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Effects of increasing fuzziness on analytic hierarchy process for spatial multicriteria decision analysis

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Abstract

Multicriteria decision analysis (MCDA) involves techniques which relatively recently have received great increase in interest for their capabilities of solving spatial decision problems. One of the most frequently used techniques of MCDA is Analytic Hierarchy Process (AHP). In the AHP, decision-makers make pairwise comparisons between different criteria to obtain values of their relative importance. The AHP initially only dealt with crisp numbers or exact values in the pairwise comparisons, but later it has been modified and adapted to also consider fuzzy values. It is necessary to empirically validate the ability of the fuzzified AHP for solving spatial problems. Further, the effects of different levels of fuzzification on the method have to be studied. In the context of a hypothetical GIS-based decision-making problem of locating a dam in Costa Rica using real-world data, this paper illustrates and compares the effects of increasing levels of uncertainty exemplified through different levels of fuzzification of the AHP. Practical comparison of the methods in this work, in accordance with the theoretical research, revealed that by increasing the level of uncertainty or fuzziness in the fuzzy AHP, differences between results of the conventional and fuzzy AHPs become more significant. These differences in the results of the methods may affect the final decisions in decision-making processes. This study concludes that the AHP is sensitive to the level of fuzzification and decision-makers should be aware of this sensitivity while using the fuzzy AHP. Furthermore, the methodology described may serve as a guideline on how to perform a sensitivity analysis in spatial MCDA. Depending on the character of criteria weights, i.e. the degree of fuzzification, and its impact on the results of a selected decision rule (e.g. AHP), the results from a fuzzy analysis may be used to produce sensitivity estimates for crisp AHP MCDA methods.

Keywords: Multicriteria decision analysis, Analytic hierarchy process, Fuzzy logic, Sensitivity analysis, Geographical information systems

1. Introduction

Multicriteria decision analysis (MCDA) supports analysts in making sound decisions for complex problems involving conflicting criteria. In MCDA, decision-makers define their subjective preferences between different criteria. A variety of MCDA techniques are available to find an optimal result based on the ranked criteria. Non-spatial MCDA methods assume spatial homogeneity of the criteria within the study area. However, the evaluation criteria often vary across space. As a consequence, there is a need to combine the MCDA with Geographical information systems (GIS) to include the spatial dimension in the analysis process (Malczewski, 1999). In GIS-based (or spatial) multicriteria decision analysis approaches, geographical data and value judgments are brought together to obtain more information for decision-makers in problems involving a large set of feasible alternatives and multiple evaluation criteria (Malczewski, 1999, 2006a).

One of the most widely used methods in spatial multicriteria decision analysis is the Analytical Hierarchy Process (AHP), introduced and developed by Saaty (1977, 1980). Applying the AHP in spatial multicriteria decision analysis, as described in Malczewski (1999), involves three major steps. In the first step, AHP decomposes the decision problem into a hierarchy of essential elements like e.g. goals and objectives, criteria, and sub-criteria. Decision-makers then, in the second step, compare these elements on a pairwise basis to estimate the relative importance of each element over each other, and make a comparison matrix of the ranks for each hierarchical level. In the third step, these matrices are combined in order to form composite weights representing the ratings of alternatives with respect to the goal. A wide variety of techniques exist for computing the alternative weights, e.g. the lambda max and geometric mean techniques (discussed in Section 2.2.3).

Although the AHP is a powerful technique in the MCDA for combining and evaluating attributes and objectives, some researchers claim that the AHP shows limited applicability in complex spatial decision situations. Warren (2004) pointed out some of the problems associated with the AHP;

among them possible rank reversal. The possible rank reversal in the AHP means that the addition and deletion of a criterion might change the relative ranking of the other criteria (Wang & Elhag, 2006). Yang and Chen (2004) mentioned that the uncertainty associated with the conversion of decision-makers judgments to the numbers in verbal judgment of preference is not considered by the AHP and personal judgment of the decision-makers may also have a huge effect on the AHP's results.

To overcome these types of problems, some researchers attempt to modify the original methods by incorporating the concept of fuzzy linguistic quantifiers into the GIS-based MCDA (Malczewski, 2006b). For instance some authors integrate fuzzy theory with the AHP to use the flexibility of the modified model in formulating uncertainty. For example, Yager and Kelman (1999) introduced an extension of the AHP using ordered weighted averaging operators for structuring and modelling decision problems. Buckley (1985a) considered a trapezoidal membership function for comparison ratios in the AHP and Chang (1996) developed a new approach for the triangular case. However, some researchers are against fuzzifying the AHP. For example, Saaty, the developer of the AHP, believes that due to some uncertainties in the nature of the AHP, the method is already fuzzy and there is no theoretical proof that making the AHP fuzzier will lead to better results (Saaty, 2006; Saaty & Tran, 2007).

Quite a few case studies exist where MCDA has been used in a spatial context, but reported use of fuzzy MCDA where AHP has been applied in GIS is still relatively scarce. One of the early case studies was carried out by Sui (1992), who treated urban land evaluation, and among the recent ones there are e.g. Gemitzi, Tsihrintzis, Voudrias, Petalas, and Stravodimos (2007) who dealt with municipal solid waste landfills, Vahidnia, Alesheikh, and Alimohammadi (2009) who treated hospital site selection, and Vadrevu, Eaturu, and Badarinath (2010) who looked at fire risk evaluation. But so far, there have not been enough empirical studies on conceptual and operational validation of the MCDA's ability in solving real-world spatial problems (Malczewski, 2006a). Nor are there many empirical studies on comparisons between different GIS-based multicriteria decision methods; previous attempts to compare the methods have mostly been done in theory. Moreover, almost no previous research considers different degrees of uncertainty for pairwise comparison ratios. In most of the previous work, different methods have been compared when the uncertainty in comparison ratios has been constant; e.g. fuzzy ratios described by constant triangular membership functions have theoretically been compared with the logarithmic least squares method in van Laarhoven and Pedrycz (1983). Therefore, there is a need for a comparative analysis of the GIS-based multicriteria decision methods in the context of real-world spatial decision problems. The general scope of this paper is to empirically test how results of the AHP, as one of the most used GIS-based multicriteria decision methods, change through the introduction of fuzzy functions. This is done in the context of a hypothetical GIS-based decision problem of locating a dam in Costa Rica using real-world data. The specific aims of the paper are to see whether there is a significant difference in the results:

- When different methods are used to determine the weights, i.e. lambda max and the geometric mean methods.

- Between different degrees of fuzzification of the AHP.

A spin-off of the study is that the procedure applied can also function as a guideline on how to perform a sensitivity analysis in spatial MCDA given the subjective uncertainty related to weighting. Although some examples on sensitivity analyses exist (e.g. Chen, Yu, & Khan, 2010), "Spatial sensitivity analysis is an underdeveloped component of sensitivity analysis in spatial multicritiera evaluation" (Ligmann-Zielinska & Jankowski, 2008, p. 225). Ligmann-Zielinska and Jankowski (2008) not only give a full discussion on spatial sensitivity analysis, they also present a framework for sensitivity analysis. Therefore, the procedure presented here in this paper can be seen as part of their "spatiality" axis on their sensitivity analysis cube where "the stability of solution to changes in weights (weighting)" is one of that axis' major factors (Ligmann-Zielinska & Jankowski, 2008, p. 220). Section 2 gives an overview on the essential concepts of spatial MCDA in general and the AHP in particular. The differences between the conventional and the fuzzy AHP are outlined in order to understand the implications of the present work. Section 3 presents the case study and the data used for testing the different types of the AHP. Section 4 demonstrates the results of the comparison of the different methods, and Section 5 gives a discussion of the results and their implications for a spatial decision-making process.

2. Methodology

2.1. Multicriteria decision analysis (MCDA)

Multicriteria decision analysis (MCDA) is based on a number of pivotal evaluation criteria, defined according to the conditions of the problem being considered. In most decision-making problems, the management team has already a well-defined goal. In order to reach this goal, it is necessary to choose from a number of options. In MCDA, these options are referred to as alternatives. The potential alternatives have different attributes and characteristics. The decision-makers try to choose the best among them by considering the effects these alternatives have on the quality of the final result. The relative importance of each alternative is based on a number of different criteria. Decision-makers rank these criteria over each other in order to determine a weight for each criterion.

In GIS-based (or spatial) MCDA, each of the criteria is represented as an information layer, either as a constraint or as a factor map (Malczewski, 1999). Constraint maps are Boolean maps representing restrictions or limitations of the decision-making problem, prohibiting certain decisions to be taken. Factor maps describe the opportunity criteria and represent the quality of achieving an objective through a spatial distribution (Malczewski, 1999). How to combine information from several criteria layers to form a single index of evaluation is the primary issue in multicriteria evaluation (Eastman, 2003). Combining the criteria is done using one of the different decision rules. A decision rule allows for assessing the preference of all the alternatives and finding the optimal decision (Malczewski, 1999). The decision rule combines the available information on the alternatives with the preferences of the decision-makers into an overall composite index. In spatial MCDA, the additive decision rules are most commonly used. The additive decision rules with a weighted linear combination of factors form a suitability map. The suitability map may be a raster layer that contains the final scores of the pixels thereby indicating how suitable each pixel is regarding the goal of the problem. The suitability is then defined as $S = \sum_i w_i x_i$, where w_i is the weight of factor i, and x_i is the factor map i. In cases where constraints are also applied in the procedure, simply the product of them is multiplied by the suitability calculated from the factors (Eastman, 2003), i.e. $S = \sum_i w_i x_i \cdot \prod_j c_j$ where c_j = constraint map j.

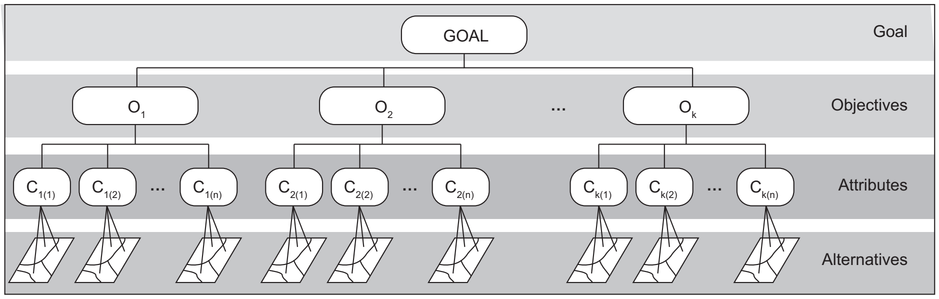

Fig. 1. Structure of the AHP.

One of the most commonly used decision rules in MCDA is the Analytical Hierarchy Process (AHP). It is an additive decision rule which allows structuring the problem in a hierarchy and provides a powerful tool for the MCDA procedure.

2.2. Analytical Hierarchy Process (AHP)

With AHP, the decision-making process starts with defining the problem, determining the goal, and finding out objectives or criteria that are important to the problem. It continues with structuring and dividing the problem into a hierarchical form of ordered levels and sub-levels. Fig. 1 illustrates the structure of the AHP elements. At the top is the final goal, followed by a number of objectives, attributes, and alternatives. In the AHP the alternatives are represented in GIS databases while each layer contains the attribute values assigned to the alternatives and each alternative is related to the higher-level attributes (Malczewski, 1999).

2.2.1. Pairwise comparison matrix

In AHP, the weights for the criteria are calculated separately for each hierarchical level. The AHP allocates a pairwise comparison matrix for each hierarchical level. These comparison matrices are defined by ranking the criteria of the corresponding level over each other. Decision-makers determine the ranking of the criteria on the basis of the importance of each criterion over the other, considering their significance in the problem. Saaty (1977, 1980) developed a system of numbers combined with a verbal judgment range for aiding the decision-makers in defining the ranks of the criteria (Table 1). The verbal judgment range stretches from equality to extreme importance of one

Table 1
Scales in pairwise comparisons (Saaty, 1980).

Intensity of importance	**Verbal judgment of preference**
1	Equally important
3	Moderately more important
5	Strongly more important
7	Very strongly more important
9	Extremely more important
2, 4, 6, 8	Intermediate values between adjacent scale values

Table 2
Random Inconsistency Index (RI) for n = 1, 2,..., 9 (Saaty, 1980).

N	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

factor over another. These verbal values correspond to the odd-numbered numerical judgments between 1 and 9 with some intermediate even-numbered values in between (Saaty, 1990). If the elements of the pairwise comparison matrix (the comparison ratios) are shown with a_{ij}, which indicates the importance of ith factor over jth, then a_{ji} can be calculated as $1/a_{ij}$ (Boroushaki & Malczewski, 2008).

2.2.2. Consistency ratio

For measuring the consistency of the subjective ranking, a consistency ratio is computed for each pairwise comparison matrix. A matrix is called consistent if and only if $a_{ik} \cdot \ a_{kj} = a_{ij}$, where a_{ij} is the ijth element of the matrix (Buckley, 1985a). In problems with a large number of criteria, expressing subjective judgments generally lead to matrices that are not quite consistent. A measure of how far a matrix is from consistency is performed through the consistency ratio (CR). Using the biggest eigenvalue, λ_{max}, the consistency index (CI) is determined by $CI = (\lambda_{max} - n)/(n - 1)$, where n is the number of criteria (Malczewski, 1999). The final consistency ratio is calculated by comparing the CI with a random index (RI) depending on the number of elements being compared, $CR = CI/RI$ (Table 2). RI is the consistency index of a randomly generated pairwise comparison matrix (Malczewski, 1999). The ratio indicates a reasonable level of consistency in the pairwise comparison if $CR < 0.10$. If $CR \geq 0.10$, the values of the ratio indicate inconsistent judgments.

2.2.3. Calculating the final weights

A wide variety of techniques exist that calculate the final relative weights of each criterion. Two of the most commonly used are the lambda max and the geometric mean techniques. For obtaining the final weights using the lambda max technique, a vector of weights is defined as the normalized eigenvector corresponding to the largest eigenvalue, λ_{max}. If the weights are shown as a vector w consisting of w_i ($i = 1, \ldots, n$), they are calculated as follows: $C \times w = \lambda_{max} \times w$, where C is the pairwise comparison matrix of the criteria, w is the vector of weights, and λ_{max} is the largest, or principal, eigenvalue of C. To simplify calculating eigenvectors in the lambda max technique, Malczewski (1999) has developed an approximation of the eigenvector associated with the maximum eigenvalue. In this procedure, normalized pairwise weights are calculated by dividing each pairwise weight by the sum of pairwise weights in each column. Then the arithmetic average of each row of the normalized matrix gives the resulting relative weight of the corresponding criterion or alternative. The accuracy of this approximation is increased when the pairwise comparison matrix has a low consistency ratio.

In the geometric mean technique, as Buckley (1985a) explained, first the geometric mean of each row of the pairwise comparison matrix is computed as

$$r_i = \prod_{j=1}^{n} (a_{ij})^{1/n} \tag{1}$$

where a_{ij} ($i, j = 1, ..., n$; n is the number of rows and columns and i and j are identifiers of a row and column, respectively) are the comparison ratios (element of the pairwise comparison matrix), and n is number of the criteria. Then, by using the following formula, the weights of each criterion are calculated:

$$w_i = \frac{r_i}{\sum_j r_j} \tag{2}$$

In the scientific literature, there has been some controversy whether to use the lambda max or the geometric mean techniques (see e.g. Buckley, Feuring, & Hayashi, 2001; Chang et al., 2008). As Buckley (1985a) stated, calculating the eigenvector is sometimes problematic in the lambda max technique and a decision cannot be made on selecting one criterion over another. Crawford and Williams (1985) and Barzilai (1997) also prefer to use the geometric mean instead of the lambda max. Barzilai (1997) even believes that the geometric mean is the only acceptable solution to the problem of deriving weights from pairwise comparisons in the AHP. Buckley (1985a) stated that in the case of a consistent pairwise comparison matrix, the geometric mean method always produces the same weights as the lambda max technique. Also if $n = 3$ (n is number of criteria), then both methods give the same weights; and when $n > 3$ then the results for the weights in both methods are close (Buckley, 1985a). In this paper we have used both lambda max and geometric mean techniques to show how they differ in the context of fuzzy AHP.

2.3. Fuzzy AHP

The pairwise comparison matrix is created based on the subjective judgments of the decision-makers. As Yang and Chen (2004) mentioned, there might be some uncertainty associated with this judgment that is not taken into account by the AHP. In some cases differences in the subjective judgment can have an important effect on the outcome of the AHP. One possibility to overcome these problems might be through modifying the original AHP by incorporating the concept of fuzzy numbers with different levels of uncertainty. In this approach, the crisp numbers in the comparison ratios are replaced by fuzzy numbers. Using fuzzy numbers gives more flexibility to decision-makers while defining the comparison ratios. It also allows integrating the decision-makers uncertainty in ranking the criteria into the AHP process and it might moderate the rank reversal problem.

2.3.1. Fuzzy pairwise comparison matrix

The comparison ratios can be fuzzified through the use of trapezoidal membership functions (Buckley, 1985a). Suppose that a fuzzy number is described as $(\alpha, \beta, \gamma, \delta)$, then the membership function for this fuzzy number is $\mu(x)$ where these two numbers form an ordered pair $(x, \mu(x))$. The graphic description of the fuzzy membership function shown in Fig. 2 is determined as follows: zero to the left of α, a line segment from $(\alpha, 0)$ to $(\beta, 1)$, a horizontal line segment from $(\beta, 1)$ to $(\gamma, 1)$, a line segment from $(\gamma, 1)$ to $(\delta, 0)$, and zero to the right of δ (Buckley, 1985a). The wideness $(\gamma - \beta)$ defines the level of uncertainty; a larger $\gamma - \beta$ represents a higher uncertainty. Note that if $\gamma = \beta$ then the trapezoidal fuzzy membership function shape is converted to a triangular shape. The crisp comparison ratio a_{ij} (i.e. the ratio in the ith row and jth column of the normal pairwise comparison matrix) can be

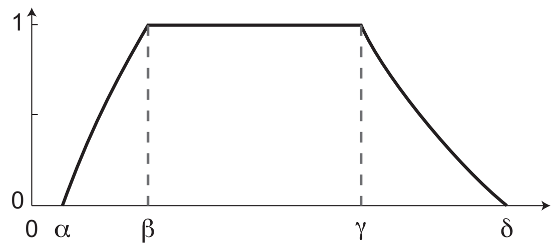

Fig. 2. Trapezoidal fuzzy membership function of $(\alpha, \beta, \gamma, \delta)$.

represented by a fuzzy comparison ratio $\bar{a}_{ij} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$, where $\bar{a}_{ij}$ is the ratio in the ith row and jth column of the fuzzy pairwise comparison matrix.

2.3.2. Consistency ratio for fuzzy weights

Buckley (1985a) proved the following theorem: Consider the fuzzy comparison matrix $\bar{A} = [\bar{a}_{ij}]$, where $\bar{a} = (\alpha_{ij}, \beta_{ij}, \gamma_{ij}, \delta_{ij})$ are the fuzzy comparison ratios, and let $(\beta_{ij} \leq a_{ij} \leq \gamma_{ij})$ for all i, j; If the crisp comparison matrix $A = [a_{ij}]$, where the a_{ij} are the crisp comparison ratios, is consistent: then fuzzy comparison matrix $\bar{A}$ is also consistent. So if the conditions mentioned in the theorem are satisfied and the consistency ratio of the main matrix A is low, then the fuzzified matrix $\bar{A}$ can also be considered as a consistent matrix.

2.3.3. Calculating the final fuzzy weights

The lambda max technique is not easily extendable to fuzzy matrices for determining the weights (Buckley, 1985a). In this study we have used the geometric mean technique which can be uncomplicatedly extended to fuzzy pairwise matrices. For calculating the final weights based on the fuzzy pairwise comparison matrix, the membership function μ_i for the fuzzy weights is determined by the following formulas (Buckley, 1985b). Let

$$f_i(y) = [\prod_{j=1}^{n}((\beta_{ij} - \alpha_{ij})y + \alpha_{ij})]^{1/n} \tag{3}$$

and

$$g_i(y) = [\prod_{j=1}^{n}((\gamma_{ij} - \delta_{ij})y + \delta_{ij})]^{1/n} \tag{4}$$

$for\ 0 \leqslant y \leqslant 1$.

Define

$$\alpha_i = [\prod_{j=1}^{n} \alpha_{ij}]^{1/n} \tag{5}$$

and

$$\alpha = \sum_{i=1}^{n} \alpha_i \tag{6}$$

Similarly, define β_i and β, γ_i and γ, δ_i and δ. Finally let

$$f(y) = \sum_{i=1}^{n} f_i(y) \tag{7}$$

and

$$g(y) = \sum_{i=1}^{n} g_i(y) \tag{8}$$

The final fuzzy weights w_i are determined as $(\alpha_i/\delta, \beta_i/\gamma, \gamma_i/\beta, \delta_i/\alpha)$, where the graph of μ_i is zero to the left of α_i/δ, on the interval $[\alpha_i/\delta, \beta_i/\gamma]$ is $x = f_i(y)/g(y)$, a horizontal line from $[\beta_i/\gamma, 1]$ to $[\gamma_i/\beta, 1]$, on the interval $[\gamma_i/\beta, \delta_i/\alpha]$ is $x = g_i(y)/f(y)$ and zero to the right of δ_i/α. Here the x-axis is horizontal and the y-axis is vertical, therefore the graph of $x = f_i(y)/g(y)$ starts at $[\alpha_i/\delta, 0]$ for $y = 0$ and monotonically increase to $[\beta_i/\gamma, 1]$ as y grows from 0 to 1.

2.3.4. Defuzzification

In order to use the obtained fuzzy weights in GIS software and to compare them with the non-fuzzy weights, it is necessary to convert the fuzzy weights to ordinary crisp numbers. The procedure of approximating fuzzy numbers to crisp numbers is referred to as defuzzification. Different defuzzification methods exist and according to, e.g. Liu (2007) and Opricovic and Tzeng (2003), one of the most commonly used is the centre of gravity method (COG). This method computes the centre of gravity of the area under the membership function, i.e.

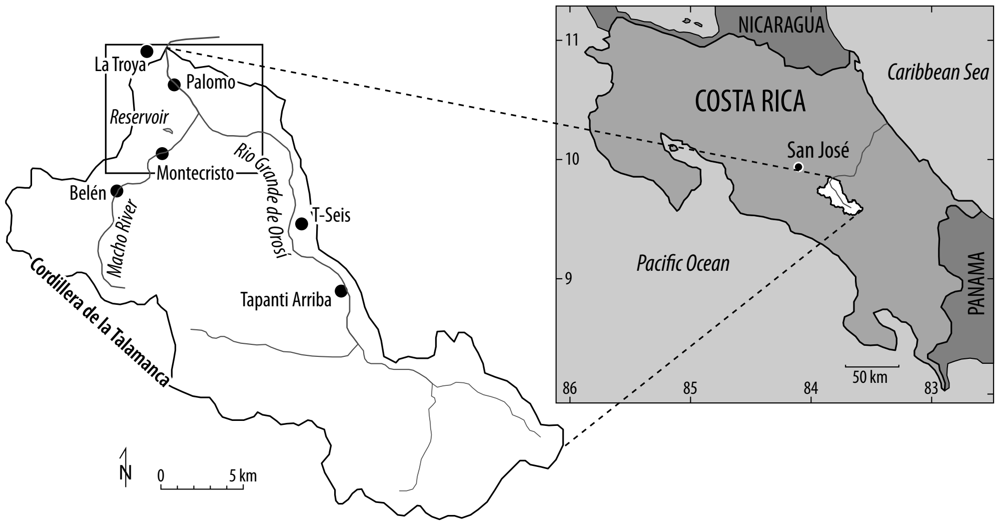

Fig. 3. Map of the study area, covering the watershed of the Río Grande de Orosí and its tributary the Macho River.

$$COG(x_0) = \frac{\int_{x=\alpha}^{\delta} \mu_X(x).xdx}{\int_{x=\alpha}^{\delta} \mu_X(x)dx} \tag{9}$$

in which $\mu_X(x)$ is the membership function of the fuzzy number, and $\alpha \leq x \leq \delta$ (Liu, 2007).

3. Case study: locating a dam in Costa Rica

To be able to practically test the probable differences in results of different types of the AHP in spatial-MCDA, we decided to use a case study of finding the best location for a hydroelectric dam in Costa Rica. The study area covers a small portion (approximately 384 km^2) of Costa Rica and is located in the middle of the country, slightly south of Cartago city. It contains the Orosí River, its tributary the Macho River, some small towns and villages such as Orosí and San Rafael, and abundant areas of rain forest (Fig. 3).

3.1. Data

The main source of data for this work is the Shuttle Radar Topography Mission (SRTM) 90 m Digital Elevation Model (DEM) (available from the CGIAR Consortium for Spatial Information website http://srtm.csi.cgiar.org).

These data were used to derive most of the hydrological features. We also used land information data (Saborio, no year), rainfall data for different rain stations in the area (Vahrson, 1992), and topographic maps of the area (scale 1:50,000). The purpose of the latter was to check the derived hydrological features.

3.2. Considered constraints and factors

Choosing the optimal location for a hydroelectric dam involves several evaluation criteria. The general rule for selecting evaluation criteria, which is problem specific, is that criteria should be identified with respect to the decision situation (Malczewski, 1999). According to Baban and Wan-Yusof (2003), evaluation criteria for dam and reservoir site selection include: economy, hydrology and hydraulics, topography, geology, construction related issues, and environmental considerations. Because the purpose of this study is not to produce a map including all aspects involved for actual dam-site planning, but rather to show the implications for issues regarding selection of the AHP in spatial MCDA, we only used a subset of criteria. We decided to use three constraints and seven factors to make the study more realistic. Criteria considered in this study are itemized and described in Tables 3 and 4. Thus: the main goal is to locate a new dam site; the objectives or main criteria can be described as e.g. economic, land conservation, and hydrology; the attributes are represented by different GIS factor layers (e.g. degree of undulation as a measure of cost for building a dam, or distance to natural parks as a measure of land conservation); and finally the alternatives are represented by all the raster cells (cf. Fig. 1 and Boroushaki & Malczewski, 2008, p. 401). However, to be able to focus on the influence on the different degrees of fuzzification and not to make the dependence on different criteria unnecessary complicated, this case study has treated all factor layers as if there was only one objective and that all attributes are on the same AHP hierarchy level below that objective. This approach makes the results from this study of use to both simpler linear weighted combination methods, where no hierarchy levels are included, as well as to AHP, due to the use of Saaty's approach of calculating weights from pairwise comparisons. The pairwise comparisons were used to generate the relative AHP weight for each of the factor layers, while the individual raster cell values were given a value between 0 and 255 to represent each pixels rank or suitability for each of the considered factor criteria (see Eastman, Jin, Kyem, & Toledano, 1995). All constraint and factor maps needed for the spatial analysis in this work

Table 3
Constraint types considered in the work.

Constraint map	Description
River	The dam must be built on a river. River = 1, others = 0
Reservoir	The dam cannot be built over the existing reservoir. Reservoir = 0, others = 1
National Park	National Parks in the study area are protected and the dam cannot be built in them. National Park = 0, others = 1

Table 4
Factor types considered in the work.

Factor map	Description
Undulation	The higher undulation, the better
Road	The closer to roads, the better
City	The closer to cities, the better
Water discharge	The larger the water discharge, the better
Hydraulic head	The higher the value of hydraulic head (i.e. steeper), the better
Agriculture	The farther away from agricultural land, the better
Forest	The farther away from virgin rain forest, the better

have been created using standard GIS software; each map is represented by a raster layer.

We evaluated the importance of the factors over each other by comparing the impact of the factors on locating the dam as a hydroelectricity power station. One of the most important factors is the degree of undulation because of its economic effect and influence on the generation of energy. Higher undulation and larger variation in terrain surrounding the river give a possibility for building a shorter dam wall. Distance between the dam and the cities and roads in the study area are the two next important considerations economically, because of the effect of easier power distribution to the users. Two other significant factors are water discharge and hydraulic head because they are directly related to the availability of hydropower at a given location (Gismalla & Bruen, 1996). Agriculture and forest consideration are the last two factors considered in the work due to croplands development and environmental laws in Costa Rica. The resulting pairwise comparison matrix for crisp ratios in the work is presented in Table 5. The consistency ratio (CR) calculated for this comparison matrix is fairly low, equal to 0.055, which indicates an acceptable level of uncertainty in the matrix (see Section 2.2.2).

3.3. Degrees of fuzzification

In order to make comparable results, we considered four different levels of fuzzification in membership functions in this work. The four different membership functions for the comparison ratios are categorized to triangular, narrow trapezoidal, medium trapezoidal and wide trapezoidal. The wideness of the membership functions is set to:

- $\gamma_{ij} - \beta_{ij} = 0$ and $\delta_{ij} - \alpha_{ij} = 2$ for the triangular;
- $\gamma_{ij} - \beta_{ij} = 1$ and $\delta_{ij} - \alpha_{ij} = 2$ for the narrow trapezoidal;
- $\gamma_{ij} - \beta_{ij} = 1.5$ and $\delta_{ij} - \alpha_{ij} = 3$ for the medium trapezoidal;
- $\gamma_{ij} - \beta_{ij} = 2$ and $\delta_{ij} - \alpha_{ij} = 3$ for the wide trapezoidal type.

The larger $\gamma_{ij} - \beta_{ij}$ and $\delta_{ij} - \alpha_{ij}$ represents a bigger level of uncertainty. Considering the crisp ratios presented in Table 5 and applying the four different levels of membership functions, we calculated four fuzzy pairwise comparison matrices for each level of uncertainty. A combined fuzzy pairwise comparison matrix for different levels of uncertainty is presented in Table 6. Empty elements in these matrices contain the corresponding fuzzy positive reciprocal values, i.e. $\bar{a}_{ji} = (\bar{a}_{ij})^{-1} = [\delta_{ij}^{-1}, \gamma_{ij}^{-1}, \beta_{ij}^{-1}, \alpha_{ij}^{-1}]$ and $\bar{a}_{ii} = [1, 1, 1, 1]$ for all $i = 1, \ldots, n$ (Buckley, 1985a). As the consistency ratio (CR) for the crisp comparison matrix in this work is fairly low (see Section 3.2), and the condition of the theorem presented in Section 2.3.2 is completely satisfied, the same consistency ratio of the crisp comparison matrix is considered for the fuzzy pairwise comparison matrix.

4. Results

The factors' weights in this work are computed by running MATLAB codes written for this purpose. The factors' weights for non-fuzzy lambda max and geometric mean are calculated based on the description of calculating the final weights in Section 2.2.3, considering the crisp pairwise comparison matrix presented in Table 5. Fuzzy weights of the factors are computed according to the equations and descriptions of calculating the final fuzzy weights in Section 2.3.3, considering the fuzzy pairwise comparison matrices for the four different membership functions (Table 6). The computed fuzzy

Table 5
Pairwise comparison matrix for crisp ratios.

	Forest	**Agriculture**	**Hydraulic head**	**Water discharge**	**City**	**Road**	**Undulation**
Forest	1	1/3	1/4	1/4	1/5	1/6	1/7
Agriculture	3	1	1/3	1/3	1/4	1/4	1/5
Hydraulic head	4	3	1	1/2	1/3	1/4	1/4
Water discharge	4	3	2	1	1/3	1/2	1/3
City	5	4	3	3	1	1/2	1/2
Road	6	4	4	2	2	1	1/2
Undulation	7	5	4	3	2	2	1

Table 6
Pairwise comparison matrix for trapezoidal fuzzy ratios. $\Theta = 1$ for triangular and narrow trapezoidal functions, $\Theta = 1.5$ for medium and wide trapezoidal functions. $\theta = 0$ for triangular, $\theta = 0.5$ for narrow, $\theta = 0.75$ for medium, and $\theta = 1$ for wide trapezoidal functions.

	Forest	Agriculture	Hydraulic head	Water discharge	City	Road	Undulation
Forest	[1,1,1,1]						
Agriculture	[3-Θ;3-θ;3+θ;3+Θ]	[1,1,1,1]					
Hydraulic head	[4-Θ;4-θ;4+θ;4+Θ]	[3-Θ;3-θ;3+θ;3+Θ]	[1,1,1,1]				
Water discharge	[4-Θ;4-θ;4+θ;4+Θ]	[3-Θ;3-θ;3+θ;3+Θ]	[2-Θ;2-θ;2+θ;2+Θ]	[1,1,1,1]			
City	[5-Θ;5-θ;5+θ;5+Θ]	[4-Θ;4-θ;4+θ;4+Θ]	[3-Θ;3-θ;3+θ;3+Θ]	[3-Θ;3-θ;3+θ;3+Θ]	[1,1,1,1]		
Road	[6-Θ;6-θ;6+θ;6+Θ]	[4-Θ;4-θ;4+θ;4+Θ]	[4-Θ;4-θ;4+θ;4+Θ]	[2-Θ;2-θ;2+θ;2+Θ]	[2-Θ;2-θ;2+θ;2+Θ]	[1,1,1,1]	
Undulation	[7-Θ;7-θ;7+θ;7+Θ]	[5-Θ;5-θ;5+θ;5+Θ]	[4-Θ;4-θ;4+θ;4+Θ]	[3-Θ;3-θ;3+θ;3+Θ]	[2-Θ;2-θ;2+θ;2+Θ]	[2-Θ;2-θ;2+θ;2+Θ]	[1,1,1,1]

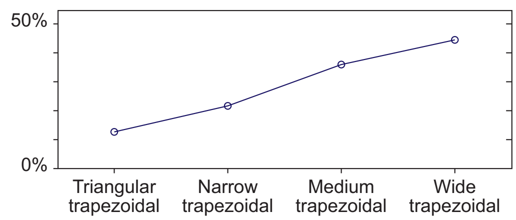

Fig. 4. Percentage difference between the weights obtained from various levels of uncertainty in the fuzzy AHP and geometric mean method.

weights for the fuzzy AHP have to be defuzzified before being applied in the GIS software (see Section 2.3.4), so the fuzzy weights are converted to crisp numbers based on the defuzzification procedure described in Section 2.3.4. The defuzzified weights then are normalized by dividing each of them by the sum of all factors weights. The final resulting factor weights of the AHP (lambda max and geometric mean) and the final defuzzified and normalized weights of the fuzzy AHP for four different degrees of fuzzy functions are shown in Table 7.

A comparison of the final weights in Table 7 shows some differences in the weights obtained from the different methods of the AHP. The differences between weights obtained by the lambda max and geometric mean methods are small while by increasing the level of the uncertainties in the fuzzy methods, the resulting weights show more significant differences.

To illustrate the effect of increasing uncertainties, Fig. 4 shows the increasing differences between the weights obtained from the geometric mean and the weights obtained from each fuzzified method. The difference is calculated by first subtracting the weight vector of the non-fuzzy geometric mean method from each fuzzified method's vector and then dividing by the geometric mean weight. Finally the sum of the absolute values of all elements in the normalized weight difference vector gives the final difference value. The percentage difference $D\%$ is:

$$D\% = 100 \times \sum_i \frac{|w'_i - w_i|}{w_i} \tag{10}$$

where w'_i is the defuzzified and normalized weight number i which has been calculated through the fuzzy AHP geometric mean method and w_i is the weight number i calculated from the geometric mean method.

Table 7
AHP weights obtained from the AHP lambda max and geometric mean methods and defuzzified weights from the different degrees of fuzzy functions.

Factor	**Lambda-max**	**Geometric-mean**	**Fuzzy-triangular**	**Fuzzy-narrow-trapezoidal**	**Fuzzy-medium-trapezoidal**	**Fuzzy-wide-trapezoidal**
Undulation	0.3069	0.3126	0.3022	0.2953	0.2833	0.2766
Road	0.2259	0.2293	0.2292	0.2292	0.2292	0.2291
City	0.1869	0.1864	0.1919	0.1961	0.2039	0.2075
Water discharge	0.1132	0.1128	0.1159	0.1179	0.1211	0.1232
Hydraulic head	0.0846	0.0804	0.0822	0.0833	0.0851	0.0862
Agriculture	0.0526	0.0495	0.0498	0.0498	0.0495	0.0496
Forest	0.0298	0.0290	0.0288	0.0285	0.0279	0.0277

We have implemented the weights in the GIS, based on the described procedure for spatial-MCDA in Section 2.1, to produce the final maps of optimum locations for dam construction. The comparison of the final maps shows some differences in representing optimum locations of the dam. These differences are due to the differences in the weights obtained from different methods of the AHP. In order to be able to compare the results of the different methods, the suitability maps of different methods are shown in Fig. 5 and to make the comparison of the maps more viewable, only the northernmost part of the study area which contains the highest suitability scores, i.e. with values over 150, is shown (see Fig. 3 for location). Parts outside this area contain lower ranked pixels which indicate not very suitable locations for dam construction and therefore are not of interest. As described in Section 2.1, the final scores in suitability maps show the location preferences for the dam. In Fig. 5, the highest suitability scores represent the best locations for the dam construction.

To analyze the effect of different degrees of uncertainty and to illustrate how this procedure can function as a sensitivity analysis, the resulting maps are compared with the maps obtained by the AHP geometric mean, which served as a basis for the fuzzified methods. In order to compare the methods, the suitability map of the geometric mean method has been subtracted from the suitability maps of the each other methods (Fig. 6). In MCDA usually only the highest ranked pixels, i.e. in this case the classes of highest suitability scores in Fig. 5, are the ones of interest as differences in high suitability values may lead to different decisions. In Fig. 6, a positive difference in the final scores indicates that the location has obtained a lower score by the geometric mean method than by the compared method, and a negative difference in the final scores means the geometric mean method considers this location as more optimal for dam construction.

Fig. 7 shows the difference of each pixel score for every method subtracted by the corresponding pixel score of the geometric mean method. The plots show increasing differences with increasing level of uncertainty in the fuzzy methods. For higher pixel scores, in some cases, the differences can be quite considerable and therefore lead to divergent decisions in the location choice.

To examine if the relationship between the degree of uncertainty in fuzzification and the differences between the resulting maps is statistically significant, a hypothesis was tested using a Chi-Square test. The suitability scores of the resulting maps were classified and the differences in number of observations per priority class were tested, where the significant level was set to

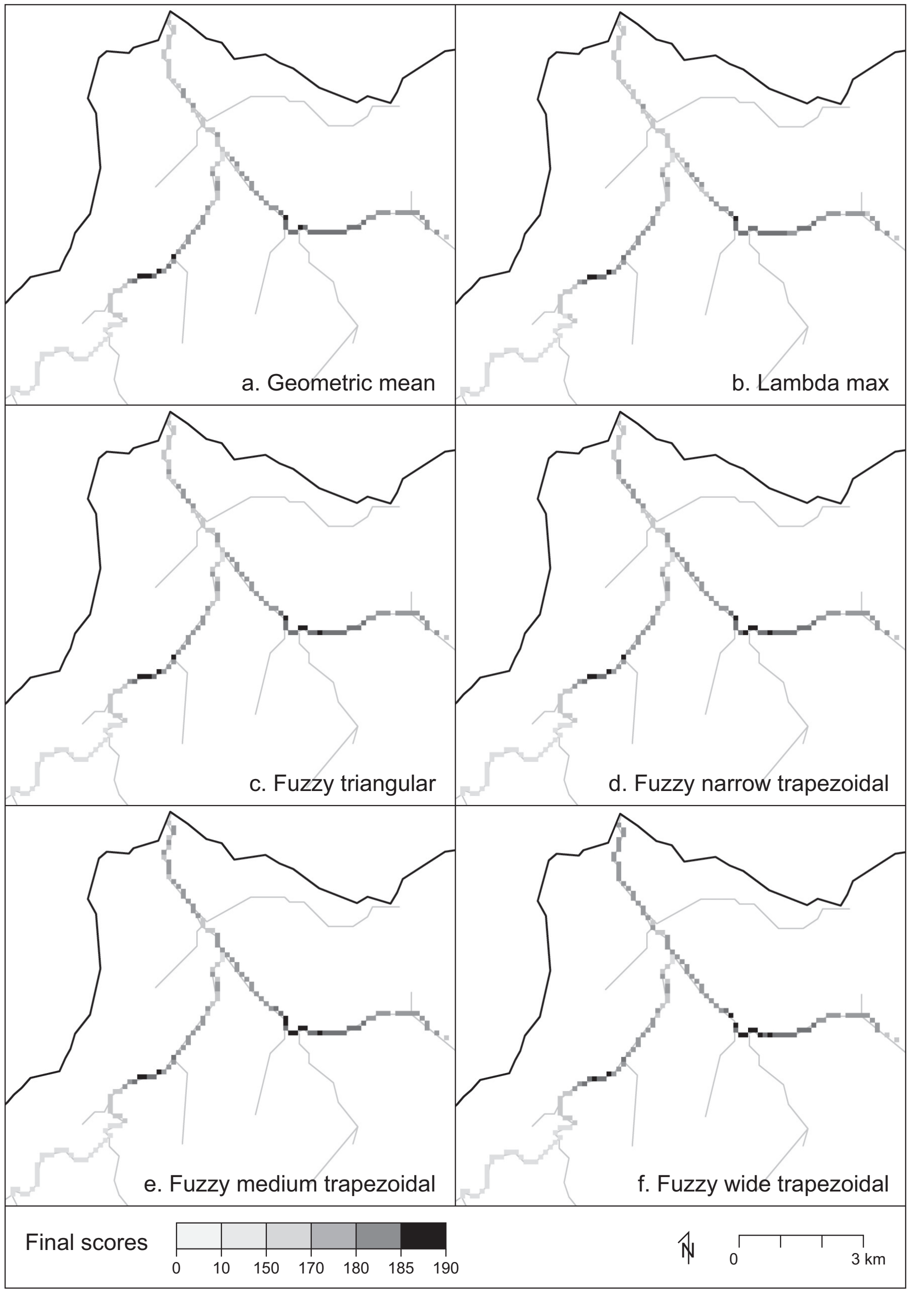

Fig. 5. Suitability maps representing the final scores of pixels for the: (a) geometric mean, (b) lambda max, (c) fuzzy triangular, (d) fuzzy narrow trapezoidal, (e) fuzzy medium trapezoidal, and (f) fuzzy wide trapezoidal methods. See Fig. 3 for location.

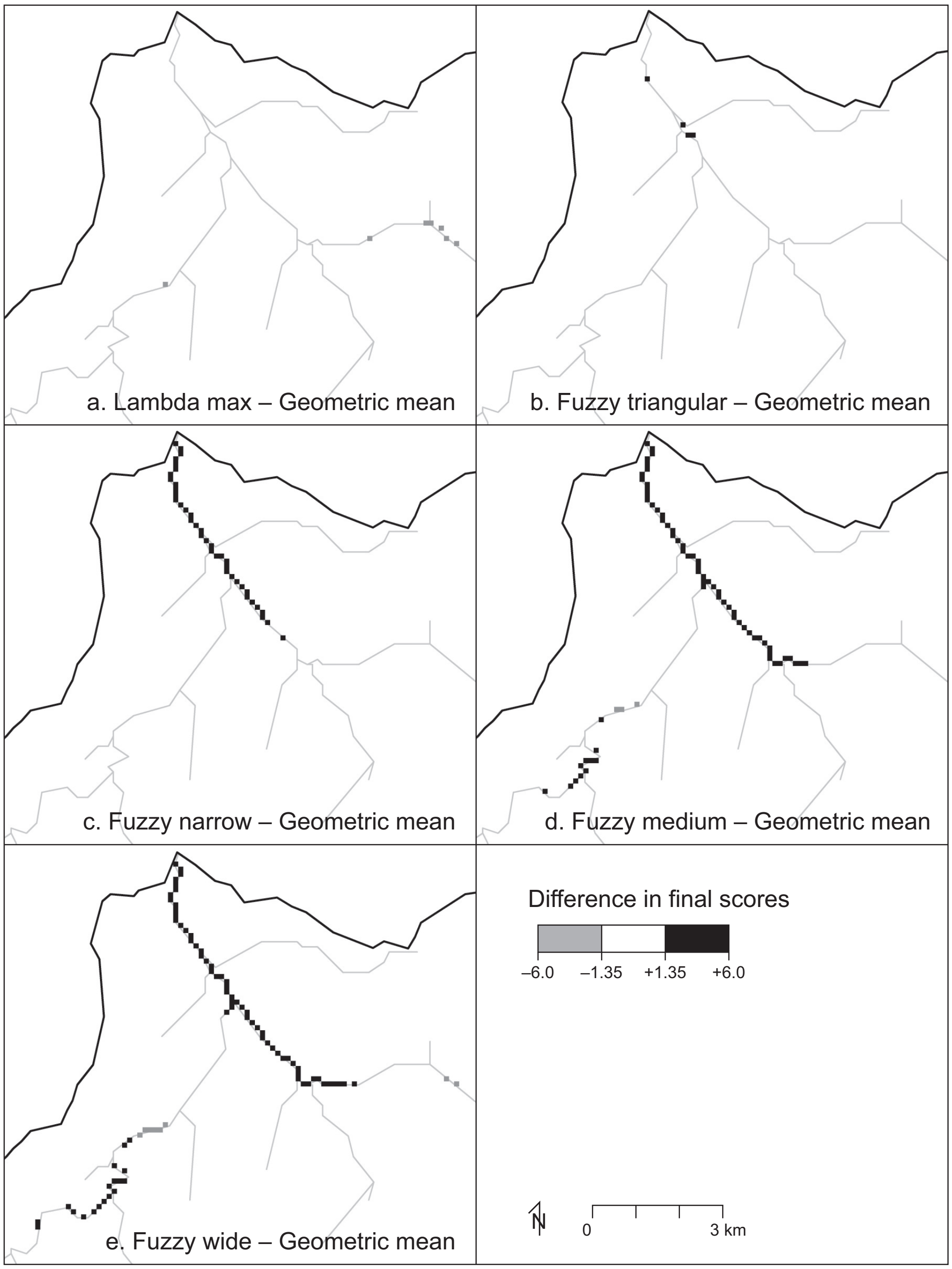

Fig. 6. The final scores of the suitability map of each method subtracted by the geometric mean.

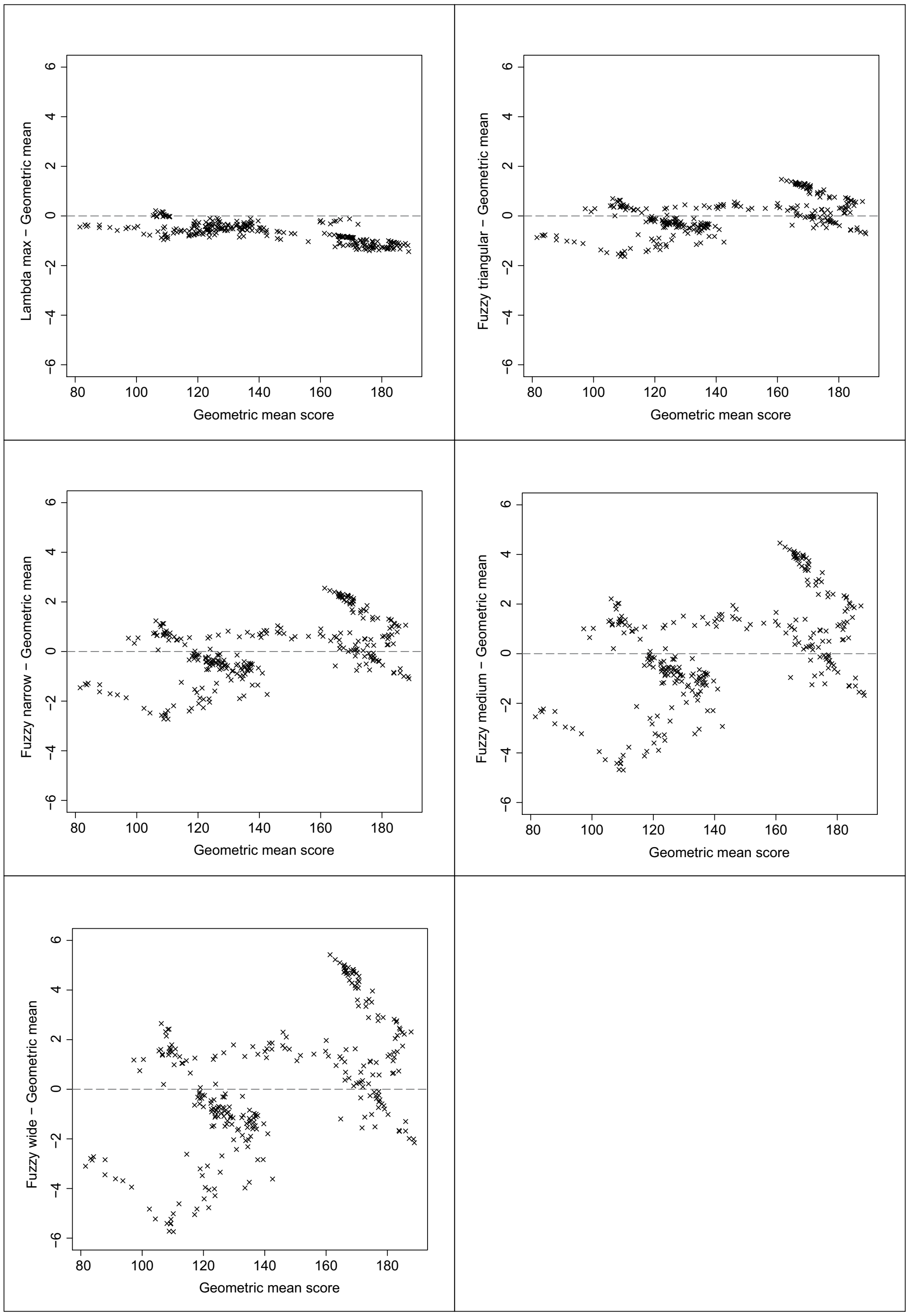

Fig. 7. The final scores of each method subtracted by the geometric mean plotted against the final scores of the geometric mean method.

0.05 with four degrees of freedom (DF) considering five classes in each map. The Chi-Square test was repeated for each of the uncertainty levels of the fuzzy AHP. The test shows that the difference grows between the resulting maps of classified suitability for the AHP geometric mean and the fuzzy AHP methods by increasing the uncertainty to a level that may become significant. When comparing with the geometric mean method, the Chi-Square values increase from 0.61 for the triangular, over 2.44 and 4.21 for the narrow and medium trapezoidal, to 6.56 for the wide trapezoidal methods. Although the result of each individual test to some extent is dependent on the number of priority classes and the significant level chosen as well as that this test ignores the impact of spatial autocorrelation on the expected agreement between the resulting maps (as is described in Hagen-Zanker (2009)), the overall conclusion remains intact that by increasing the level of uncertainty to some certain level, the Chi-Square value may eventually exceed a critical value.

5. Discussion and conclusion

When using only a crisp pairwise comparison matrix the results differ only slightly between the geometric mean and the lambda max AHP methods. It is not easy to say which method gives more acceptable results, since there are not any reference results, so the results obtained from the different methods are equally acceptable. Furthermore, using the AHP for a problem with applying only one of the techniques, e.g. geometric mean, to different pairwise decision matrices gives results that are unlikely to be similar. For example, applying the AHP geometric mean to a decision-making problem by different decision-makers for calculating weights of the criteria could lead to different results. Different decision-makers have typically distinct preferences and priorities; consequently, the resulting pairwise comparison matrices are different, which might have a huge effect on the AHP results. On the other hand, in complex problems with different criteria under consideration, ranking criteria over each other and making reasonable and consistent pairwise comparison matrices can be very complicated. Subjective judgments by humans are mostly represented by uncertain and vague patterns and lead to matrices that are not quite consistent. Considering the mentioned issues, we can conclude that there are some intrinsic uncertainties in the nature of the AHP which is in good agreement with Saaty's consideration of the AHP as an already fuzzy process (Saaty, 2006; Saaty & Tran, 2007).

The fact that comparison ratios are not exact numbers makes the development of a pairwise comparison matrix in the AHP difficult. The AHP uses a rather imprecise scale of judgments ranking. Mixing concepts of importance and exact numbers is difficult. The numbers 1, 3, 5, 7 and 9 used in the pairwise comparison matrix only indicate the importance of the corresponding factors. The way in which the ratios in pairwise comparison matrix are defined is different from the meaning of numbers in algebra. For example, if a factor A is ranked by the number 3 over factor B, it does not mean that factor A is 3 times more important than factor B, but only that factor A is moderately more important than B (see Table 1). Additionally, decision-makers in the AHP are rather free to vary their choices of values for the judgment scale, because the judgment values are flexible verbal judgments of performance rather than exact or crisp values. These numerical representations of judgments are already fuzzy (Saaty & Tran, 2007), and making fuzzy judgments fuzzier does not necessarily lead to a better result (Saaty, 2006; Saaty & Tran, 2007). Therefore any interpretation of the comparison ratios or even weights by the decision-makers should be done carefully in the AHP.

Although the AHP may already be a fuzzy procedure, some researchers consider a number of reasons to fuzzify it further. One reason for this could be the need for greater flexibilities in decision-makers judgments when ranking the criteria over each other. Using the fuzzy concept, decision-makers are more flexible in expressing their judgments by applying different degrees of fuzzification or uncertainty in fuzzy ratios. In fact, different levels of uncertainty in fuzzy ratios can help decision-makers to have wider judgments. For example, it gives the possibility for overlapping preferences of criteria if the judge is not sure about the degree of importance of them over each other. Another reason is when different decision-makers have to agree upon the preferences in ranking the criteria. Instead of combining diverse opinions and judgments, they can agree on an interval expressed as a fuzzy trapezoidal membership function. The fuzzy concept can also be useful for problems where the uncertainty is expected to increase in time; for example in a case where different conditions are supposed to change in future, but it is uncertain how.

In this work, the comparison of the fuzzy AHP with different levels of uncertainty with the conventional AHP shows differences in resulting weights and suitability maps. Further, these differences grow by increasing the level of uncertainty in comparison ratios. Specifically, when the wideness of the fuzzy membership functions is increased, the differences become more significant. These differences can potentially change the final decisions in decision-making processes. Based on the previously described results and discussion the following findings are to be emphasized:

- The AHP is sensitive to the level of uncertainty in fuzzification, which calls for attention of the decision-makers that they should be aware of this sensitivity while using the fuzzified AHP.

- The increasing differences in results due to widening fuzzy membership functions show that the methodology described can also function as a guideline on how to perform a sensitivity analysis in spatial MCDA, given the subjective uncertainty related to the weighting, especially when analyzing the results from a crisp AHP or any other related linear weighted combination MCDA method.

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