN DBRs is no longer lattice matched and strain relaxation issues arise in the active region. Thus, most of researchers agree to use Al contents 30% for DBRs at the price of a reduced optical stop band [24]. The numerous groups already have demonstrated Al$_x$Ga$_{1-x}$N/GaN based DBRs with different composition of aluminum at a range of ultraviolet to blue-green spectrum [25], [43]. Someya et al. [25] demonstrated 43 pairs of Al$_{0.34}$Ga$_{0.66}$N/GaN as a bottom DBRs operating at ~400nm. Zhou et al. [25] employed 60 pairs of Al$_{0.25}$Ga$_{0.75}$N/GaN based bottom DBRs operating at 383.2nm. All these AlGaN/GaN DBR structures required a large numbers of periods due to relatively low refractive index
contrast between AlGaN and GaN layers [25] and H. Dartsch et al. [26] demonstrated strain compensated 20 Al$_{0.41}$Ga$_{0.59}$N/GaN periods lasing at 500nm Bragg wavelength [26]. After the AlGaN/AlGaN and AlGaN/GaN DBRs demonstration, AlN/GaN DBRs have been attractive because it has large refractive index contrast (Δn/n=0.16) that make possible to achieve highest reflectivity with minimal number of periods and large stop bandwidth. However the AlN/GaN layers have relatively large lattice mismatch (~2.4%) and the difference in thermal expansion coefficients b/w GaN (5.59×10$^{-6}$/K) and AlN (4.2×10$^{-6}$/K) that leads to formation of cracks and dislocations that could result in the reduction of reflectivity and increase in scattering loss. A number of research groups have already reported the fabrication of AlN/GaN based DBRs with possible peak reflectance in the range of ultraviolet (UV) to blue-green region using minimal number of periods. Chinh-Chiang Kao et al. [25] reported AlN/GaN based bottom DBRs of VCSEL operating at 450nm producing 94% reflectance peak with 18nm stop bandwidth and 25 periods [25]. Both H. M. Ng and T. D. Moustakas [22], [43] also reported the significant success in the fabrication of AlN/GaN based DBRs. First they reported that 20.5 periods of AlN/GaN based DBRs are required to obtain 95% reflectivity at 392nm. After that they reported 98%, 99% and 97% reflectivity at 410nm, 467nm and 560 nm Bragg wavelength with 25.5, 20.5 and 20.5 periods respectively. 99% reflectivity is highest possible reflectivity achieved with 45nm stop bandwidth. In order to achieve cracking free AlN/GaN DBRs, AlN layers are grown thicker than GaN that increases the mechanical yield strength of the AlN layers [43], [22].

4.1.1 Refractive Index Model
An accurate knowledge of refractive index and absorption coefficient are mandatory to design the optoelectronics devices. Since the nitride family of semiconductors is still not enough mature, significant inconsistencies found in published data for the refractive index as a function of the photon energy and the optical wavelength [30].

The refractive index contrast between layers plays an important role in determining the amount of light that will be confined in the active region. But unfortunately in practice, index steps available in III-N materials are limited due to difficult growth process, due to doping requirement and due to lattice mismatching. The large step in refractive index that is our key requirement is limited by the amount of composition of content [30].

There are two commonly used models to calculate refractive index of nitride materials, i.e.

1. Sellmeier Model
2. Adachi’s Model
4.1.1.1 Sellmeier Model
Sellmeier equation is used to calculate the refractive index. The 2\textsuperscript{nd} order Sellmeier equation is given below:

\[
n(\lambda) = \sqrt{A_0 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2}} \quad (4.1)
\]

Where \( A_0, A_1, \) and \( A_2, \) are the fitting parameters and \( \lambda_1, \lambda_2 \) are the wavelength.

The coefficients for the calculation of refractive index of GaN and AlN are given in Table 1, Table 2 and in Table 3.

The fig. 23 shows the refractive index of GaN and AlN calculated by parameters mentioned in Table 1 and the fig. 24 also shows the refractive index of GaN and AlN calculated by parameters mentioned in Table 2 whereas refractive index of GaN shown in fig. 25 is calculated by parameters mentioned in Table 3. If we compare fig. 23 and fig. 24 then fig. 23 shows very lower values of refractive index over wavelengths so the values of refractive index shown in fig. 24 seem more accurate.

Table 1: Sellmeier parameters for GaN and AlN [27]

<table>
<thead>
<tr>
<th>Materials</th>
<th>( A_0 )</th>
<th>( A_1 )</th>
<th>( A_2 )</th>
<th>( \lambda_1 (\text{nm}) )</th>
<th>( \lambda_2 (\text{nm}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaN</td>
<td>1</td>
<td>2.37e-2</td>
<td>1.254</td>
<td>352.1</td>
<td>141.8</td>
</tr>
<tr>
<td>AlN</td>
<td>1</td>
<td>5.91e-2</td>
<td>0.981</td>
<td>190.5</td>
<td>109.5</td>
</tr>
</tbody>
</table>
Figure 23: GaN and AlN Refractive index versus wavelength

Table 2: Sellmeier parameters for GaN and AlN [28]

<table>
<thead>
<tr>
<th>Materials</th>
<th>$A_0$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$\lambda_1$(μm)</th>
<th>$\lambda_2$(μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaN</td>
<td>3.6</td>
<td>1.75</td>
<td>4.1</td>
<td>0.256</td>
<td>17.86057</td>
</tr>
<tr>
<td>AlN</td>
<td>3.14</td>
<td>1.386</td>
<td>3.861</td>
<td>0.1715</td>
<td>15.0333</td>
</tr>
</tbody>
</table>
The parameters for GaN and AlN are given in Table 2 valid for wavelength range 0.35-10μm and 0.22-5μm respectively [28].

GaN refractive index can also be calculated by Sellmeier first order equation given below:

$$n(\lambda) = \sqrt{\frac{A_0^2 + \frac{A_1^2}{\lambda^2 - A_2^2}}{A_0^2}}$$  \hspace{1cm} (4.2)$$

The parameters used to calculate the refractive index of GaN shown in fig. 25 are given in Table 3:
Table 3: Sellmeier parameters for GaN [29]

<table>
<thead>
<tr>
<th>$A_0$</th>
<th>$A_1$ (nm²)</th>
<th>$A_2$ (nm²)</th>
<th>Thickness d (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.27±0.02</td>
<td>304.7±7.8</td>
<td>294.0±4.5</td>
<td>1250.2±16</td>
</tr>
</tbody>
</table>

Figure 25: Refractive index of GaN versus wavelength

4.1.1.2 Adachi’s Model

Several models already have been reported to calculate refractive index but up to now Adachi’s model is considered most accurate model. Several research groups have proposed own values of Adachi’s fitting parameters that give different level of accuracy for a range of composition of Al and In in nitride alloys. The continuous equation to calculate the refractive index for all nitrides material proposed by Adachi is given below [31]:

...
Where $E_g$ is the bandgap energy of the material, $E$ is photonic energy can be expressed as $E=hf$, where $h$ is Planck’s constant and $f$ is the frequency of the laser emission and $A(x)$ and $B(x)$ are the fitting parameters [31].

Peng and Piprek proposed fitting parameters $A(x)$ and $B(x)$ are given below obtained from spectrally resolved refractive index measurement of the binary compounds (AlN, GaN and InN) using Adachi model to calculate refractive index for all the nitride ternary alloys [30].

\[
A(x) = 13.55x + 9.31(1-x) \quad (4.4)
\]
\[
B(x) = 2.05x + 3.03(1-x) \quad (4.5)
\]

Despite Peng and Piprek used a greater amount of experimental data to produce $A(x)$ and $B(x)$ but still there is lack of accuracy in refractive index because Peng and Piprek used refractive index of AlN is $n=2.36$ was measured in 1966. Using the inaccurate value of AlN refractive index leads inaccuracy in fitting parameters that produce inaccurate values of refractive index of $\text{Al}_x\text{Ga}_{1-x}\text{N}$ [30].

Brunner et al. [32] and Ambacher et al. [32] have published the most comprehensive data on the refractive indices of the $\text{Al}_x\text{Ga}_{1-x}\text{N}$ alloy, covering the entire composition range. They observed the consistent growth method, measurement techniques for determining the refractive index, bandgap energy for entire range of composition for all the samples [32]. Brunner et al. [32] and Ambacher et al. [32] used fitting parameters are given below to calculate $\text{Al}_x\text{Ga}_{1-x}\text{N}$ for entire range for composition of Al [32]:

\[
A(x) = (3.17 \pm 0.39)x^{1/2} + (9.98 \pm 0.27) \quad (4.6)
\]
\[
B(x) = (2.66 \pm 0.12) - (2.20 \pm 0.2)x \quad (4.7)
\]
\[
E_g(\text{Al}_x\text{Ga}_{1-x}\text{N}) = 6.28x + 3.42(1-x) - 1.3(1-x)x \quad (4.8)
\]

Using above parameters the refractive index show decline for lower percentage aluminum range (i.e., for $x<0.05$). The decline of refractive index at low amount of aluminum is important to take into account because the majority of the laser cladding layers contains small percentages of aluminum [30]. The fig. 26 and fig. 27 show the refractive index versus wave length and photon energy respectively using Brunner et al. [32] and Ambacher et al.[32] fitting parameters.
In order to avoid the error in model for low amount of aluminum that is visible in fig. 27, the improvement of the fitting parameters of Brunner et al. [32] is necessary. Hence, the refractive index data of Brunner et al. [32] are digitized and fitting parameters are reevaluated. The best fits for fitting parameters \( A(x) \) and \( B(x) \) are given below were found by using second order polynomial expressions over the range \( 0 < x < 0.38 \) [30].

\[
A(x) = 9.82661 - 8.21608x - 31.590x^2 \quad (4.9)
\]

\[
B(x) = 2.73591 + 0.84249x - 6.29321x^2 \quad (4.10)
\]

The fig. 28 shows the refractive index using above mentioned error free parameters \( A(x) \) and \( B(x) \) in equation (4.9) and equation (4.10).

**Figure 26:** Refractive index of \( \text{Al}_x \text{Ga}_{1-x} \text{N} \) using \( 0 < x < 1 \) versus wavelength
Figure 27: Refractive index of Al$_x$Ga$_{1-x}$N using 0<x<1 versus photon energy

Figure 28: Refractive index of Al$_x$Ga$_{1-x}$N versus photon energy
4.1.2 Absorption coefficient and modeling of absorption

The accurate knowledge of absorption coefficient is equally important to design the any optoelectronic devices. O. Ambacher et al. [32] and J.F. Muth et al. [33] work on AlGaN alloy for absorption is very attractive. Absorption coefficient is measured above the bandgap, where the multiple reflections of light could be absorbed by the high absorption coefficient. In order to calculate accurate absorption the film thickness is kept less than 1µm which permits the direct calculation of absorption coefficient well above the bandgap [33].

![Figure 29](image1.png) ![Figure 30](image2.png)

**Figure 29:** Absorption versus photon energy [32]  **Figure 30:** Absorption versus photon energy [33]

We calculate absorption coefficient in our model based on work done by O. Ambacher et al. [32] shown in fig. 29 and J.F. Muth et al. [33] shown in fig. 30 by

If, \( E < E_g \)

Then,

\[
\alpha = 10^2 E^{1/2} \]

(4.11)
And if, $E > E_g$ [27]

Then

$$\alpha = 10^{\frac{1.5e5-4e4}{0.5} E - E_g}$$

(4.12)

The fig. 31 shows the absorption of Al$_x$Ga$_{1-x}$N alloy using varying composition of Al versus photon energy. In ideal condition, the absorption level should be or less than $10^3$ cm$^{-1}$ but as we can see in fig. 31 as mole fraction of Al increases, absorption also increases so it limits the use of high mole fraction of Al in Al$_x$Ga$_{1-x}$N alloy. We also observe that the absorption is considerable high on bandgap energy and near bandgap energy which increases with increase of Al composition that’s why we design DRBs on wavelength at least 40 nm ahead from the wavelength corresponding to bandgap of both layers, see fig. 32. We also plotted Al composition in Al$_x$Ga$_{1-x}$N alloy versus wavelength and bandgap in fig. 32 we observe that as Al composition increases band gap energy also increases.

**Figure 31:** Absorption versus photon energy
4.1.3 AlGaN/AlGaN reflective spectrum

We implement our model on Al\textsubscript{x}Ga\textsubscript{1-x}N/Al\textsubscript{x}Ga\textsubscript{1-x}N distributed Bragg reflectors. We find that 37\textsuperscript{th} number of periods for Al\textsubscript{0.9}Ga\textsubscript{0.1}N/Al\textsubscript{0.7}Ga\textsubscript{0.3}N distributed Bragg reflectors with thickness 27.44nm and 24.21nm respectively are required to produce 99.95% reflectivity on 250nm Bragg wavelength shown in fig. 33. Sapphire (Al\textsubscript{2}O\textsubscript{3}) is used as substrate. In fig. 33 the Blue color plot is taken without dispersion and without absorption and we obtain Red plot with dispersion and with absorption. The Red plot that is more realistic gives us parameters more close to real parameters when Al\textsubscript{0.9}Ga\textsubscript{0.1}N/Al\textsubscript{0.7}Ga\textsubscript{0.3}N distributed Bragg reflectors are fabricated. The thing that is very important to take into account in fig. 33 the effects of dispersion and absorption that can reduce the reflectivity and stop bandwidth. In our thesis work, we show reflectivity in percentage (%) and wavelength is shown in nanometer (nm).
We also used $\text{Al}_{0.9}\text{Ga}_{0.1}N/\text{Al}_{0.5}\text{Ga}_{0.5}N$ DBRs and plotted reflectivity versus wavelength on various Bragg wavelengths or design wavelengths and obtain approximately 99.67% reflectivity with the adjustment of number of periods shown in fig. 34. We tabulated Bragg wavelengths and corresponding number of periods to obtain approximated 99.67% reflectivity in Table 4.
Table 4: Reflectivity on various Bragg wavelength with number of periods for Al_{0.9}Ga_{0.1}N/Al_{0.5}Ga_{0.5}N DBRs

<table>
<thead>
<tr>
<th>Reflectivity (R) (%)</th>
<th>Number of Periods (p)</th>
<th>Bragg wavelengths ((\lambda_B)) (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>~99.67%</td>
<td>24</td>
<td>300</td>
</tr>
<tr>
<td>~99.67%</td>
<td>31</td>
<td>350</td>
</tr>
<tr>
<td>~99.67%</td>
<td>35</td>
<td>400</td>
</tr>
<tr>
<td>~99.67%</td>
<td>37</td>
<td>450</td>
</tr>
<tr>
<td>~99.67%</td>
<td>39</td>
<td>500</td>
</tr>
</tbody>
</table>

The fig. 34 also shows the variation of refractive index over wavelength. We observe layer with composition 0.91 having high refractive index and layer with composition 0.51 having low refractive index constituting the DBRs. We also observe that refractive index reduces with the increase of wavelength and refractive index contrast become small as we move towards higher wavelength. Small refractive contrast is a reason requiring higher number of periods with increasing Bragg wavelength to obtain same reflectivity shown in Table 4.

Figure 34: Reflectivity versus wavelength over Bragg wavelengths 300nm, 350nm, 400nm, 450nm, and 500nm and variation of refractive index of layers with composition 0.91 and 0.5 over wavelength
4.1.4 AlGaN/GaN reflective spectrum
The 48\textsuperscript{th} number of periods of Al\textsubscript{0.34}Ga\textsubscript{0.66}N/GaN DBRs are required to obtain 99.50% reflectivity on 400nm Bragg wavelength. The fig. 35 shows the reflectivity versus wavelength using Al\textsubscript{0.34}Ga\textsubscript{0.66}N/GaN DRBs.

![Reflectivity versus wavelength using Al\textsubscript{0.34}Ga\textsubscript{0.66}N/GaN DBRs](image)

Figure 35: Reflectivity versus wavelength using Al\textsubscript{0.34}Ga\textsubscript{0.66}N/GaN DBRs

4.1.5 AlN/GaN reflective spectrum
The 17\textsuperscript{th} number of periods of AlN/GaN DBRs are required to obtain 99.66% reflectivity. The fig. 36 shows the reflectivity versus wavelength on 450nm Bragg wavelength.
4.1.6 AlGaN/AlN reflective spectrum
The 20th number of periods of Al$_{0.15}$Ga$_{0.85}$N/AlN DBRs with thickness 37nm and 41nm respectively are required to obtain 99.98% reflectivity at 370nm Bragg wavelength shown in fig. 37 and the fig. 38 shows the transmissivity versus wavelength we observe that almost 1.15% light is reflected it means almost all light is transmitted and the phase variation over wavelength is depicted in fig. 39, we observe phase changes around design wavelength very smoothly. The fig. 40 shows reflectivity using both upper and bottom DBRs. The parameters used for fig. 40 are given in Table 5.
Table 5: Parameters used to obtain reflectivity spectrum for upper and bottom DBRs

<table>
<thead>
<tr>
<th>Reflectivity $R$ (%)</th>
<th>No. of periods $(p)$ (Upper layers)</th>
<th>No. of periods $(p)$ (Bottom layer)</th>
<th>$\text{Al}<em>{0.15}\text{Ga}</em>{0.85}\text{N}$ thickness $(d_1)$ (nm)</th>
<th>$\text{AlN}$ thickness $(d_2)$ (nm)</th>
<th>Bragg wavelength (nm)</th>
<th>Refractive index (SQW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.98</td>
<td>20</td>
<td>20</td>
<td>41</td>
<td>37</td>
<td>370</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Figure 37: Reflectivity versus wavelength using $\text{Al}_{0.15}\text{Ga}_{0.85}\text{N}/\text{AlN}$ DBRs
Figure 38: Transmissivity versus wavelength using $\text{Al}_{0.15}\text{Ga}_{0.85}\text{N/AlN}$

Figure 39: Phase versus wavelength
4.1.7 Bandwidth of Al$_x$Ga$_{1-x}$N DBRs in the UV range and dispersion effect

The larger bandwidth of the primary reflectance peak is one of the requirements of nitride-based VCSEL structures because the active region of the nitride lasers is most commonly based on InGaN heterostructures or multiple quantum wells, whose emission spectra are very fragile to small variations in growth or process parameters [43].

The width of the bandgap can be calculated by the following formula [4]:

$$B_{\text{th}1} = \left| (4\lambda_{th}/\pi) \sin^{-1}\left((n_2 - n_1)/(n_2 + n_1)\right) \right|$$

(4.13)
The approximated form of above formula also has considerable accuracy [4]:

\[ B_{th2} = \left( \frac{2\lambda_b}{\pi} \right) \left( \frac{n_2 - n_1}{n} \right) \]  

(4.14)

The above mentioned formulas in equation (4.13) and (4.14) are not applicable for nitrides based DBRs because of strong dispersion near the bandgap of both AlN and GaN [27].

So we use transmission matrix method for better understanding of bandwidth’s variation with and without dispersion over different design wavelengths i.e. 300nm, 350nm, 400nm, 450nm and 500nm. The fig. 41 shows the comparison of bandwidth’s simulations with and without dispersion, theoretical formula and its approximated form. The comparison between simulated bandwidth with and without dispersion is quite interesting we observe that bandwidth without dispersion decreases from 300nm to 350nm and then it increases with increasing wavelength till 500nm and in dispersion bandwidth increases with increasing wavelength that intersect at a point when bandwidth are same for dispersion and without dispersion on a particular wavelength. The bandwidth calculated with theoretical formulas show different variation than versus Bragg wavelengths.

![Figure 41: Bandwidth versus Bragg wavelength](image)
We would like also to investigate the relation between bandwidth and reflectivity with and without dispersion on various Bragg wavelengths with increasing number of periods.

The fig. 42 shows the relation between bandwidth and reflectivity without dispersion. We consider 11th numbers of periods to explain the bandwidth trend on Bragg wavelengths 300nm, 350nm, 400nm, 450nm and 500nm. We calculated the bandwidth 45.900nm, 45.500nm, 49.200nm, 53.550nm and 58.500nm at Bragg wavelength 300nm, 350nm, 400nm 450nm and 500nm respectively. We observe bandwidth is decreasing with the increase of wavelength range from 300nm to 350nm but after that bandwidth increases in the range of Bragg wavelength from 350nm to 500nm. The fig. 42 also shows as number of periods increasing, reflectivity also increases.

Figure 42: Reflectivity versus bandwidth on various Bragg wavelengths without dispersion
The fig. 43 shows an interesting effect of dispersion on bandwidth. Here again we consider 11th numbers of periods that could be useful to make comparison of both fig. 42 and fig. 43. We observe bandwidths 31.800nm, 37.800nm, 43.600nm, 49.050nm and 54.500nm on Bragg wavelengths 300nm, 350nm, 400nm, 450nm and 500nm respectively. As we can see the bandwidth increases in the range of Bragg wavelengths from 300 nm to 500nm.

Now if we compare both figs. 42 and 43, and observe bandwidths with same Bragg wavelength and same number of periods then we can see dispersion effect appears to decrease the bandwidth in fig. 43.

**Figure 43:** Reflectivity versus Bandwidth on various Bragg wavelengths with dispersion

### 4.1.8 Al$_x$Ga$_{1-x}$N DBRs design curves for ultraviolet range

For practical realization of DBRs for VCSELs in the UV range it could be handy to find minimum wavelength and Al composition of 2nd layer over number of periods.
In fig. 44 we plot various curves for operating wavelength versus number of periods with fix Al composition in first layer and fix 99.50% reflectivity. From the fig. 44 we can select a curve having particular composition in first layer and we can choose a minimal operating wave length corresponding to a specific number of periods for 99.50% reflectivity. To obtain the minimum operating wavelength, we have always wavelength 40nm superior than bandgap to avoid absorption. As we have seen in fig. 31 absorption at bandgap and near the bandgap is high that can influence the performance of device so in order to avoid the absorption we always design the VCSEL at Bragg wavelength superior to minimum bandgap of both layers constituting the DBRs.

Similarly, in fig. 45 we plot various curves for Al composition in 2nd layer versus number of periods with fix Al composition in first layer and fix 99.50% reflectivity. From the fig. 45 we can select a curve having particular composition of Al in first layer and we can choose minimum composition of Al in 2nd layer for specific number of periods. As we have already mentioned above to avoid the absorption we always have value of wavelength superior to minimum bandgap energy of both layers constituting the DBRs.

![Figure 44: Minimum operating wavelength versus period](image-url)
Figure 45: Al compositions in 2nd layers versus number of periods

4.2 $\text{Al}_{x}\text{Ga}_{1-x}\text{N}/\text{In}_{y}\text{Ga}_{1-y}\text{N}$ DBRs for UV VCSEL

AlGaN has proved to be promising nitride alloy for UV DBRs. But it suffers lowest refractive index contrast, lattice mismatching and stress between the layers that leads to formation of creaks and dislocations in layers [34].

To overcome the shortcomings of AlGaN, other nitride alloys like InGaN and AlInN have obtained much attention for researchers from few years. Lattice matching issue can be overcome by using $\text{Al}_{0.82}\text{In}_{0.18}\text{N}/\text{GaN}$ DBRs with larger refractive index contrast and AlGaN/InGaN DBRs could be useful to overcome the stress issue in layers [34]. The fig. 46 is useful to understand relation between these alloys.
The expressions shown in equation (4.15), (4.16) and (4.17) are used to calculate bandgap energy of all three nitride alloys in Electronic volt (eV) and lattice constant are calculated using equation (4.18), (4.19) and (4.20) in Angstrom (Å) are given below\cite{32}, \cite{35}, \cite{36}, \cite{37}, \cite{38}:

\[ E_g(Al_xGa_{1-x},N) = 6.28x + 3.42(1 - x) - 1.3(1 - x)x \]  \hspace{1cm} (4.15)

\[ E_g(In_yGa_{1-y},N) = 1.994y + 3.42(1 - y) - 1.3(1 - y)y \]  \hspace{1cm} (4.16)

\[ E_g(Al_zIn_{1-z},N) = 6.28z + 1.994(1 - z) - 1.3(1 - z)z \]  \hspace{1cm} (4.17)

Where 6.28eV, 3.42eV and 1.994eV are bandgap energies of AlN, GaN and InN respectively and 1.3 is bowing parameter.
\[
\begin{align*}
\rho(Al_x Ga_{1-x}N) &= 3.11x + 3.19(1-x) \\
\rho(In_y Ga_{1-y}N) &= 3.5446y + 3.19(1-y) \\
\rho(Al_z In_{1-z}N) &= 3.11z + 3.5446(1-z)
\end{align*}
\]

(4.18) (4.19) (4.20)

Where, 3.11Å, 3.19Å and 3.5446Å are lattice constants of AlN, GaN and InN respectively.

In this section, we will consider the possibility to design balanced compressive and tensile stress AlGaN/InGaN DBRs. We plotted some graphs that provide us useful information for designing the stress free AlGaN/InGaN DBRs.

The fig. 47 shows that stress increases with the increase of composition of Al and In for Bragg wavelengths 550nm. From the fig. 47, we can select particular mole fraction of Al and In to balance the compressive stress and tensile stress. We use the following expression to calculate the stress in Al\(_x\)Ga\(_{1-x}\)N and In\(_y\)Ga\(_{1-y}\)N layers.

\[
\begin{align*}
\text{stress}(Al_x Ga_{1-x}N) &= \frac{\text{Thickness}(Al_x Ga_{1-x}N) \times (\text{Lattice}(GaN) - \text{Lattice}(Al_x Ga_{1-x}N))}{\text{Lattice}(GaN)} \\
\text{stress}(In_y Ga_{1-y}N) &= \frac{\text{Thickness}(In_y Ga_{1-y}N) \times (\text{Lattice}(In_y Ga_{1-y}N) - \text{Lattice}(GaN))}{\text{Lattice}(GaN)}
\end{align*}
\]

(4.21) (4.22)

The fig. 48 shows the relation between layer’s thickness and composition of Al and In in Al\(_x\)Ga\(_{1-x}\)N and In\(_y\)Ga\(_{1-y}\)N alloys respectively. In fig. 48, In\(_y\)Ga\(_{1-y}\)N layer’s thickness is close to experimental values for a range from y = 0 to y = 0.3 while for higher composition of In it strongly deviates.
Figure 47: Calculation of stress for Al$_x$Ga$_{1-x}$N and In$_y$Ga$_{1-y}$N for values of $x$ and $y$ from 0 to 1
If we consider only curves for Bragg wavelength 550nm and we find that Al composition 0.332 for Al$_x$Ga$_{1-x}$N layer and In composition 0.08 for In$_x$Ga$_{1-x}$N layer that can give us $5.01 \times 10^{-10}$ compressive stress and tensile stress with thickness about 60nm and 56nm respectively. So before we used these values in our model first we need to calculate refractive index of InGaN. We already calculated refractive index of AlGaN in section 4.1.1.

4.2.1 Refractive index model for InGaN material

There is very few experimental data available for refractive index of InGaN alloy because high composition of indium in alloy leads to inhomogeneities in the composition of material that make very difficult to calculate accurate refractive index [30].
Peng and Piprek [30] published a continues equation for InGaN alloy, based on work by Adachi [8]. This model is recommended for lower composition of In. The refractive index of InGaN for indium composition from $y = 0$ to $y = 0.30$ is shown in fig. 49. The fitting parameters used in calculations are given below [30].

\[
A(x) = 13.55x + 9.31(1 - x) \quad (4.23)
\]
\[
B(x) = 2.05x + 3.03(1 - x) \quad (4.24)
\]
\[
E_g(In_y Ga_{1-y} N) = 1.99y + 3.42(1 - y) - 1.3(1 - y)y \quad (4.25)
\]

Figure 49: Calculation of refractive index for $In_y Ga_{1-y} N$ material for different values of $y$

### 4.2.2 Reflectance spectrum using $Al_x Ga_{1-x} N/In_y Ga_{1-y} N$ DBRs

Using the calculated values of $x = 0.332$ and $y = 0.08$, we obtain 99.51% reflectivity, shown in fig. 50 with parameters given in Table 6:
Table 6: Parameters used to obtain reflectivity spectrum for Al$_x$Ga$_{1-x}$N/In$_y$Ga$_{1-y}$N DBRs

<table>
<thead>
<tr>
<th>Reflectivity ($R$) (%)</th>
<th>Thickness (Al$<em>x$Ga$</em>{1-x}$N) x=0.332 (nm)</th>
<th>Thickness (In$<em>y$Ga$</em>{1-y}$N) y=0.08 (nm)</th>
<th>Stress (Al$<em>x$Ga$</em>{1-x}$N) x=0.332</th>
<th>Stress (In$<em>y$Ga$</em>{1-y}$N) y=0.08</th>
<th>Number of periods ($p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.51</td>
<td>60</td>
<td>56</td>
<td>5.01e$^{-10}$</td>
<td>5.01e$^{-10}$</td>
<td>53</td>
</tr>
</tbody>
</table>

Figure 50: Reflectivity versus wavelength using Al$_{0.332}$Ga$_{0.668}$N / In$_{0.08}$Ga$_{0.92}$N DBRs
4.3 Lattice matched Al$_{x}$In$_{1-x}$N/GaN based DBRs

Despite AlInN have attractive properties like wide bandgap range (0.7-6.2eV) and lattice matched with GaN. The third Nitride AlInN alloy has not been intensively studied as compared to ternary AlGaN and ternary InGaN alloys, because the growth of high quality film is hampered by phase separation issues [39] [40].

AlInN alloy exhibits the largest variation in bangap and shows a significant bowing in bandgap measurements [35]. AlInN based film has been fabricated using molecular beam epitaxy (MBE), Metal-organic chemical-vapor deposition (MOCVD) and reactive sputtering and then bandgap and refractive index are measured. However there were discrepancies in reported results [20] [36]. Refractive index calculations based on these measurements show strong deviation from the linear estimation [35].

There is no data available or expression to calculate the accurate AlInN refractive index as function of optical wavelength or photon energy. Because of lack of information about optical properties and physics of AlInN material system the research communities have been shifting towards fabrication of lattice matched AlInN/GaN DBRs. The fig. 51 shows the nitride alloys bandgap energy versus lattice constant that gives us a better view of lattice matched AlInN/GaN.

J. Dorsaz et al. [39] have reported lattice matched crack free Al$_{0.84}$Ga$_{0.16}$N/GaN DBRs with 10th and 20th number of periods, at design wavelength 455nm and 515nm and achieved 76% and 90% reflectivity with 35nm and 41nm FWHM (full width at half maximum) respectively [39], J.-F. Carlin et al. [41] also reported same results and after this 99.4% reflectivity is reported with a 30nm stopband centered at 450nm with 40th number of periods [39], Eric Feltin et al. [20] also reported same results in École Polytechnique Federale De Lausanne (EPFL).

If reader interested in more details regarding fabrication of lattice matched AlInN/GaN DBRs then please read PhD thesis is written by Julien DORSAZ from university of École Polytechnique Federale De Lausanne [1] and Final thesis is written by Reem Omer Rahmatalla Muhammad Boota from university of Linköping, Department of Technology, Sweden [42].

We took parameters given in Table 7 from [39] for lattice matched Al$_{0.83}$In$_{0.17}$N/GaN and implemented on our model and we found 99.43% reflectivity shown in fig. 52 at 450nm Bragg wavelength with 37th number of periods and 32nm stopband that is agree with fabricated Al$_{0.83}$In$_{0.17}$N/GaN DBRs deposited by Metal-organic Vapor Phase Epitaxy (MOVPE) [39].
Table 7: Parameters used for lattice matched Al$_{0.83}$In$_{0.17}$N/GaN DBRs

<table>
<thead>
<tr>
<th>Reflectivity ($R$) (%)</th>
<th>Thickness (Al$<em>{0.83}$In$</em>{0.17}$N) (nm)</th>
<th>Thickness (GaN) (nm)</th>
<th>Number of periods ($p$)</th>
<th>Bandwidth (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.43</td>
<td>50.44</td>
<td>46.03</td>
<td>37</td>
<td>32</td>
</tr>
</tbody>
</table>

Figure 51: Bandwidth versus Lattice constant and lattice matched Al$_{0.83}$In$_{0.17}$N/GaN
Figure 52: Reflectivity versus wavelength using lattice matched Al$_{0.83}$In$_{0.17}$N/GaN DBRs
5 Conclusion

In this project, we demonstrated third-Nitride-based DBRs for VCSEL in UV wavelength range. We made a model for DBR using the famous transmission matrix method. We implemented this model to all three nitrides alloys (Al$_x$Ga$_{1-x}$N, In$_y$Ga$_{1-y}$N and Al$_z$In$_{1-z}$N) and try to find highest possible reflection of primary peak with optimization of parameters like number of periods, design wavelength and composition of content. We also calculated refractive index and absorption coefficient of these nitrides alloys that are backbone for designing of any optoelectronics devices. We also discussed the difficulties for accurate calculations of optical parameters (bandgap, refractive index and absorption coefficient) and the effect of dispersion on bandwidth and on reflectivity of primary peak.

We found, AlGaN alloy has been well investigated and lots of information is already published. Up to now AlGaN has proved to be most promising material for nitride-based DBRs on the basis of best available information of optical parameters like refractive index and absorption coefficient and bandgap energy.

In$_y$Ga$_{1-y}$N alloy is commonly used in active region for quantum wells (QWs). We discussed difficulties to measure accurate refractive index of Indium rich alloy. We demonstrated AlGaN/InGaN DBRs that could be useful to overcome the stress issue in layers.

Despite Al$_z$In$_{1-z}$N has one of most potential candidate, providing lattice matched Al$_{0.83}$In$_{0.17}$N/GaN DBRs is less investigated compared to Al$_x$Ga$_{1-x}$N and In$_y$Ga$_{1-y}$N alloys. Probably the reason is strange nature of AlInN that make difficult to measure its refractive index, bandgap and absorption coefficient. We didn’t find any published experimental data and expression to calculate AlInN refractive index but some of research groups have fabricated lattice matched Al$_{0.83}$In$_{0.17}$N/GaN DBRs that we have discussed with details.
6 Future work

GaN and its ternary alloy AlGaN has proved to be a promising material for DBRs on the basis of available information but it still suffers from lowest refractive index contrast and highest lattice mismatch that leads to cracks and additional dislocation in Bragg mirrors that result in reduced reflectance and reduced FWHM (full width at half maximum).

These shortcomings of AlGaN based DBRs can be overcome using other nitride ternary alloys, i.e., InGaN and AlInN. The main challenge in future should be fabrication of pure homogeneous crystalline layers of InGaN and AlInN that makes possible to measure optical parameters like band gap energy, refractive index and absorption correctly. These experimental data could be used to form fitting parameters for Adachi model and Sellmeier model to calculate optical parameters for entire range of composition in In$_y$Ga$_{1-x}$N and Al$_z$Ga$_{1-z}$N alloys. The physics of AlInN and InGaN alloys are not well studied yet. It should be investigated intensively in future.
7 References


(Access April, 2009)


[43] H. M. Ng and T. D. Moustakas, “High reflectance III-Nitride Bragg reflectors grown by molecular beam epitaxy,” Electrical and Computer Engineering Department and Center for photonics Research, Boston University, USA, pp. 1-6


