A STUDY OF A PROBLEM SOLVING ORIENTED LESSON STRUCTURE

IN MATHEMATICS IN JAPAN

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This paper presents and analyses "Mondaikaiketsu no jugyou" which translates to "the problem solving oriented – approach". It is a set of didactic techniques with the aim of motivating the students' positive attitude toward engaging in mathematical activities and fostering mathematical thinking. As an analytical tool, The Anthropological Theory of Didactics (ATD) will be applied.

Key-words: Problem solving, Japanese mathematics class, ATD

INTRODUCTION

Teaching methods were developed differently in Japan compared to other industrialized countries. Hiebert, Stigler and Manaster (1999) argue that Japanese teachers emphasize mathematical thinking rather than mathematical skills. This goal is reached by having the students discuss with the teacher and peers on the settlement options of problems presented to the whole class. I will call this type of didactic techniques, where students work on whole-class problem solving, for *problem oriented lesson structure* (POLS).

A basic problem in mathematics education, and in the training of teachers, is to find ways to organise the classroom work so as to make the students active learners of mathematics, without losing the focus on the mathematical content. Japanese teaching methods, like the ones described, have attracted attention in Sweden lately (Dagens Nyheter, 2009), and it has been discussed as a possible model to develop in Swedish school system.

Kazuhiko Souma is one of the teacher educators/researchers who has proposed, introduced and elaborated on POLS. He calls his method "*The problem solving oriented*" approach (shortened to PSO; the author's translation; "*Mondaikaiketsu no jugyou*", in Japanese). Like POLS in general, it aims to enhance the students' attitude towards engaging in mathematical activity in the classroom. In Japan, there is a tradition of publishing practical books for mathematics teachers as the target group. This literature aim to present ideas and concrete lesson plans, based on well-constructed mathematical problems, according to the proposed teaching methods (Souma, 1995; Kunimune & Koseki, 1999; Tsubota, 2007). Souma has written and edited a number of such books and his method is actively and widely used by teachers in service. It has received little attention from the academic community, perhaps because of its practical attribute and the lack of clear theoretical base. The lack of theoretical overhead is perhaps part of the appeal, but becomes a problem when one is to describe and assess such approaches.

I conjecture that PSO (and POLS in general) could make a beneficial contribution for pre-service teacher education due to its distinct didactical structure, but the lack of a theoretical base is a hinder. The aim of my paper is therefore to present and analyse PSO in relation to other Japanese POLS approaches. To illustrate, I will analyse an episode of a practical application of PSO in a Japanese classroom.

As an analytical tool, I will use the anthropological theory of didactics (ATD) developed by Y. Chevallard (2006), with the assumption that ATD has the right level of abstraction for the purposes of didactic planning. The focus of this paper is on how the proposed didactic techniques and lesson structure of PSO relates to the description in ATD of the didactic process. In a follow-up paper I intend to shift focus to the mathematical content and in particular to how Souma's ideas regarding the construction of problems can be analysed in ATD.

BACKGROUND TO THE PSO APPROACH

The Anthropological Theory of Didactics

The anthropological theory of didactics (ATD) approaches learning and knowledge as institutional issues. Mathematics learning can be modelled as the construction, within a context of social institutions of interlinked *praxeologies* of mathematical activity, which we also refer to as mathematical organisations (MOs) (Chevallard, 1999 in Barbé, et al., 2005). A praxeology is described by its *tasks* and *techniques (praxis)*, together with its *technology* and *theory (logos)*. Technology constitutes the tools for discourse on and justification of the techniques and the theory provides further justification of the technology and connections to other MOs. The process, under which a mathematical praxeology is constructed within the educational institution, is called the *didactic process* (ibid.). Chevallard proposes to describe it as being organised in six "moments" that can be thought of as different modes of activity in the study of mathematics. The moments are: (FE) the moment of *first encounter* (or re-encounter) of tasks associated to the praxeology, (EX) the *exploratory* moment of finding and elaboration of techniques suitable to the tasks, (T) the *technical-work* moment of using and improving techniques, (TT) the *technological–theoretical* moment in which possible techniques are assessed and technological discourse is taking place, (I) the *institutionalisation* moment where one is trying to identify and discern the elaborated MO and (EV) the *evaluation* moment which aims to examine the value of the MO. The description of mathematical knowledge (the MO) and learning/teaching of this (the didactic process) is referred to as the *epistemological* component of ATD.

To organise the work of achieving an appropriate MO, the educator faces the task of designing and controlling the didactic process. To this end, one develops a didactical praxeology, or a *didactical organisation* (DO), consisting of didactic techniques together with a technology/theory to describe and justify those techniques.

I think it is fair to say that ATD carries both a *normative* and *descriptive* component. As an analytical framework, it holds that any didactic processes can be *described* as the construction, via the six moments, of a praxeology. Similarly, the didactic organisation can be described in terms of its praxis and logos block, independently of whether the studied DO's have ATD incorporated as an epistemological model of the learning object and the didactic process. But, when used as a design tool, ATD also carries some normative implications, so that the resulting MOs and DOs can be compared with respect to how suitable, structured, and useful (or *legitimate*) they are. A typical implication is that the local MOs should integrate with and reinforce larger superstructures in the form of global praxeologies and that all moments of the didactic process needs to be visited and such local objectives are compatible with the aims of Souma's writings. In this paper, I will attempt to use ATD to describe the DO proposed by Souma and also use the epistemological component of ATD to motivate some of Souma's didactic techniques. Thus, in a way, propose an *extension* of the DO proposed by Souma with a technological-theoretical block from ATD.

The PSO lesson template

PSO has the form of a proposed lesson structure and Souma states that it is instrumental that the PSO approach is applied with the same basic form regularly. The motivation is that familiarity with the situation makes the students feel secure in participating in the discourse and engaging in the didactic process.

Souma, like most Japanese writers of this genre, often gives general didactic advices: to be generous with positive feedback, taking care of shy students, etc., in order to handle the long-term didactic goals, such as "fostering the students to active learners of mathematics". The explicit motivations are often taken from a technological-theoretical block, which could be referred to as Japanese "didactical/pedagogical common sense", although, as stated below, Souma explicitly refers to the cognitive theories of Dewey and Polya as a motivation. The epistemological description is usually a concrete mathematical example and listing presupposed knowledge and goals for the lessons. This format is natural for this type of inspirational literature, but has its limitations when one wants to discuss the generalities. A central recurring term is that of "mathematical activity", which measures the degree of interest, independence and motivation with which students are carrying out the mathematical work.

In order to make a fair description of Souma's approach, it does not seem reasonable to eliminate all such psychological aspects. Therefore, in this description below, I will make qualifications when talking about the didactic process: like the degree of *participation* in the didactic process and "*invigorate* the didactic process" to mean, "increasing the *mathematical activity*" of the didactic process.

According to Souma's example from his book (1995), a typical POLS lesson starts with a teacher giving a problem, for instance, "Show that the difference of the squares of two integers that follow each other is equal to the sum of the two numbers ($5^2 – 4^2 = 9 = 5 + 4$, $24^2 – 23^2 = 24 + 23$ and so on)." The students try to solve the task and some students write their solutions on the blackboard and explain their solution orally. Souma wonders (pp. 103-104) if the students in this situation will feel a "necessity" to reflect upon the task. Furthermore, some students might not get any ideas on how to solve the problem and will therefore become alienated from the discourse. As an alternative, he proposes the following variation of the problem formulation: The teacher writes down expressions on the blackboard without any comments;

$$5^2 - 4^2 = 9, \;\; 24^2 - 23^2 = 47, \;\; (-9)^2 - (-10)^2 = -19$$

and asks the students what they can observe. All students are supposed to be able find such observations, perhaps working in groups. Students may answer, "It becomes odd numbers", "The differences equal the sum of the integers", "The differences equals the first integer times two minus one", "The last integer times two plus one". After the response of the students, the teacher then controls that all proposals are correct on the blackboard and says; "Now we try to prove each of the statements". Ideally, the formulated problems have many possible roads to solutions: Several students may use the formula for expanding the square of a sum; and several others, using x to the first integer and y for the second integer, the rule of the conjugate.

Souma proposes to use a didactic technique, which I refer to as *guessing*. One should, regularly, let all students guess an answer or formulate hypotheses about the phenomena. It is implied that the "guess" is something that all students can participate in. In the example the students are not asked to simply guess an answer, but they are invited to, discover patterns by themselves, make hypotheses about the phenomena and by implication set their own tasks. By committing to make a guess or a hypothesis, especially in the social context of the class, the student will have a stronger motivation to study the task and follow it up.

We can find explicit theoretical motivation of the DO in Souma's description (1997) where he declares that he is inspired by John Dewey's theory of reflective thinking. Dewey (1933) presents five cognitive phases of problem solving. 1. Recognize the problem. 2. Define the problem. 3. Generate hypotheses about the phenomena. 4. Use reasoning if the hypotheses are viable to solve the problem. 5. Test the most credible hypotheses. Dewey's theory has a general scope and is applicable to any problem context. It is also concerned with the cognitive dimensions, rather than the didactic process as such. Souma states that educators in mathematics may have a tendency to hurry up to address the later phase to "use reasoning". In this way, the development of reflective thinking and motivation may be impeded. Souma thus feels that it is necessary to pay attention especially to the first three phases. He expresses that (1997), from Dewey's theory, we may infer that it is important that we should "(a) have an aim for why we solve the task, (b) feel a necessity to solve the task and (c) have made hypotheses before starting the reasoning process." (p. 34) Souma also refers to Polya's (1957) cognitive theories on problem solving and, in particular, Polya's insistence on the importance of guessing. Polya states that our hypothesis may of course be wrong, but the process of examining the guess should lead to improved hypotheses and a deeper understanding.

The focus on motivation on the first encounter and the exploration, together with the insistence on a well defined and controlled mathematical content, is perhaps the point that, most distinctively, sets PSO apart from other proposed DO's in the POLS tradition. Souma states that the teacher much take care to plan how the problem is presented and reflect on how students will to act. Souma names (1987) the type of tasks a teacher should aim at, as *"open-closed" tasks* that stimulate conjecture and application of guessing. In ATD terms one can say the task should be "closed" so as to give a *controlled* vector from the (FE), the moment of first encounter, to (EX) and (T), and also a (somewhat) predictable outcome during the following discourse, which usually would concern the establishing of the technological-theoretical environment (TT). The task should also be "open", by giving the student a chance to make individual choices during the exploration (EX), and later give ample material for discussion, so as to invigorate the didactic process.

Souma means that, starting from standard tasks in the ordinary textbooks of mathematics, the teacher can modify parts of the tasks or change the way of stating them as in the example we saw. If the tasks presented during a sequence of lessons, are carefully constructed, it lead to conjectures, new problems and methods that, in ATD terms, productively connects the local MO's covered with more global ones and inspire to technological and theoretical discourses on higher-level MOs. The insistence on open-endedness of the task is common with the "open approach method" (Nohda, 1991), which is another proposed variant of POLS. The open approach method is used and analysed by Japanese educators (Hino, 2007). Open-ended problems often take the form of formulating a mathematical model for some phenomenon that lead to multitude of problem formulations, techniques and solutions. The intent is to let students develop and express different approaches and let them reflect on their own ideas by seeking to grasp those of their peers (Miyakawa & Winsløw, 2009). Souma (Personal Communication, 2010) judges the open-approach method as something that cannot be used in everyday school mathematics. POLS lessons applying too ambitious open-ended problems might be isolated from ordinary lessons that, for instance, aim to train students' basic mathematical skills, but Souma (1987) acknowledge this type of projects at the end of a course. Nohda also notifies that "We do the teaching with the open-approach once a month as a rule" (Nohda, 1991, p. 34). Bosch et al. (2007) have discussed the danger with open-ended activities, which are introduced at school without any connection to a specific content or discipline. They state that this type of didactic technology suffers the risk of causing the construction of very punctual mathematical organisations, since this is what students are trained to study.

If we return to the lesson template and the example, the teacher should let students who have different types of solutions present their problem in class. The teacher then leads the class to discuss the reason behind each method and have all students determine which of the techniques they have used and why. This is the didactic technique of *whole class discussion* of solutions, which PSO has in common with POLS in general. The discussion of alternative solutions gives an opportunity to introduce, establish and reinforce technological and theoretical components of the MO studied, like in this case, the expansion of the square, the rule of the conjugate and the different use of variables. The primary motivation is to steer the didactic process into (TT), where new methods and techniques are approved. Solutions and motivations given by the students are sometimes unexpected and may make more sense for other students. The class discussion also serves the purpose of increasing the participation in the didactic process.

After this, Souma recommends that the students have an opportunity to reflect upon the mathematical theory. The teacher can point out what they have learned by having a student read out from the textbooks explanations of the theory relevant for the lessons. During this *theoretical reflection*, the teacher can steer the didactic process towards, say, (I) institutionalisation or (EV) evaluation. Souma states firmly (1997) that studies in mathematics should be organised and based on a well-written textbook that gives a clear explanation of the mathematical definitions and theories. The classroom discourse is only one form of the study process, the study of mathematics will always entail individual studies and individual problem solving inside and outside school. Moreover, the textbook allows the students to recognize and get familiar with the theory, which the textbooks usually explain in more full detail. In other words, the *textbook technology* is proposed, for the purpose of further covering of all six moments.

A MATHEMATICAL PROBLEM ORIENTED CLASS IN JAPAN

The following episode illustrates a mathematical problem oriented lesson where the teacher practices the PSO approach. This study take place during a lesson study in grade eight at a lower secondary school affiliated to the School of Education in Asahikawa, Japan, 2009. The teacher is a former Masters student of Souma. The number of students in this class is 40. The lesson is about how to solve a system of linear equations and is the third lesson on this topic. The students have already studied the addition method by solving linear equations obtained from word problems with an everyday life character. The lesson plan was written and distributed by the teacher to us observers beforehand. Posing the mathematical tasks and problems presented during the lesson is common with the POLS based lesson plans, but distinct to PSO is, that it is always written “students possible conjectures” and “students possible solutions”, so that teachers always prepare different didactic responses depending on which act students take (Souma, personal communication, 2010).

In the guidebook of Japanese national curriculum standards “The curriculum guidelines” (2008) for mathematics for Japanese secondary school, a system of linear equations with two variables is described (p. 90) as follows: “Solving a linear equation with two unknowns is to make clear that this can be done by using a method that eliminates one of the two variables and then solve equations with one unknown, which is a method students already know". Thus, the didactic transposition of the praxeology "System of linear equations" to the knowledge to be taught in class (Chevallard, 1985 in Bosch & Gascón, 2006), focuses here on the technique of elimination; reducing the pair of two variables equations to one equation with one unknown. Techniques and technological terms present are substitution, row operations, isolation, coefficients, variables, etc. which are collected from the theoretical base of "Elementary algebra".

The lesson

As the first step, the teacher shows the problem by verbally reading out a system of linear equations; $\{7x + 3y = 30,\ x - 5y = 26\}$ and the students are asked to copy this in writing. He asks: "There are two boys, Taro and Jiro, who both solved this problem. Taro said, "I eliminate *x*". Jiro said, "I eliminate *x as well*". Their answers were the same, but their methods of the solutions were different. Today's task is to consider how they solved the problem differently". The teacher does not show the techniques; the students must consider the possible techniques, which obviously is not only one.

The teacher gives them a few minutes ("individual thinking activity" –according to the lesson plan) and encourages them to find as many solutions as possible. He states in lesson plan that this is especially meant for the gifted students who find solutions quickly. The teacher picks up two students who have obtained different techniques and lets those two students write their solutions on the blackboard. The teacher asks the class how many of them used the technique one of the two students has used. The students raise the hands and it is 37 of them. The teacher asks what is the name of this technique and gets the answer "the addition method" which the class already learned at the previous lessons. The teacher asks the class how this technique works. A student answers "Change the coefficient to the same and erase one of the variable". The student who has written the solution on the black board explain her reasoning how she has "changed the coefficient". She says, "*x*'s coefficient must be changed, so I multiplied it by 7". The teacher responds, –"OK, you multiplied by 7 and got the same coefficient for all the *x:*s". He changes his voice tone a little and then asks "And then, (looks around the class) *what can you do* with the *x*?" Several students respond, "We can *eliminate* the *x*".

They later discuss the other solution technique called the "substitution method", He inquires again how many of the students came up with an example of that technique (17; –many of them used both methods), and asks for the name of the technique, and then lets the students explain how the technique works. (Some students might already have learned about the technique at "Juku" – a private school offering special classes held on weekends and after regular school hours.) The teacher later asks if there are any students who found variants of the addition method, with an intention to let the students be aware to variation of techniques of the addition method. One student presents his solution by multiplying with –1/7 to $7x + 3y = 30$, instead of multiplying by –7 to $x - 5y = 26$. This presentation awakes a big discussion in the class if it is not a bit too complicated. The teacher concludes the discussion by encouraging the student with: “But it worked? Didn’t it?”

After the class has had this look at the two different techniques, the teacher lets one student read out loud a passage from the chapter in the textbook, explaining the substitution method. The students work out three to five textbook problems using the substitution method from the book. Afterwards, the teacher asks the class “In which types of problem do you use the addition method and in which types do you use the substitution method?” He lets the students write down their reasoning. The students are then encouraged to *create* several examples of problems they think fit each technique and different proposals are then later discussed.

ATD analysis of the lesson

The purpose of this lesson is to introduce the substitution method and compare it with it to the addition method and to show that both methods reduce the system to the one-variable one-equation case. In his lesson plan, the teacher writes that “The aim of the task” is to “make the students *find out* that there is an another method than addition method through mentioning that two boys use different methods”. He asks how to reconstruct the solution of two boys, instead of asking them “Solve this system of linear equations using the substitution method”. This is an instance of the Souma’s guessing technique, since all students are assumed to be able to use the addition method and students are requested to make proposals rather than final answers. This is also an example of an “open–closed” task; with alternative solutions, but a limited number of possible outcomes. As intended, the task steer the didactic process from (FE) to (EX) and (TT), since it is about finding a new technique, where (TT) is mainly covered during the whole class discussion. The task will also entail (T), technical work, since the students should solve the system with the chosen method. The teacher stimulates *participation* by having all students report which method they have followed. In the discourse, the teacher takes care to make the students use the correct technological terms, like “addition method”, “substitution method”, and the use of “eliminate” rather than “erase”. Much of the same holds for the final task when they are asked to construct problems that are suitable for each method. As proposed by Souma, reflection on theory is carried out when one student reads out loud from the textbook. This steers the didactic process to the moment of (I), so that the class now verifies what they have done during the lesson. More (T) is covered when the students work on problems in the textbook.

DISCUSSION AND CONCLUSION

One can summarise Souma’s approach as one firmly grounded in the POLS tradition. PSO, in particular, focuses on how to start up the didactic process using the guessing technique by adding the elements of conjecture, construction and choice from the start, stimulate students’ curiosity to tackle with the mathematical tasks. Souma argues for the didactic techniques of presenting problems followed by whole-class discussion, theoretical reflection and the use of textbooks. The main difference with POLS in general is *guessing* and that Souma stress the need for *open-closedness* when it comes to task construction. Souma holds that a DO based on open-ended activities needs to be established as a long-term *didactical contract* (Brousseau, 1998) and that it should be used as the regular lesson structure.

In this paper I *describe* PSO using the analytical framework of ATD and also use ATD motivate the didactical techniques: By using the *guessing* technique, the teacher allows *all* the students in the class to *participate*. *Open-closed* tasks, *textbook* and *theoretical reflection* are techniques with the dual purpose of both *invigorating* and *control* the didactic process. Some qualitative predicates, like "participation", "activity" and "invigorate", regarding the didactic process were introduced to cover the psychological/cognitive motivations, which are implicit and explicit in PSO. ATD is of course a theory with a wide scope and is also a "radical" theory that problematises the content of mathematical curricula, and, in this respect, Souma's insistence on following a textbook and the focus on closedness is perhaps a contradictory "conservative" trait. But, on the whole, I think that most normative implications of ATD are in line with PSO and with POLS in general. Fundamentally, by attacking problems and following up by whole-class discussion, one links praxis with logos.

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