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Proofs of Derivations in *Memory Polynomial Baseband Modeling of RF Power Amplifiers*

Per N. Landin, Kurt Barbé, Wendy Van Moer,
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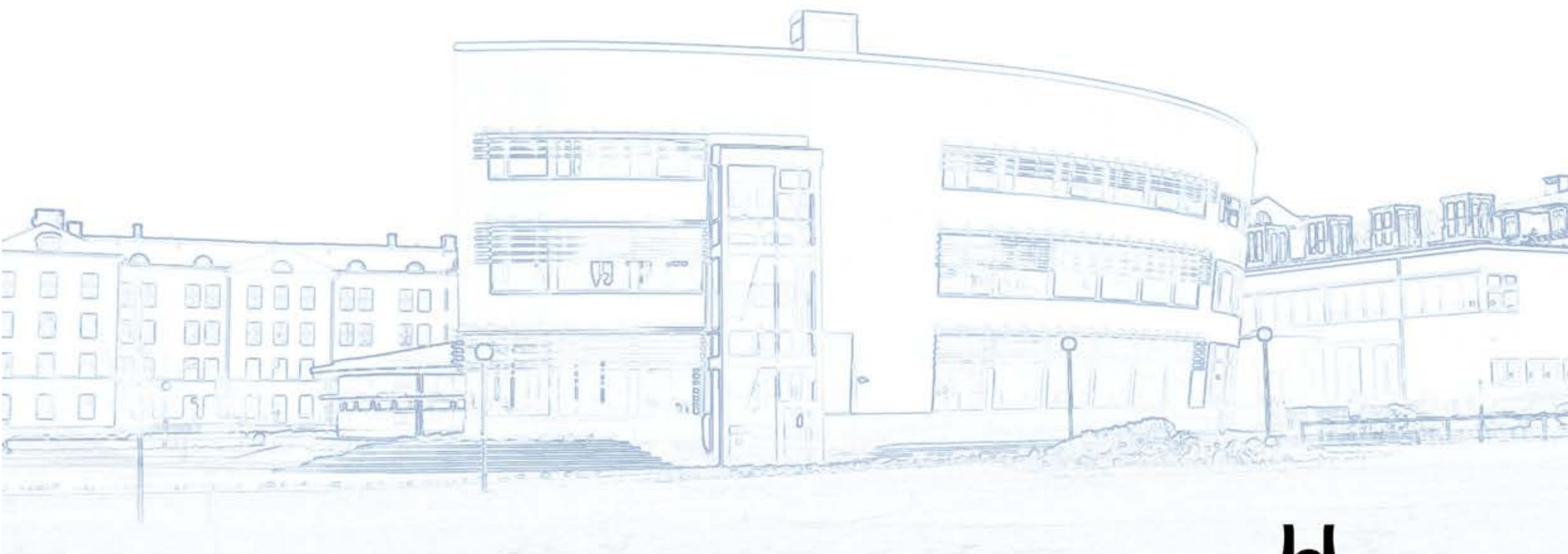

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I. Contents

This paper contains supporting derivations for the paper *Memory Polynomial Baseband Modeling of RF Power Amplifiers*. All references to numbered equations, propositions and assumptions are to the corresponding number in *Memory Polynomial Baseband Modeling of RF Power Amplifiers*.

II. Proof of Proposition 4

Starting from (9), using the formula for binomial expansion and reordering signal components with even orders of $u(t)$ into a term $v_{DC}(t)$ and signal components with odd orders of $u(t)$ into $v_{CC}(t)$ results in

$$\begin{aligned}
v_1(t) &= a_0 + a_1 u(t) - a_1 f * v(t) \\
&+ a_2 u(t)^2 - 2a_2 u(t) f * v(t) + a_2[f * v(t)]^2 \\
&+ a_3 u(t)^3 - 3a_3 u(t)^2 f * v(t) + 3a_3 u(t)[f * v(t)]^2 \\
&- a_3[f * v(t)]^3 \\
&+ a_4 u(t)^4 + 4a_4 u(t)^3 f * v(t) + 6a_4 u(t)^2 [f * v(t)]^2 \\
&+ 4a_4 u(t)[f * v(t)]^3 + a_4[f * v(t)]^4 \dots \\
&+ \mathcal{O}([f * v(t)]^2) + \mathcal{O}(u(t)[f * v(t)]^2) = \\
&\sum_{p=1}^{P} \sum_{r=1}^{\lfloor \frac{p+1}{2} \rfloor} a_p \binom{p}{(2r-1)} u^{2r-1}(t) \left[-f * v(t)\right]^{p-(2r-1)} \\
&+ \sum_{p=1}^{P} \sum_{r=1}^{\lfloor \frac{p+1}{2} \rfloor} a_p \binom{p}{2(r-1)} u^{2(r-1)}(t) \left[f * v(t)\right]^{p-2(r-1)} + \\
&a_0 + \mathcal{O}([f * v(t)]^2) = \\
&v_{CC}(t) + v_{DC}(t) + \mathcal{O}([f * v(t)]^2).
\end{aligned}$$

The double summation arise as a result of the mixing between the feedback term and the direct terms in the nonlinearity.

III. Proof of Proposition 5

The terms close to DC (and thus not removed by the low-pass filter $F(\omega)$) are those of even order. These can due to the band-pass to low-pass transform be expressed as

$$\begin{aligned}
f * v(t) =& f * (a_0 + a_2 u^2(t) + a_4 u^4(t) + \dots) = \\
& f * (b_0 + b_2 \alpha^2(t) + b_4 \alpha^4(t) + \dots) = \\
& f * \sum_{p=0}^{\lfloor \frac{P+1}{2} \rfloor} b_{2p} \alpha^{2p}(t).
\end{aligned}$$

Here $b_{2p} = a_{2p} \frac{1}{2^p}$ and comes from considering $a_{2p} u^{2p}(t) = a_{2p} \left\{ \frac{\alpha^2(t)}{2} \left[1 + \cos(2\omega_c t)\right] \right\}^p$.

IV. Proof of Proposition 7

Using Assumptions *11* and *12*, (14) is rewritten as

$$\begin{aligned}
v_1(t) = & \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} a_p u(t) \alpha^{2(p-1)} \\
& - \frac{a_1}{2} u(t) f * \sum_{p=0}^{\lfloor \frac{P+1}{2} \rfloor} b_{2p} \alpha^{2p}(t) \\
& + \mathcal{O}([f * v(t)]^2).
\end{aligned}$$

To reduce the number of terms in the two sums can be added resulting in

$$v_1(t) = \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor + 1} \widehat{a}_{2p-1} u(t) \widehat{f} * \alpha^{2(p-1)}(t)$$

with $\widehat{a}_{2p-1} = a_{2p-1} - \frac{a_1}{2} b_{2(p-1)}$, and $\widehat{a}_{2(\lfloor \frac{P+1}{2} \rfloor + 1) - 1} = -\frac{a_1}{2} b_{2\lfloor \frac{P+1}{2} \rfloor}$ being the new coefficients resulting from adding the two sums, and $\widehat{f} = 1 - f$.

V. Proof of Proposition 8

The signal components close to the carrier in $v(t)$ from (10) can under Assumption *13* be expressed as

$$\begin{aligned}
v_1(t) = & \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} a_{2p-1} \binom{p}{p-2} u(t) \alpha^{2(p-1)}(t) \\
& + \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} a_{2p-1} \binom{p}{p-1} u(t) \alpha^{2(p-1)}(t) \\
& \left[-f * \sum_{r=0}^{\lfloor \frac{P+1}{2} \rfloor} b_{2r} \alpha^{2r}(t) \right] + \mathcal{O}(u(t)[f * v(t)]^2) \\
= & \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} a_{2p-1} \binom{p}{p-2} u(t) \alpha^{2(p-1)}(t) \\
& - \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} \sum_{r=0}^{\lfloor \frac{P+1}{2} \rfloor} a_{2p-1} \binom{p}{p-1} b_{2r} \\
& u(t) \alpha^{2(p-1)}(t) f * \alpha^{2r}(t) + \mathcal{O}(u(t)[f * v(t)]^2).
\end{aligned}$$

Merging common terms to reduce the number of coefficients results in

$$\begin{aligned}
v_1(t) = & \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} \sum_{r=0}^{\lfloor \frac{P+1}{2} \rfloor} \widehat{\widehat{a}}_{2p-1,r} u(t) \alpha^{2(p-1)}(t) \widehat{f} * \alpha^{2r}(t) \\
& + \mathcal{O}(u(t)[f * v(t)]^2)
\end{aligned}$$

with $\widehat{\widehat{a}}_{2p-1,0} = a_{2p-1}\binom{p}{p-2} - a_{2p-1}\binom{p}{p-1}b_0$, $\widehat{\widehat{a}}_{2p-1,r} = a_{2p-1}\binom{p}{p-1}b_{2r}$ if $r \geq 1$ and $\widehat{f} = 1 - f$.

VI. Details of EPH Derivation

Substitute $u(t) = h_I * x(t) = (c_I + \gamma_I) * x(t)$ in (17) to get

$$y(t) = (c_I + \gamma_I) * x(t) + \sum_{p=2}^{\lfloor \frac{P+1}{2} \rfloor} \widehat{a}_{2p-1}\,(c_I + \gamma_I) * x(t)\,\widehat{f} * \left\{ [c_I * x(t)]^2 + 2(c_I + \gamma_I) * x(t) + [\gamma_I * x(t)]^2 \right\}^{p-1}.$$

Now using *Assumptions 7* and *8* results in the approximation

$$[c_I * x(t)]^2 + 2(c_I + \gamma_I) * x(t) + [\gamma_I * x(t)]^2 = [c_I * x(t)]^2 + \mathcal{O}(c_I * x(t)).$$

Using this results in

$$y(t) = (c_I + \gamma_I) * x(t) + \sum_{p=2}^{\lfloor \frac{P+1}{2} \rfloor} \widehat{a}_{2p-1}\,(c_I + \gamma_I) * x(t)\,\widehat{f} * [c_I * x(t)]^{2(p-1)}.$$

Applying the output filtering $H_O(\omega)$, transforming to base-band and sampling gives

$$y_{\mathrm{LP}}(n) = \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} h_I(m_1) h_O(m_2)\, \widehat{a}_{2p-1}\, x_{\mathrm{LP}}(n - m_1 - m_2) \sum_{m_3=0}^{M_3} |x_{\mathrm{LP}}(n - m_2 - m_3)|^{2(p-1)}$$

with $h_I(m_1)$ symbolizing the sampled low-pass equivalent FIR representation of the filter $H_I(\omega)$, and similar for $h_O(m_2)$ and $\widehat{f}(m_3)$.

Parallelizing the terms above gives

$$y_{\mathrm{LP,\,EPH}}(n) = \sum_{p=1}^{\lfloor \frac{P+1}{2} \rfloor} \sum_{m_1=0}^{M_1} \sum_{m_2=0}^{M_2} g_{m_1,m_2}^{2p-1}\, x_{\mathrm{LP}}(n - m_1)\, |x_{\mathrm{LP}}(n - m_2)|^{2(p-1)}$$

which is the desired EPH model.

VII. Details of EEMP Derivation

Assumptions *14* and *15* are strictly given by the following:

Assumption 14: The frequency dependence of the filter $F(\omega)$ is much larger than the frequency dependence of $H_O(\omega)$ in the sense that

$$\min_{g_{H_O},\,\tau_{H_O}} ||H_O(\omega)W(\omega) - g_{H_O} e^{-j\omega\tau_{H_O}} U(\omega)||_2 << \min_{g_F,\,\tau_F} ||F(\omega)U(\omega) - g_F e^{-j\omega\tau_F} W(\omega)||_2$$

with g_{H_O} and g_F being gain constants, and τ_{H_O} and τ_F time delays. $W(\omega)$ is the frequency domain representation of a signal $w(t)$ with the power equally split in two spectral regions of bandwidths larger than 0. One region is at DC and the other is at the carrier.

Assumption 15: The power in the linear parts of the model, i.e. all signal components that can be described as a linear filtering of the input signal, is larger than the nonlinear signal components in the sense

$$||h * u(t)||_2 \gg ||h * \sum_{p=2}^{\lfloor \frac{P+1}{2} \rfloor} \widehat{a}_{2p-1}\,\widehat{f} * \alpha^{2(p-1)}(t)||_2$$

for some stable LTI filter $H(\omega)$ with impulse response h. $||\cdot||_2$ denotes the signal 2-norm.

Assumption 14 implies that the deviation from constant gain-linear phase of the filter $F(\omega)$ is much larger than that of $H_O(\omega)$, within relevant bandwidths.

Considering that the power of $u(t)$ is "large" from *Assumption 15* and the relative variation of the filter characteristics from *Assumption 14*, leads to applying the filtering of $H_O(\omega)$ only to the linear term $u(t)$ but not to the summation. This is expressed as

$$y(t) = \widehat{a}_1\, h_O * u(t) + \sum_{p=2}^{\lfloor \frac{P+1}{2} \rfloor} \widehat{a}_{2p-1}\, u(t) \widehat{f} * \alpha^{2(p-1)}(t).$$

Transformation to low-pass equivalent, sampling and parallelization of the filters results in the desired model in (28).

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