

# VISUALIZATION OF ISOTHERMAL LOW-REYNOLDS CIRCULAR AIR JET USING COMPUTED TOMOGRAPHY

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## ABSTRACT

The intention of this paper was to demonstrate the principle and usefulness of computed tomography for concentration field measurements. Radial extinction coefficient profiles have been reconstructed using the LTD approach in the transition region of an isothermal jet of air at Reynolds number of 2 600. Reconstructed profiles were compared against velocity profiles at axial distances ranging from 2 to 20 nozzle diameters downstream. Results indicate that the width parameter of the reconstructed scalar distribution is around 23 % larger than the velocity distribution for distances between 10 and 20 nozzle diameters downstream. This finding is in good agreement with the results of other investigators. This technique has evidently yielded an accurate description of the scalar field of the round isothermal free jet.

The quality of the reconstructions is very promising considering the relatively few measurement data, projection angles and low pixel resolution used in this study.

Computed tomography is superior for monitoring chemical concentrations over larger areas (whole room) when PMS and PLIF are unfeasible.

## 1. INTRODUCTION

Today, the concentration of gases in indoor air is sampled by time-integrated point-sampling devices. The point-samplers are usually integrated over a long period, placed at fixed locations in a test-region. Thus, the result only gives a rough estimation of the gas-concentration distribution because the spatial resolution is limited to the discrete location of the sampling devices. These point-measuring techniques are very time-consuming and information about short-term fluctuation is lost when concentrations are integrated for a long time. Today traditional techniques are also intrusive and disturb the airflow and concentration distribution in the test-region.

Tomography is best known for its use in medical X-ray absorption imaging where it is a well-established diagnostic technique. The mathematical description of the process in absorption tomography is applicable to both medical application and indoor climate testing. For indoor climate applications,

computed tomography is a process for reconstructing a spatial concentration profile of a plane through a room, using a network of line-integrated concentration measurements. The basic components of any tomographic system are a remote sensing system and a tomographic reconstruction algorithm. Reconstruction of chemical concentration is troublesome, because of fluctuations in both time and space, which make the sampling and reconstruction of e.g. plumes in air much more difficult than the reconstruction of organs in a body.

Computed tomography based on optical sensing offer numerous advantages over probe-based techniques for the study of concentration fields. Optical measurement techniques are non-perturbing to the flowfield being studied. Another advantage is the much higher temporal resolution.

This paper demonstrates the usefulness of computed tomography for indoor climate applications. In this paper scattering tomography is applied, which enables the use of small and inexpensive measurement systems. The Least Third Derivative (LTD) approach is used to reconstruct the concentration profile of a horizontal circular axisymmetric air jet. The reconstructed concentration profiles are compared to the velocity profiles measured by a hot-wire anemometer.

## 2. METHODOLOGY AND EXPERIMENTAL SETUP

Media, such as aerosol particles or chemical gases, illuminated by a beam of light will scatter and absorb some of that light, thereby diminishing the intensity of the beam along its axis. This process is often called *attenuation* or *extinction* and the total attenuation of light intensity is the sum of attenuation due to scattering and absorption. For aerosol particles this attenuation depends on the wavelength of the incident beam, the chemical composition of the particles, particle size and shape, number of particles, and orientation.

Laser remote sensors form a special class of optical remote sensors. Laser remote sensing systems, such as tunable lasers and Fourier transform infrared (FTIR) instruments, has been used extensively for outdoor applications as well in industries to measure a variety of gases. The intensity loss due to absorption

( $\ln I_0/I$ ) at a certain frequency is directly proportional to the concentration. Via a calibration curve the measurements can be converted to path-integrated concentrations (ppm-m). For non-absorbing monodisperse aerosol particles the intensity loss due to scattering is directly proportional to the number of particles per unit volume.

In chemical and particle concentration applications, tomographic technology involves acquisition of measurement signals from detectors, located mostly around the boundaries of an investigated region, revealing information about the concentration distribution within the region.

For parallel beams of light sent through a medium in cross-section, the ratio of the light intensity traversing the medium,  $I$ , to that incident on the medium;  $I_0$ , is given by the Lambert-Beer law:

$$\frac{I}{I_0} = e^{-\int \sigma \cdot dL} \quad (1)$$

where

$$\sigma = \sigma_s + \sigma_a \quad (2)$$

$\sigma$  is the total extinction coefficient,  $\sigma_s$  extinction coefficient due to scattering,  $\sigma_a$  extinction coefficient due to absorption and  $L$  is the path length of the light beam through the medium. Simply stated, the law claims that when a sample is placed in the beam, there is a direct and linear relationship between the amount (concentration) of its constituent(s) and the amount of energy absorbed and scattered.

Optical path-integrated measurements provide only one-dimensional attenuation values, with no information about the distribution of  $\sigma$  along the beam path. An exception is the light detection and ranging (LIDAR) technique for detecting pollutants in the atmosphere<sup>(x)</sup>.

## 2.1 Computed tomography

Computer tomography is the process of converting 1-dimensional data into two-dimensional information. Applied to air monitoring, computed tomography is the process of mapping the concentration distribution of a plane through a region, using a network of path-integrated data and a reconstruction algorithm, see Figure 6. Concentration can thereby be resolved in real-time using far fewer measurements than would be required to obtain the same level of details using point samples.

The strategy in computed tomography is to obtain parameter values, which would most likely yield the same path-integrated data as those obtained from the measurements.

In air monitoring, computed tomography is a continuous inverse problem which can be converted into a discrete one with the assumption that the attenuation coefficient function can be

represented by a finite number  $N$  of values. Considering all  $M$  beam paths

$$\ln I_0/I_i \approx \sum_{j=1}^N a_{ij} \cdot \sigma_j \quad (3)$$

where  $a_{ij}$  denotes the length of path  $i$  passing through pixel  $j$ .

Using tomographic algorithms, the section of the object is reconstructed from these multi-angular measurements. It is often necessary to reconstruct gas concentrations using few beams from few angles. Assuming no symmetry in the gas concentration distribution being reconstructed, images that are distinguishable when a complete set of data are given may not be distinguishable when only a few beams from few views are used. Therefore reconstruction errors can arise on account of insufficient data but also due to the presence of random noise in the measurements. Hence, it is essential that the reconstruction algorithm not only performs well for limited data, but also in the presence of measurement noise, especially if real-time reconstruction is desirable.

In this study the Low Third Derivative (LTD) approach is used to convert the data. The LTD algorithm, first developed by Price et al. [1], is not an iterative process; instead, it performs a weighted linear least-squares solution by direct matrix inversion of the system matrix. In order to be able to make a linear least-square solution, prior information has to be added to the system matrix. In the LTD method, one item of prior information is that the third spatial derivative in each direction is assumed to be close to zero, which imposes spatial smoothness.

A modified version of the LTD method has been proposed [2, 3], where an extra constraint is placed on the reconstructed concentration field. It states that pixels contained within beams whose path-integral is below a cut-off limit (for example below the detection-limit of the system) are forced to assign a concentration value of zero or at least very close to zero. This constraint is incorporated by dividing up each "zero" path into shorter paths and by splitting the path by the cell faces. This improves the method significantly, by minimizing artifacts and streaks when very sharp concentration gradients are present.

The weights of the prior information,  $w_p$ , and the path-integrated data,  $w_b$ , has to be assigned differently. The relationship between the path-integrated data and the prior information is controlled by the weight ratio,  $\eta$ :

$$\eta = w_b/w_p \quad (4)$$

It has been found numerically that the optimal weight ratio is around 500-5000 for low measurement noise level [3].

## 2.2 Experimental setup

The isothermal jet flow under consideration is issued from a small nozzle of diameter 40 mm, at the exit section, located 1.25 m above the floor. The jet is a low Reynolds jet, intended for personalised ventilation applications. The air velocity at the exit

of the nozzle,  $U(0)$ , had a mean velocity of 1 m/s and a relative turbulence intensity close to 1%. The inlet Reynolds number  $Re(0)$  was then around 2 600 when using the diameter at the exit of the nozzle,  $D$ , as characteristic length. The nozzle design and its main components is illustrated in Figure 1. The slope of the nozzle was designed as a fifth degree polynomial where the exit section coincides with the point where the tangent is parallel to the nozzle axis. This design was used with the aim of obtaining a satisfactory sharp top-hat profile as initial conditions for the jet evolution. The jet flow was injected in a room with dimensions (length by width by height) of 3.5×3.0×2.5 m.

The frame for remote optical detection of the air jet was composed of 62 diode-lasers and 62 detectors divided over two projection angles, see Figure 2. The frame was rotatable to enable more path-integrated measurements from different views. Figure 3 shows the path configuration when using four views. The reconstruction area was 0.394m × 0.394m and images were constructed typically on a 31 × 31 grid. Measurements with the frame were performed for different vertical planes;  $x/D = 2, 4, 10, 15, 20$  and 25 from the nozzle outlet.

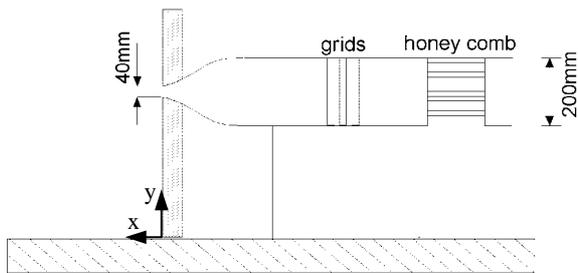


Figure 1. Nozzle design.

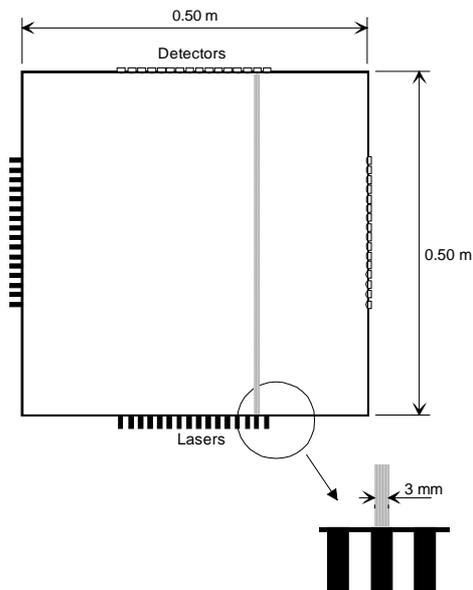


Figure 2. Measurement frame.

Smoke with a mean diameter of 1.5 microns was introduced into the duct just before the honey-comb. The diode-lasers were operating at a wavelength of 650 nm, and the width of each beam was close to 3mm. Smoke in the beam paths reduces the intensity of the radiation arriving at the detectors. The detector signal was between 2-10 Volt (10 Volt corresponds to zero smoke). The accuracy of the measurements was 0.1 Volt. The minimum detection level was set to 0.1 V, meaning that all signals with a change of intensity below 0.1 V were set to zero. The measurements over each view were performed over totally 60s with a sampling rate of 100 Hz.

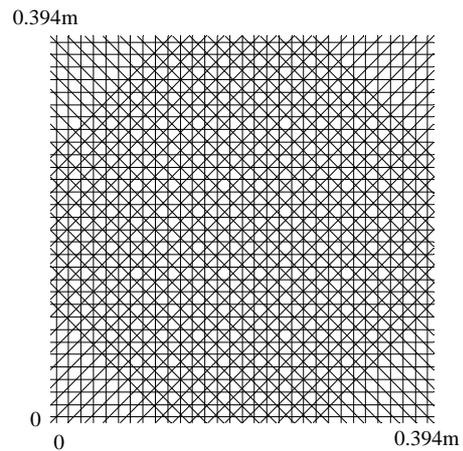


Figure 3. Path configuration when using 4 measurement views.

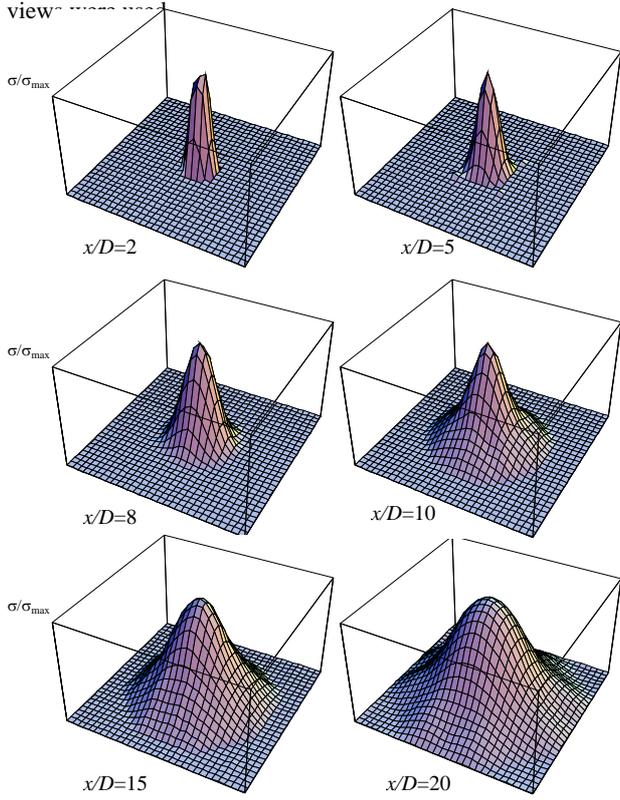
The 1D measurements were converted to 2D information with the modified LTD method in the Mathematica (Wolfram Research Inc.) programming environment.

The velocity measurements were carried out by use of a single fibre-film probe, DANTEC 55R76 with a temperature-compensating sensor. The velocity sensor was nickel film deposited on 70 µm diameter quartz fibre, overall length 3 mm, sensitive film length 1.25 mm, copper and gold plated at the ends and the film was protected by a quartz-coating approximately 0.5 µm in thickness. A temperature-compensated bridge of type DANTEC 56C14 was used. The data acquisition was done with the Lab-View System from National Instruments. The board used for data acquisition was AT-MIO-16DE-10 with 16 bits. All the measurements were performed with a sampling rate of 3000 Hz for a time duration of 180 seconds.

### 3. RESULTS AND DISCUSSION

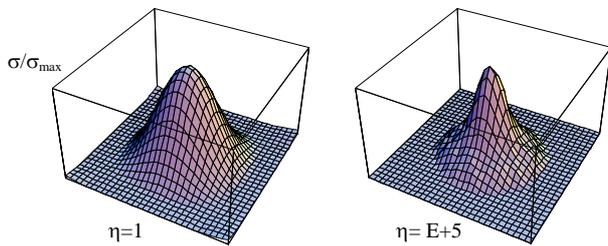
Figure 4 demonstrates a visualization of the free jet flow from the present study. It shows the reconstructions of the distribution of the smoke extinction coefficient distribution using experimental data of the air jet at six different distances downstream the nozzle exit. These reconstructions are based upon time-averaged path-integrated measurements from four views. According to the results, the algorithm produced promising non-artifactual reconstructions using experimental

data when time-averaged path-measurements from only four views

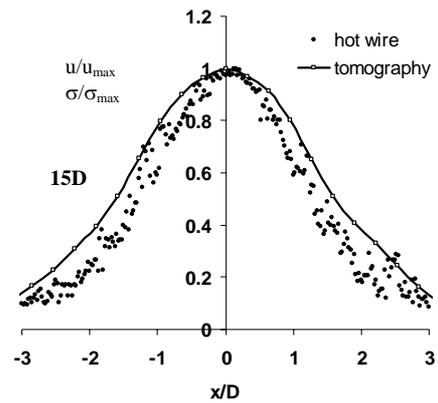
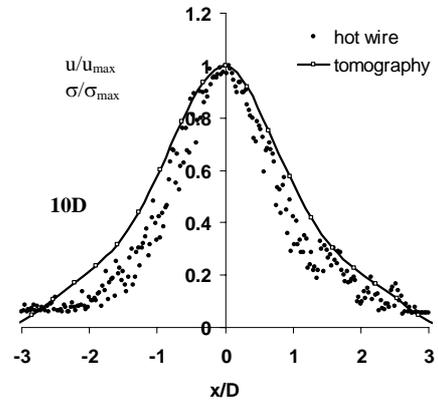
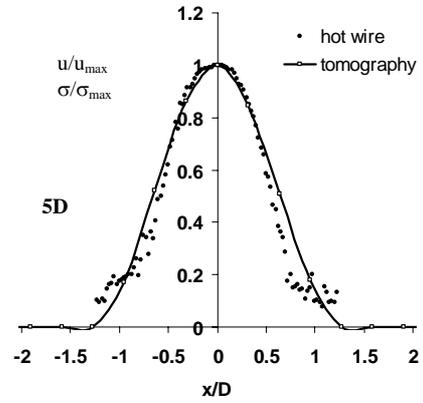


**Figure 4.** Reconstruction with a 31 x 31 pixel resolution of the smoke extinction coefficient distribution using only four views.

The quality of the reconstructions is very affected by the weight ratio. High ratio can result in noisy reconstructions while low ratio force to solution to follow the prior information, see Figure 5. The weight ratio used for the reconstructions in figure 4 were 2500 except for  $x/D=2$  where the weight ratio had to be higher (10000).



**Figure 5.** Reconstructions of the extinction coefficient at  $x/D=10$  using non-optimal weight ratio.



**Figure 6.** Radial reconstructed extinction coefficient distribution compared to the radial velocity profile at three distances downstream the nozzle exit.

In Figure 6 the radial reconstructed extinction coefficient distribution is compared to the radial velocity profile at 3 distances downstream the nozzle exit. The profiles are wider than the velocity profile, which agrees well with the previous results. Rodi [4] presents a comprehensive compilation of studies

regarding width parameters,  $b$ , for turbulent round jets. All studies showed that the scalar profile is wider than the velocity profile, and the scalar width parameter was between 8 to 32 % larger.

For comparison with results of previous works, a Gaussian smoke concentration and velocity profile was assumed:

$$\frac{\sigma}{\sigma_{max}} = e^{-\frac{r^2}{b_\sigma^2}} \quad (5)$$

$$\frac{u}{u_{max}} = e^{-\frac{r^2}{b_u^2}} \quad (6)$$

In this study the width parameter of the extinction coefficient distribution is around 23 % larger than the velocity distribution for distances between 10 and 20 nozzle diameters downstream.

Due to the relatively low measurement data, projection angles and pixel resolution, the reconstructed profiles can be slightly smoothed out, especially close to the nozzle exit. This is clearly the case for the reconstruction at  $x/D=2$ .

The projection data can also be analyzed including fluctuating characteristics of each path-integral. Path-integrated measurements give information not only about the mean concentration. Their rms values also give information about the turbulence structure of the flow. However, in this study the fluctuating characteristics are not analyzed since the measurement then has to be performed over longer time to achieve accurate information.

The results presented here are very promising taking into account the relatively few measurements, projection angles and low reconstruction pixel resolution. The intention of this paper was to demonstrate the principle and usefulness of computed tomography for concentration field measurements. In order to concentrate on quantitative values, especially for complex concentration distributions, a very high number of path-integrated measurements must be performed for different views. Therefore, concentration field measurements over such a small region (sub-area of room) as in this study is more suitable performed with planar Mie scattering (PMS) of smoke particles and planar laser-induced fluorescence (PLIF) of acetone. Instead, computed tomography is superior for monitoring over larger areas (whole room) when PMS and PLIF are unfeasible. Recently, experiments were performed with only 28 fixed sources and detectors placed non-symmetrically around the room boundaries over 4 views allowing a complete measurement cycle of 28 optical paths in only 7 sec Fischer et al.[5]. Reconstructions using their data show that it is actually possible to examine transient  $\text{CH}_4$  concentration transport indoors over a large area with reasonably good accuracy from as few as 28 path-measurements placed over only 4 views, Price et al.[1].

There are some drawbacks using scattering tomography instead of absorption tomography for the case of passive scalar transport.

The first is noise from non-seed particles, since all particles in the flow field will scatter light. Another problem is inertia of a particulate tracer, meaning that it might not follow the fluid motion accurately. However, Becker et al.[6] observed that smoke particles less than 2 microns in diameter were able to follow a sinusoidal fluctuation. A third problem is that forward scattered and multiple scattered light might reach the detector and for that case the Lambert-Beer law does not hold. Therefore the particle concentration has to be sufficiently low and one has to use lenses to ensure that only the attenuated parallel light reach the detectors and

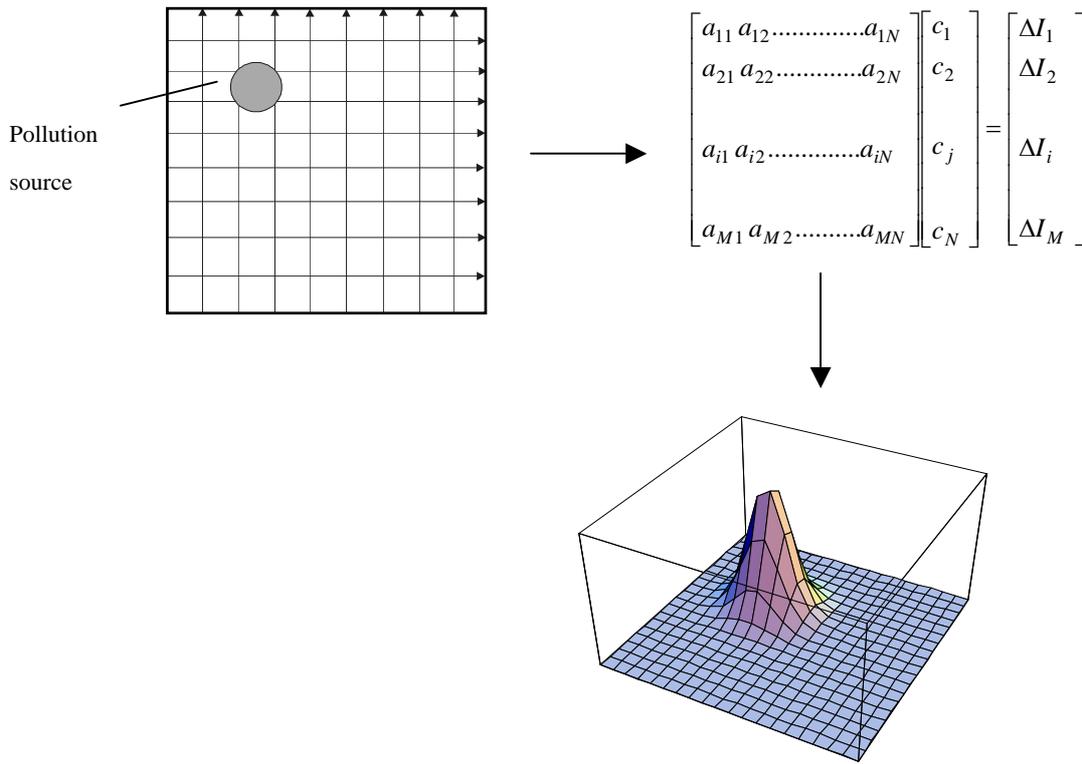
#### 4. CONCLUSION

By using computed tomography radial extinction coefficient profiles have been measured in the transition region of an isothermal jet of air at Reynolds number of 2 600. This technique has evidently yielded an accurate description of the scalar field of the round isothermal free jet. Results indicate that the width parameter of the smoke concentration distribution is around 23 % larger than the velocity distribution for distances between 10 and 20 nozzle diameters downstream. This finding is in good agreement with the results of other investigators.

The quality of the reconstructions is very promising considering the relatively few measurement data, projection angles and low pixel resolution used in this study.

#### 5. REFERENCES

1. P.N. Price, M.L. Fischer, A.J. Gadgil and R.G. Sextro, An algorithm for real-time tomography of gas concentrations, using prior information about spatial derivatives, *Atmospheric Environment*, vol.35(16),pp.2827-2835, 2000.
2. M. Cehlin and M. Sandberg, Monitoring of a Low-Velocity Air Jet Using Computed Tomography, *Proceedings of 8th International Conference Air Distribution in Rooms*, pp. 261-364, 2002.
3. M. Cehlin, Computed Tomography for Gas Sensing Indoors using a Modified Low Third Derivative Method, submitted to *Atmospheric Environment*, 2004.
4. W. Rodi, *Turbulent buoyant jets and plumes - HMT-6*, Pergamon Press Ltd, 1982.
5. M.L. Fischer, P.N. Price, T.L. Thatcher, C.A. Schwalbe, M.J. Craig, E.E. Wood, R.G. Sextro and A.J. Gadgil, Rapid Measurements and Mapping of Tracer Gas Concentration in a Large Indoor Space, *Atmospheric Environment*, 35(16), 2837-2844, 2000.
6. H.A. Becker, H.C. Hottel and G.C. Williams. On the light-scatter technique for the study of turbulence and mixing, *Journal of Fluid Mechanics*, vol. 30(2), pp. 259-284, 1967.



**Figure 7.** Tomographic reconstruction process. A network of optical beams is sent through a plane in the test-region. These one-dimensional concentration data are converted to two-dimensional information via a tomographic reconstruction algorithm. The result is displayed in a two-dimensional concentration map for the plane of interest.