FACULTY OF ENGINEERING AND SUSTAINABLE DEVELOPMENT

Studying Noise Contributions in Nonlinear Vector Network Analyzer (NVNA) Measurements

Feng Tianyang

September 2012

Master’s Thesis in Telecommunications

Master’s program in Electronics/Telecommunications

Examiner: Dr. Per Landin

Supervisors: Prof. Wendy Van Moer and Prof. Niclas Björsell
Acknowledgment

I would like to thank everybody who has helped me with this thesis. I would like to thank my supervisors, Wendy Van Moer and Niclas Björsell for giving me the advice and guidance. Special thanks go to Per Landin and Efrain Zenteno for their precious time and wise suggestions.

To my friends, those who are in distance and nearby, thanks a lot for all the fun we had together and for sharing the ups and downs with me. I know you will be there when I needed.

Finally to my beloved parents, thanks for your unconditional support and endless love. I love you!
Abstract

Noise contribution in nonlinear systems is very different from that in linear systems. The noise effects in nonlinear systems can be complicated and not obvious to predict. In this thesis, the focus was on the noise contribution in nonlinear systems when measuring with the nonlinear vector network analyzer (NVNA). An additional noise source together with a single sinewave signal was fed into the input of the amplifier and the performance was studied. The input power of the amplifier is considered to be the sum of the noise power and the signal power. The variation of the 1 dB compression point and the third order interception point as functions of the added noise power were studied. From the measured results in this thesis, the 1 dB compression point referred to the output power will decrease when increasing the added noise power at the input of the amplifier. The contribution of the added noise to the 1 dB compression point of an amplifier is considered dual: with the added noise the linear regression lines of the AM/AM curves are changed, and due to hard clipping the useful output power is reduced. As a result of those two effects, the added noise made the compression start at a lower power level. When the added noise reaches a certain level, the 1 dB compression point is hard to measure.

Thus when performing nonlinear measurements, the noise effects should be taken into considerations and further studies are required to get better understanding of the system’s behavior in noisy environment.
# Table of Contents

Acknowledgment ................................................................................................................. i

Abstract .............................................................................................................................. iii

1 Introduction ......................................................................................................................... 1
  1.1 Background .................................................................................................................... 1
  1.2 Problem statement ........................................................................................................ 1

2 Nonlinear High Frequency Measurements: Theory ........................................................... 3
  2.1 Linear Systems and Nonlinear Systems ......................................................................... 3
  2.2 Power Amplifier ............................................................................................................ 5
  2.3 The NVNA ................................................................................................................... 9
  2.4 Calibration .................................................................................................................. 12
     2.4.1 Vector Calibration ................................................................................................. 12
     2.4.2 Phase Calibration for the NVNA ......................................................................... 13
     2.4.3 Power Calibration for the NVNA ....................................................................... 16
     2.4.4 Source Power Calibration for the NVNA ............................................................. 17
  2.5 Additive White Gaussian Noise .................................................................................. 18
  2.6 Rice Distribution ......................................................................................................... 18
  2.7 Uncertainty Analysis .................................................................................................... 19
     2.7.1 Standard Deviation ............................................................................................... 19
     2.7.2 Confidence Region ............................................................................................... 20
  2.8 Input Noise Influence on the 1 dB Compression Point of an Amplifier ................. 23

3 Nonlinear High Frequency Measurements: Measurement Setup .................................... 27
  3.1 Practical Considerations ............................................................................................... 27
  3.2 Classical Measurement Setup ..................................................................................... 28
  3.3 Noise Measurement Setup ........................................................................................... 29

4 Measurement Results and Discussion .............................................................................. 33
  4.1 Internal Source Test ...................................................................................................... 33
  4.2 Classical Measurements ............................................................................................... 36
  4.3 Noise Measurements .................................................................................................... 43
  4.4 Methods Discussion ..................................................................................................... 56

5 Conclusions and Future Work ......................................................................................... 59
  5.1 Conclusions .................................................................................................................. 59
  5.2 Future Work ................................................................................................................. 60

Appendix ............................................................................................................................... 61

References ............................................................................................................................ 63
1 Introduction

1.1 Background

Nowadays, the linear framework is very well-studied and well-known by engineers. The measurement instrument, measuring techniques, and modeling methods for linear devices are various. Even though active systems are not purely linear, their nonlinear behavior was considered to be a perturbation and eliminated by engineers. Since we know that for a linear device, single tone input sinewave will result in a single tone output sinewave, it is enough to do relative measurements using vector network analyzer (VNA). However, when it comes to a nonlinear device, it is difficult to study the system’s behavior. The main reason is that one should have all the information of the absolute phase and amplitude for all harmonics to reconstruct the time domain waveform of the output signal of a nonlinear device. Thus special measurement instruments are needed for measuring nonlinear devices.

With the help of nonlinear measurement instruments, the nonlinear behavior can be further studied. Nonlinear measurement instruments such as the large-signal network analyzer (LSNA) and the nonlinear vector network analyzer (NVNA) [1] are able to measure the nonlinear time domain waveforms correctly. Different design ideas were used in these two network analyzers, the LSNA is a sampler-based methodology and the NVNA is a mixer-based methodology [2]. Good knowledge of nonlinear system measurements is a start for nonlinear modeling which is necessary for nonlinear device simulation and design.

Since a nonlinear device treats input noise in a completely different way compared to a linear device, it is worth doing measurements with input noise added to a nonlinear device and study the noise contribution in those measurements.

1.2 Problem statement

Previous study with an amplifier input consisting of a CW signal and amplified thermal noise was done in [3], where the input and output filters had the same bandwidth. The influence of the CW signal to the noise behavior was studied. The results showed that the signal gain and the noise gain are both dependent on the input CW signal peak amplitude in a very different fashion.
In this thesis, the focus is put on studying the input noise contribution of a nonlinear system with limited receiver bandwidth when measuring with a NVNA. This is illustrated in Figure 1.1. A power amplifier is chosen as the device under test (DUT) for this project. A noise source with additive white Gaussian noise (AWGN) is added to the input of the DUT, and the measured results will be analyzed as a function of the noise power to study the noise contribution. The input of the amplifier is the combination of the noise signal and the single tone sinewave, while the actual received noise power is limited by the narrow bandwidth of the receivers. The added noise will affect the nonlinear device and cause deviations to the measured results. But the measured results only contain a small amount of noise. In this way, the contributions from the added input noise can be studied instead of that from the output noise.

![Figure 1.1: The illustration of the noise measurements.](image)

The problem can be solved in three steps: firstly, the nonlinear measurements should be performed using nonlinear measurement instrument, which is NVNA in this case; secondly, a noise source with additive white Gaussian noise (AWGN) is added to the input of the DUT, and the noise measurements are performed with varying noise power; finally, the measurement results as a function of the added input power are processed and analyzed.

All the information about the nonlinear system comes from measurements, thus it is extremely important to get accurate results. To maximize the accuracy of the measurement results, a reliable instrument, suitable measurement setups, good calibrations, careful measurement, and finally the correct data analysis are required.
2 Nonlinear High Frequency Measurements: Theory

2.1 Linear Systems and Nonlinear Systems

A system is defined as a physical device that performs an operation on a signal [4]. A linear time invariant system is a system whose properties do not change with time, and also obeys the superposition principle. In other words, a system is linear if a linear combination of the input signals results in the same linear combination of the output signals:

Let \( f(x) \) be the function of the linear system,

then

\[
f \left( \sum_{i=1}^{N} k_i u_i(t) \right) = \sum_{i=1}^{N} k_i f(u_i(t))
\]  

(1)

where \( u_i(t) \) represent the \( i \)th input signal of the system as a function of time \( t \), and \( k_i \) is a constant. If the system does not fulfill this principle, the system is nonlinear.

The fast Fourier transform (FFT) [5] is a linear operator, thus the response spectrum of a linear system \( Y(\omega) \) only contains those frequency components that are present in the input spectrum \( U(\omega) \) [2]. In the case in Figure 2.1 (a), the frequency component is at angular frequency \( \omega_0 \), and no extra harmonics are created. For measuring a linear system, a ‘relative single frequency wave meter’ can be used, such as a vector network analyzer [6]. The relative phase and amplitude information between the input and output of the DUT is measured and is enough for a linear system. Let a sinusoidal signal \( u(t) \) be the input signal

\[
u(t) = A \cdot \cos(\omega_0 t)
\]  

(2)

the output signal of the linear system can be expressed as

\[
y(t) = k_1 (A \cdot \cos(\omega_0 t)).
\]  

(3)
Systems that do not behave as a linear system are nonlinear systems. A weak nonlinear system will be studied in this project. The output of the system modeled with a third-degree polynomial is written as

\[ y(t) = H[u(t)] = k_1 u(t) + k_2 u^2(t) + k_3 u^3(t) \quad (4) \]

The input and output spectrum of this nonlinear system is shown in Figure 2.1 (b).

Let a sinusoidal signal \( u(t) = A \cdot \cos(\omega_0 t) \) be the input signal, then the output signal of the nonlinear system can be expressed by

\[ y(t) = k_1 (A \cdot \cos(\omega_0 t)) + k_2 (A \cdot \cos(\omega_0 t))^2 + k_3 (A \cdot \cos(\omega_0 t))^3. \quad (5) \]

Applying the triple-angle formula as

\[ \cos(2\omega_0 t) = \cos(\omega_0 t)^2 - \sin(\omega_0 t)^2 = 2 \cos(\omega_0 t)^2 - 1 \quad (6) \]
\[ \cos(3\omega_0 t) = 4 \cos(\omega_0 t)^3 - 3 \cos(\omega_0 t) \quad (7) \]

the output signal can be calculated, and is shown in...
\[ y(t) = k_1 A \cdot \cos(\omega_0 t) + k_2 \frac{A^2}{2} (\cos(2\omega_0 t) + 1) + k_3 \frac{A^3}{4} \cos(\omega_0 t) + k_3 \frac{3A}{4} \cos(\omega_0 t) \]

\[ = k_2 \frac{A^2}{2} + \left( k_1 A + \frac{k_3 3A^3}{4} \right) \cos(\omega_0 t) + k_2 \frac{A^2}{2} \cos(2\omega_0 t) + k_3 \frac{A^3}{4} \cos(3\omega_0 t). \quad (8) \]

The output of this system not only has fundamental signal at \( \omega_0 \), but also the second harmonic and DC generated by \( k_2 u^2(t) \), and the third harmonic generated by \( k_3 u^3(t) \). This nonlinear effect is not possible in linear systems. In linear systems, with single tone input signal, only single tone output signal will be generated at the same frequency. Four terms are presented in the output signal polynomial, and their contributions at DC, fundamental frequency \( \omega_0 \), \( 2\omega_0 \), and \( 3\omega_0 \) respectively depend on the input signal amplitude \( A \) in a nonlinear fashion.

The nonlinear effects can be studied by comparing the output signal for nonlinear system in Equation (8) with that for linear system in Equation (3). The nonlinearity of a system can be considered in two ways: one is a generation of new spectral components; the other is an amplitude-dependent offset of the signal gain. Common measurements for these two effects of an amplifier are the 1 dB compression point and the third order interception point.

### 2.2 Power Amplifier

A power amplifier is an active component and it has both linear and nonlinear region. The output obtained from a power amplifier may vary according to the varying input signal. An ideal power amplifier should have a linear transfer characteristic as shown in Figure 2.2 (a), which can be described as Equation (9) with a certain constant gain factor \( K \) in dB.

\[ P_{OUT}(t) = K + P_{IN}(t) \quad [dBm] \quad (9) \]

However, a power amplifier in a real world would operate differently than just in a linear fashion. In fact, the transfer characteristic of a power amplifier should have three zones as shown in Figure 2.2 (b): the cut-off region where the amplifier acts as an open circuit, the linear region, where the amplifier gain is constant and the shapes of the input and output signals are the same, and the saturation region, where the power amplifier acts as a nonlinear system, producing harmonics and varying the gain of the fundamental tone [7].
Both the cut-off region A and the saturation region C in Figure 2.2 (b) are nonlinear regions, and we focus on the saturation region in this project.

![Transfer characteristic of an ideal linear amplifier](image1)

As seen from Figure 2.2 (b), for nonlinear system, which is what a power amplifier is when operating in saturation region, single frequency input will result in additional spectral components. Furthermore, the constant gain factor for the fundamental angular frequency $\omega_0$ no longer exists. The gain at the fundamental frequency component is constant in the amplifier’s linear region, and it starts to compress when it comes to its nonlinear region.

For a weak nonlinearity, the third-degree polynomial model in Equation (4) is suitable for the amplifier. If the same model is used here for a RF power amplifier, let a sinusoidal

![Nonlinear amplifier](image2)
signal \( u(t) = A \cdot \cos(\omega_0 t + \varphi_0) \) be the input signal and then the output signal of the amplifier will have contributions at three frequencies as shown in Equation (8).

Looking at the effects of the nonlinearity as a function of signal amplitude, the amplitude of spectral components generated by a single-tone sinusoidal signal is listed in Table 2.1.

Table 2.1: Amplitude of spectral components generated by a single-tone sinusoidal signal and nonlinearities up to the third degree.

<table>
<thead>
<tr>
<th>Angular frequency</th>
<th>DC</th>
<th>( \omega_0 )</th>
<th>2( \omega_0 )</th>
<th>3( \omega_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>( k_2 \frac{A^2}{2} )</td>
<td>( k_1 A + \frac{k_3 3A^3}{4} )</td>
<td>( k_2 \frac{A^2}{2} )</td>
<td>( k_3 \frac{A^3}{4} )</td>
</tr>
</tbody>
</table>

The amplifier gain at its fundamental frequency is \( k_1 + \frac{k_3 3A^2}{4} \). Since the output power is compressed in saturation region, the additional gain factor caused by the nonlinearity \( \frac{k_3 3A^2}{4} \) is negative, which mean \( k_3 \) is negative. The 1 dB compression point of the power amplifier is caused by this nonlinear effect. The 1 dB compression point plot is shown in Figure 2.3. As seen from Figure 2.3, the 1 dB compression point is the point where the output power of the amplifier is 1 dB less than the linear gain would give. Assume that the linear gain of the amplifier is \( K \) dB; the output power at the 1 dB compression point for this amplifier is represented by \( P_{1dB\_OUT} \), while the input power at the 1 dB compression point for this amplifier is represented by \( P_{1dB\_IN} \). Then the 1 dB compression point can be found by fitting

\[
P_{1dB\_OUT} = K + P_{1dB\_IN} - 1 \quad [dBm].
\]
Figure 2.3: Illustration of nonlinear effects. The fundamental output begins to change from its linear 1:1 slope at high amplitude levels and the generated spectral component (third harmonic) increases as a function of signal amplitude.

Studying the generated harmonics, it is clear that both gain factors of the fundamental harmonic and of the third harmonic depend on two terms: the input signal power and the negative coefficient $k_3$. This means that if we consider the output of the amplifier as function of the input power, as shown in Figure 2.3, the slope of the third harmonic will be three times the slope of the fundamental harmonic. In other words, if the input power increases a certain number of dB, the output power at the third harmonic will increase three times as fast as the power of the fundamental frequency component. Thus the linear extension lines of the fundamental and third harmonics will meet when the input power is high enough, as seen in Figure 2.3. The third order interception point is the point where the extrapolated linear and distortion products cross [8]. Notice that the TOI is defined when the input power of the amplifier is at a lower level and the amplifier is working in its linear region. The third order interception point is noted as TOI in Figure 2.3, the output power at the third order interception point is noted as $TOI_{OUT}$ while the input power is noted as $TOI_{IN}$. By using Equation (8) with negative $k_3$ and single tone input, a common approximation states that the 1 dB compression point is around 10 dB less than the third order interception point referred to its input power [9].
From the simulated output signal results in Equation (8), the phase information for the input and output harmonics is not included. When measured the input and output signals, there will be different phase information for the input signal and output harmonics. The output phase relations between harmonic components are difficult to predict. However for a nonlinear system, it is important to know the phases between the spectral components. For two signals that have identical amplitude spectra and different phase relation between harmonics, the time waveforms of them can be very different. This is illustrated in details in [10]-[11]. Thus to reconstruct the time domain signal, both the amplitude and phase information of all harmonics are needed to be measured. Having accurate time waveforms is essential for good modeling and simulation, thus special measurement instruments that can provide phase as well as amplitude are needed.

2.3 The NVNA

A normal vector network analyzer (VNA) is a relative single frequency wave meter, which measures the relative S-parameters of the electrical networks [12]. For linear time invariant (LTI) systems, a VNA is enough for analyzing LTI systems. However, the absolute phase information for each harmonics is necessary to construct the wave in time domain when it comes to nonlinear systems. To measure the phase relation between harmonics, and to study a DUT’s nonlinear behavior, the NVNA can be used.

The NVNA gives opportunity to measure both amplitude and phase information for incident, transmitted, and reflected waves of a DUT for a specified number of harmonics when a continuous wave excitation is applied to that DUT. The phase of the fundamental harmonic is considered to be the phase reference, and all phase information about the other harmonics is given by the NVNA referred to the reference.
The NVNA is based on a VNA, and thus both of them can excite the DUT at one or all ports. The NVNA uses mixers for the down-conversion process and measures one frequency component at a time. Each port has two directional bridges in order to sample the incident (a1, a3) and reflected waves (b1, b3). For each receiver the down-conversion process is made by a mixer driven by a high-frequency oscillator. Five mixers are used in the NVNA and they are all driven by the same local oscillator, LO-source as shown in Figure 2.4 [2]. For the NVNA, general measurements can be configured in three ways when using internal input source together with PNA-X 10 MHz phase reference source, the configuration diagram is shown in Figure 2.4. Details about hardware configuration of the phase reference are presented in the calibration section.

Ideal mixers are basically multipliers, which translate the modulation around one carrier to another [13]. The down conversion procedure is illustrated in Figure 2.5.
Figure 2.5: Down-conversion procedure using a mixer.

Assume the two inputs of a mixer are $\cos(\omega_1 t)$ and $\cos(\omega_2 t)$, the output of the mixer is:

$$\cos(\omega_1 t) \times \cos(\omega_2 t) = \frac{1}{2}\cos((\omega_1 - \omega_2)t) + \frac{1}{2}\cos((\omega_1 + \omega_2)t) \quad (11)$$

Down conversion mixers keep the low-frequency terms, which is $\frac{1}{2}\cos((\omega_1 - \omega_2)t)$ in this case. The high-frequency terms, which is $\frac{1}{2}\cos((\omega_1 + \omega_2)t)$ in this case, is filtered out.

The RF signal in the NVNA simplified block schematic is the coupled signal wave for one of the incident waves (a1, a3) or reflected waves (b1, b3). The LO-source in Figure 2.4 is the local oscillator signal. The intermediate frequency signal IF will be the low-frequency terms. Finally the intermediate frequency signal will be fed into the input of the AD converters.

Four green mixers in Figure 2.4 are used for the four waves, and a fifth red mixer is used to select and to down-convert the harmonic components generated by the harmonic phase reference, which is a comb generator. This harmonic phase reference is also known as measurement phase reference. The LO-source together with the input source of the NVNA port 1 and the input of the harmonic phase reference are controlled by the 10 MHz clock, which gives opportunity to measure the four waves and the reference signal at the same time. In order to measure the phase reference during measurements, the measurement phase reference should be always connected.

The relative measurement results of the four waves (a1/ref, b1/ref, a3/ref, and b3/ref) are achieved, as well as the exact phase information of the reference. Together, the phase relationship between the harmonic components in the measured waves can be derived.
2.4 Calibration

Calibration is essential before doing any measurements. For a complete NVNA calibration, three steps of calibration are required: a vector calibration, a phase calibration and a power calibration.

2.4.1 Vector Calibration

A vector calibration is a classical relative calibration which is identical to the standard calibration used for a VNA [14]. Random errors and systematic errors both contribute in the measurement errors. The random error can be seen by comparing repeated measurements and can be reduced by averaging the repeated measurements. Systematic errors such as the effect of the test port cables, connectors, components, and the variations caused by changes in the environment, cannot be reduced by averaging. The systematic error in measurement can be canceled by a good vector calibration. For microwave frequencies, the error caused by variations in the environment is serious, and thus a calibration must be performed before every use of the NVNA in order to compensate the errors [13]. The basic principle of the vector calibration is by using the ‘raw’ measured S-parameters obtained by measuring several different calibration standard components, and evaluating the error terms of the error model, the real S-parameters can be derived correctly.

The error box model according to [15]-[16] has forward model and reverse model, and the forward model is shown in the Figure 2.6.

![Figure 2.6: 12-Term Error Model, forward model [16]](image-url)
The terms of the error box model can be calculated using the values of the ‘raw’ measured S-parameters. The six error terms in Figure 2.6 are $D$ for directivity, $M_s$ for source match, $T_r$ for reflection tracking, $M_l$ for load match, $T_t$ for transmission tracking and $X$ for isolation. In connectorized measurements, isolation is usually not a problem [13]. The six error terms can be expressed as function of the ‘raw’ measured S-parameters using Equations (12-17). The measured S-parameters were denoted with sub index ‘m’.

$$D = S_{11m\,load}$$

$$X = S_{21m\,load}$$

$$M_s = \frac{2S_{11m\,load} - S_{11m\,short} - S_{11m\,open}}{S_{11m\,short} - S_{11m\,open}}$$

$$T_r = (S_{11m\,load} - S_{11m\,short})(1 + M_s)$$

$$M_l = \frac{S_{11m\,thru} - D}{T_r + M_s(S_{11m\,thru} - D)}$$

$$T_t = (S_{21m\,thru} - X)(1 - M_s M_l)$$

From the terms of the error box model, the corrected values of S-parameters can be derived by using signal flow diagram theory [17]-[18]. For the forward model, the exact solution to correct the deviations in the system for $S_{11}$ and $S_{21}$ can be expressed with Equations (18) and (19). The corrected S-parameters were denoted with sub index ‘A’.

In similar way, the reverse model can be drawn and the other six error terms can be found, which lead to the formulas for $S_{22}$ and $S_{12}$.

$$S_{11A} = \frac{(S_{11M} - D)(1 - S_{22A} M_l - S_{12A} S_{21A} M_s) - S_{11A} M_s (S_{11M} - D)}{T_r - S_{22A} M_l T_r + M_l(1 - S_{22A} M_l)(S_{11M} - D)}$$

$$S_{21A} = \frac{(S_{21M} - X)(1 - S_{11A} M_s - S_{22A} M_l - S_{11A} S_{22A} M_s M_l) - S_{21A} M_s}{T_t + (S_{21M} - X) S_{22A} M_l M_s}$$

The NVNA uses 8-term error correction which assumes the crosstalk leakage term is zero and the port match will not be changed as it switched from forward to reverse.

2.4.2 Phase Calibration for the NVNA

In order to describe a nonlinear system, and to know the exact phase information for the incident, reflected and transmitted waves of harmonics the complex calibration factor
\( K(i) = |K(i)| \cdot e^{i \phi (i)} \) in Figure 2.7 must be determined. Two additional calibration steps are introduced: a phase calibration to obtain the phase \( \phi (i) \) and a power calibration to obtain the absolute value \( |K(i)| \) [19]-[21].

![Diagram of 12-Term Error Model, forward model for NVNA calibration [9].](image)

In Figure 2.6, the standard VNA calibration, the absolute power and phase of the transition path loss of the forward direction which is factor \( K(i) \) is not of interest, only the relations of the waves are calculated, see Equation (20) [11]. The corrected waves were denoted with sub index ‘A’. The measured waves were denoted with sub index ‘m’. A full 16 error terms are in the error matrix [11].

\[
\begin{bmatrix}
    b_2 \\
    b_1 \\
    a_2 \\
    a_1
\end{bmatrix}_m = K
\begin{bmatrix}
    e_{11} & e_{12} & e_{13} & e_{14} \\
    e_{21} & e_{22} & e_{23} & e_{24} \\
    e_{31} & e_{32} & e_{33} & e_{34} \\
    e_{41} & e_{42} & e_{43} & e_{44}
\end{bmatrix}
\begin{bmatrix}
    b_2 \\
    b_1 \\
    a_2 \\
    a_1
\end{bmatrix}_A
\]  \hspace{1cm} (20)

The error term \( e_{11} \) can be set to 1. In the standard calibration, the relations between waves are of interest. Then the \( K \) factor will be cancelled when calculating the S-parameters. However, in the calibration of the NVNA, the corrected waves are of interest, and to reconstruct the time domain wave in nonlinear systems, the exact phase and power information should be known [11]. The \( K \) factor must be determined by doing phase and power calibration. Thus the phase and power calibration are extremely important for the NVNA.

The general measurement configuration using internal input source together with PNA-X 10 MHz phase reference source is used in this project, and the configuration diagram is shown in Figure 2.8.
Figure 2.8: The general measurement configuration diagram using internal input source together with PNA-X 10 MHz phase reference source.

The NVNA port 1 is connected to the input of the DUT while port 3 is connected to the output of the DUT. The PNA-X rear panel 10 MHz ref OUT port is fed into the IN port of both comb generators using a splitter. The output port of comb generator 1 is connected to port RCVR B IN which is the input port for receiver B in the NVNA port 2. In the general measurement configuration diagram, the comb generator 2 is not necessary, and it can be removed or left open during measurements.

In the NVNA, two phase references are used, one known as measurement phase reference and the other known as calibration phase reference, and they are comb generator 1 and comb generator 2 in Figure 2.8. The two comb generators are essential for non-linear measurements. In this project, Agilent’s U9391C/F comb generators [22] are used as the NVNA’s harmonic phase reference. The comb generator can be seen as a device that, when excited by a single tone input frequency, delivers at its output harmonics of this input frequency up to about 26 GHz from factory characterization. The phase relationship between those harmonics of its output is assumed to be known. Thus the systematic phase distortions introduced by the NVNA can be determined by measuring the phase relations between the harmonics with the NVNA and comparing them to the known phase relation. A scheme to perform this calibration is shown in Figure 2.9.
It is best to perform the phase calibration on the NVNA port 1 since its test receiver usually has less attenuation than that in the NVNA port 3 [23]. As shown in Figure 2.9, the calibration phase reference, comb generator 2 is connected to the input port of the DUT. Then the NVNA will measure the calibration phase reference and the measurement phase reference at the same time to determine the phase distortions introduced by the NVNA for each measurement frequency. The phase distortions will be saved and used for future measurements.

As shown both in Figure 2.8 and 2.9, it is best to choose the reference plane at the input and output of the DUT. In this way, other components in the measurement set up would be included into the calibration and the variations over frequency should be taken care of by the calibration. Especially for measuring nonlinear devices, the components in the input or the output of the DUT may probably change the phase of the waves, which is difficult to compute and compensate in other ways if the reference plane is set somewhere else.

2.4.3 Power Calibration for the NVNA

The excitation power of the NVNA needs to be known absolutely. Thus a power calibration is needed. The power calibration is done using a power meter. In this project,
power meter E4418A is used. The measurement configuration diagram when doing power calibration is shown in Figure 2.10. The NVNA port 1 is connected to a power meter. The GPIB cable of the power meter should be connected with the GPIB controller port of the NVNA. Then the power meter can be controlled by the NVNA and for each frequency the source power will be measured by both the power meter and the NVNA.

![Diagram of measurement configuration for power calibration](image)

Figure 2.10: The measurement configuration diagram when doing power calibration.

The power calibration is performed using a calibrated power meter controlled by the NVNA software. Note that the same port should be used for both phase calibration and power calibration. Since the port 1 RF path will usually have the lowest power, it is often the best choice to perform a power calibration on port 1 [23]. These two additional calibration steps give the ability to measure the absolute power injected into the DUT, and the phase relationships between the fundamental and the harmonics in the measured spectra [24].

2.4.4 Source Power Calibration for the NVNA

The source power calibration is an extra step for the NVNA calibration which ensures that the power level is accurate at the reference plane. After a full NVNA calibration which includes a standard VNA calibration, phase calibration and power calibration, the source power calibration can be performed. This is because when performing the source power calibration, the NVNA assumes that the reference receivers have already been calibrated and the receivers are used to measure the source power and adjust it to be accurate at the reference plane.
For accurate source power, the source open loop mode should be set off which is set to be on by default.

### 2.5 Additive White Gaussian Noise

A stochastic variable $X$ is said to be Gaussian if its PDF is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

(21)

where $\mu = E\{X\}$ and $\sigma^2 = Var\{X\}$ [25]. This can be represented as: $X \sim N(\mu, \sigma^2)$.

A 2-dimensional Gaussian vector $Z$ can be formed by $X$, and $Y$ under the assumption that $X \sim N(\mu_x, \sigma_x^2), Y \sim N(\mu_y, \sigma_y^2)$ and $X, Y$ are not correlated.

$$Z = [X Y]^T$$

(22)

A white process as white noise is a process with constant Power Spectral Density (PSD) [25]. Additive White Gaussian Noise (AWGN) is a process that is both white and Gaussian, and can be generated by using a 2 dimensional Gaussian vector $Z$.

### 2.6 Rice Distribution

The Rice distribution is a distribution of the magnitude of a circular normal random variable with non-zero mean. A random variable $R$ is said to have a Rice distribution with parameters $v$ and $\sigma, R \sim Rice(v, \sigma)$, if $R = \sqrt{X^2 + Y^2}$, where $X$ and $Y$ are independent normal random distributions, $X \sim N(v \cos \theta, \sigma^2)$, and $Y \sim N(v \cos \theta, \sigma^2)$. $\theta$ can be any real number. The estimation methods such as method of moments, method of maximum likelihood, and method of least squares are used for estimating the parameters of the Rice distribution, see [26]-[28] for more details. In this project, the method of moments was used to estimate the two parameters of Rician distribution in [29]. In the situation with high SNR, the Rice distribution can be treated as Gaussian distribution.
2.7 Uncertainty Analysis

For nonlinear measurements, the measurement results are in complex numbers, such as incident, reflected or transmitted waves. The complex number representation is shown on the left in Figure 2.11 with a unit circle and angle \( \theta \), while the phase amplitude representation is shown on the right with a propagating wave. These complex-valued data or vector can be represented mainly in two ways, either in terms of magnitude and phase, or in terms of real part and imaginary part. Since the magnitude and phase representation have physical meanings, they were often chosen. However, when it comes to evaluate the uncertainty of the measurements for complex-valued data, one may not be interested in phase or magnitude, but in the combination. Here in this section, two methods of evaluating the uncertainty of complex-valued data are presented.

Figure 2.11: Waves in time domain with its corresponding complex representation.

\[ x(i) = p(i) + q(i) * j \]  \hspace{1cm} (23)

where \( j^2 = -1 \).

2.7.1 Standard Deviation

Standard deviation is commonly used to express the uncertainty of repeated measurements. The standard deviation can describe the total deviations from each measurement to the mean value of all the measurements, with a single number. In the case of complex-valued results, the vector differences from each measurement to the
mean vector are included with a positive real number. The vector differences are shown in the black lines of the example in Figure 2.12.

Standard deviation $\delta$ [25] can be calculated using equations

\[
\delta = \left( \frac{1}{N-1} \sum_{i=1}^{N} (x(i) - \mu)(x(i) - \mu)^* \right)^{1/2}
\]

where

\[
\mu = \frac{\sum_{i=1}^{N} x(i)}{N}.
\]

The operator $*$ in Equation (24) indicates the complex conjugate, and $N$ is the number of repeated measurements.

The normalized standard deviation can be calculated by dividing the standard deviation $\delta$ with the corresponding mean amplitude $P_{\mu}$, as in Equation (26). The mean value $P_{\mu}$ does not have to be the mean amplitude of the repeated measurement result $x(i)$.

\[
\delta_{Norm} = \frac{\delta}{P_{\mu}}
\]

2.7.2 Confidence Region

For a scalar measurement value, the uncertainty of the measurements can be expressed as a one-dimensional interval extending on either side of the mean of all measured values. In the case of a complex-valued measurement results, the uncertainty can be expressed as a two-dimensional region in the complex plane [30]. The confidence region can be plotted using the covariance matrix of several repeated measurement results. The covariance matrix is calculated as in Equations (27-30).

The variance of the repeated measurement for the real part and imaginary part is given by:

\[
var_{real} = \delta_{real}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (p(i) - \bar{p})^2
\]

\[
var_{imag} = \delta_{imag}^2 = \frac{1}{N-1} \sum_{i=1}^{N} (q(i) - \bar{q})^2
\]

The covariance of the real part and the imaginary part is given by:
\[ C_{pq} = \frac{1}{N-1} \sum_{i=1}^{N} (q(i) - \bar{q}) \cdot (p(i) - \bar{p}) \]  
\[(29)\]

where \( \bar{p} = \frac{\sum_{i=1}^{N} p(i)}{N} \) and \( \bar{q} = \frac{\sum_{i=1}^{N} q(i)}{N} \).

Thus, the covariance matrix \( \mathbf{X} \) is formed as in Equation (30).

\[
\mathbf{X} = \begin{bmatrix}
\text{var}_{\text{real}} & C_{pq} \\
C_{qp} & \text{var}_{\text{imag}}
\end{bmatrix}
\]
\[(30)\]

where \( C_{qp} = C_{pq} \) according to Equation (29).

The uncertainty of the repeated measurements is now expressed by the covariance matrix using three terms: the uncertainty in the real component, the uncertainty in the imaginary component, and the correlation coefficient. Comparing with the standard deviation from Equation (24), one can see that this covariance matrix in Equation (30) contains more information about how the relationship is between the real uncertainty and the imaginary uncertainty.

For example, for the six repeated measurements with complex-valued results in Table 2.2, the standard deviation and the covariance matrix are calculated, and the confidence region with 95% confidence interval is plotted in Figure 2.12.

### Table 2.2: The 6 repeated measurements results listed together with their standard deviation in the example.

<table>
<thead>
<tr>
<th>Repeated results</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.96+1.10i</td>
<td>1.00-1.10i</td>
<td>1.11-1.15i</td>
<td>1.21-1.50i</td>
<td>0.92+1.05i</td>
<td>1.00+1.15i</td>
</tr>
</tbody>
</table>
Figure 2.12: Elliptical 95% confidence region for the six repeated measurements in example. Measured values are shown as stars and the mean value is shown as a square and is the center of the ellipse.

As in Figure 2.12, an ellipse can be plotted using the covariance matrix. The ellipse represents all the points that are solutions to Equation (31)

$$(\beta - \mu)'X'X(\beta - \mu) = C$$  \hspace{1cm} (31)

where the vector variable $\beta$ represents the points on the boundary of the confidence region, and $\mu$ represent the mean vector of measurement results. It represents the center of the ellipse.

$$\mu = \begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix}$$  \hspace{1cm} (32)

The constant $C$ in Equation (31) is estimated by using a chi-squared distribution. Since the chi-squared distribution is the distribution of a sum of squares of certain independent standard normal variables, it is suitable to be used in this case.

An ellipse is a region that contains the information of the repeated measurements. The uncertainties of the real/imaginary components and the correlation coefficient are included by the size, aspect ratio and orientation of the ellipse. An ellipse with certain confidence interval such as 95% indicates that the probability of each measurement result falls into that ellipse is 95%. The standard deviation of the complex-valued results in
Figure 2.12, using Equation (24), is 1.2993. This contains less information than the confidence region plot, but is much simpler to compare with other data.

A geometric representation of a complex-valued uncertainty as demonstrated above can be very useful in observing and understanding trends in complex number uncertainties. For instance, the changes in the size, shape and orientation of the uncertainty ellipse can be examined over different measurement frequencies. Such investigation for S-parameters has been presented in [31].

2.8 Input Noise Influence on the 1 dB Compression Point of an Amplifier

When introducing random noise at the input of an amplifier, the 1 dB compression should come at a lower power level in theory. Since for the same amount of input power, the useful output power is less for noisy measurements than the noise-free measurement under compression. The compression of an amplifier is due to the current limitation of the transistor, thus with a pure sinewave as input, the output signal of an amplifier will be cut-off and become more like a squared wave when working in its saturation region.

A pure sinewave and noisy sinewave with equal total power are simulated. To get equal total power of the pure sinewave and noisy sinewave, different amplitude factors are used: $\text{sig}_K$ for pure sinewave and $\text{noise}_K$ for noise sinewave. See Figure 2.13. The pure sinewave is in blue and the noisy sinewave is in red, random noise with zero mean is added to the red waveform. The pure sinewave is generated as

$$\mu_0 = \text{sig}_K \ast \sin(2 \ast \pi \ast 0.01 \ast N)$$  \hspace{1cm} (33)

where $N$ is the sample vector.

The noisy sinewave is generated by adding random noise

$$\mu_N = \text{noise}_K \ast \sin(2 \ast \pi \ast 0.01 \ast N) + \delta \ast n_0$$  \hspace{1cm} (34)

where $n_0$ is random noise with zero mean, and $\delta$ is noise standard deviation.

In order to get equal signal power for both $\mu_0$ and $\mu_N$, the noise factor $\text{noise}_K$, and noise standard deviation $\delta$ must be set first. Then the noisy sinewave power is used to calculate the signal factor $\text{sig}_K$. In the plotted example in Figure 2.13, the two signals have $\text{noise}_K = 15$, $\delta = 5$, $\text{sig}_K = 15.0844$ and the same power 3.53 dB.
Figure 2.13: The simulated sinewave in the upper plot and the simulated sinewave under compression in the lower plot. The noisy sinewave are in red, and pure sinewave are in blue.

The simulated linear waves are plotted in the upper plot of Figure 2.13. However, in the saturation region of the amplifier, the absolute voltage above a certain threshold will be compressed. Assume that a hard clipping amplifier with a threshold level of 12 V is used. The waves with absolute voltage above 12 V will be compressed to 12 V to simulate the compression effect of an amplifier. The simulated waves under compression are plotted in the bottom plot of Figure 2.13. The power of both simulated waves under compression can be calculated. For this case, the noisy sinewave under compression has power 2.5832 dB where pure sinewave under compression has 2.6247 dB. This simulation shows that for the same amount of input power, the useful output power is less for noisy measurements than clean measurements under compression.

In a next step the noise standard deviation $\delta$ in Equation (34) is decreased from 5 to 1. The other settings for simulation remain the same as in the previous example. Then for the same amount of input power, the output power under compression for clean measurement it is 2.5524 dB while for the noisy measurement it is 2.5463 dB. It is clear from the simulation that the useful output power is less for a noisy sinewave than for a pure sinewave. Furthermore, with lower noise standard deviation $\delta$, the difference in output power with those two waves is less.
The 1 dB compression point is the point where the output power of an amplifier is 1 dB less of the ideal case, which in other words, is a measurement showing where the useful output power is 1 dB less than the ideal case. Thus, for a tested amplifier with hard clipping at certain power level, the 1 dB compression will come at a lower power level when increasing the added noise power.
3 Nonlinear High Frequency Measurements: Measurement Setup

3.1 Practical Considerations

When building a measurement setup, a clear goal is necessary and important. Other than that, some considerations about the device under test, and the measurement instrument should be thought through first. In this project, the goal is clear and has two parts: one is to know all the information about the amplifier, the other is to measure the amplifier when input noise is added. For both the classical measurement setup and noise measurement setup, the DUT and the measurement instrument are the same. Thus they have the following considerations.

The NVNA has power limitation for both excitation power and received power. High excitation power would cause the internal amplifier of the source to operate in its nonlinear region and make the excitation power unstable. The receiver of the NVNA has also a maximum receivable power. Received power that exceeds the limitation would cause damage in the receivers, and even though the NVNA would detect this and automatically turn off the measurements, it is too dangerous to do so.

Attenuators should be added at the input of the DUT to provide good input match and make sure that the input power would not exceed the maximum input power of the amplifier. However, in the meantime to achieve nonlinear operation of the amplifier, and to measure the 1 dB compression point, the input power should be bigger than the 1 dB compression point input power. Thus, suitable attenuation should be chosen to give good input match and also give enough power for the DUT within the source power limitation.

Attenuators also should be added at the output of the DUT to make sure that the power would not damage the receiver of the NVNA. The drawback of the attenuation in the measurement system is that the signal to noise ratio is reduced. The attenuation will reduce the signal power in the system, but the noise floor will remain the same. Thus, the attenuation in the system will affect the accuracy of the measurements.

For the noise measurement setup, additional considerations should be added because of the introduction of a noise source at the input of the amplifier.

In a first step, the introduction of two RF signal generators will be used in this setup. These two RF signal generators should be locked to the same frequency reference.
Locking to the same frequency reference is necessary to avoid huge frequency drifting. However, in this case, the drifting is not a big concern, due to the fact that one signal generator is used to produce noise. Good isolation must be achieved to prevent the influence from one source to the other. Hence, some electronic components should be added to the system, such as combiners, and couplers. The working frequency for those components needs to be taken into consideration.

One should also notice that for the calibration wizard in the NVNA software, the calibration frequency band is from the fundamental frequency to the desired harmonics. For each desired frequency, the source power will be calibrated and try to reach the same power level at the reference plan during the power calibration. This might be a problem for certain measurements. If the working frequency band of one component in the system can not cover the calibration frequency band, and the source power could not reach the reference level at the reference plan, then the calibration will fail.

### 3.2 Classical Measurement Setup

The classical measurement setup is for classical measurements, or in other words, without any additional noise source added on purpose. To get an idea about the internal source performance of the NVNA the following setup was used. See Figure 3.1.

![Figure 3.1: Measurement setup for the NVNA internal source test.](image)

Only cables of the NVNA port 1 and NVNA port 3 were in the setup and the DUT was chosen to be a thru connection. In this way, the internal source behaviour can be studied without influence caused by other electronic components.
When it comes to measuring the amplifier, attenuators at the input and the output of the amplifier must be added to the measurement setup, to improve source match and to protect the receiver in the NVNA port 3. The classical measurement setup for amplifier is shown in Figure 3.2.

![Figure 3.2: Classical measurement setup for amplifier measurements.](image)

### 3.3 Noise Measurement Setup

In the noise measurement setup, the additional noise source was added on purpose at the input of the amplifier. The main challenge here is to prevent the cross talk between the two sources and to combine the signal and the noise together as the total input of the DUT.

The first thought was to use circulators to prevent the cross talk from one source to the other and use combiners to combine these two signals, as shown in Figure 3.3. The two circulators used here were both 9A47-31 from RFS Ferrocom, with an operating frequency from 790-960 MHz. The combiner was a ZESC-2-11 from Mini-Circuits with an operating frequency between 10 to 2000 MHz. The signal generator for the noise source was a vector signal generator SMU 200A from Rohde&Schwarz. The fundamental frequency for all measurements and measurement setups was chosen to be 900 MHz.
The first version should work for normal VNAs. However, for the NVNA the circulator number 2 at the NVNA port 1 in the setup causes problems. Three harmonics of the incident and received waves were of interest in the measurements. Due to the frequency limitation of both combiner and circulator, an operating frequency of 900 MHz was chosen. When performing the power calibration at the input of the amplifier, the power calibration was done for all frequencies wanted, which were 900 MHz, 1800 MHz and 2700 MHz. Due to the circulator’s frequency limitation, the excited power from the NVNA port 1 was not able to pass the circulator for frequencies 1800 MHz and 2700 MHz. Thus, when performing the power calibration, the source will be pushed to reach the same reference power at the reference plane when the operating frequency is 1800 MHz and 2700 MHz, which is not possible with this band-limited combiner. The calibration for this measurement setup will fail and the measured results will be wrong.

To avoid the problem caused by the circulator number 2, a second version of noise measurement setup was presented in Figure 3.4. A directional coupler was used instead of the circulator number 2 and combiner.

The coupler used was a 4243-20 from Narda-East, with specified 20 dB coupling factor. Its operating frequency in the datasheet is guaranteed from 1.0-3.5 GHz. However, in practice, the measurement results show that this coupler operates well for a frequency of 900 MHz as desired.

Amplifier number 1 used in the measurement setup was ZHL-42 from Mini-Circuits [32]. Amplifier number 1 was biased with 15 V DC power, and was working in its linear region with constant gain of 30 dB.
A directional coupler is a component capable to differentiate signals traveling in different directions; this device is usually used to observe the power in forward mode by measuring at the coupled end.

The four ports in Figure 3.5 are named as: Input, Coupled, Isolation and Output ports, and the coupling, isolation, and directivity are defined by the Equations (35-36).

\[
C(dB) = 10 \log_{10} \left( \frac{P_{Input}}{P_{coupled}} \right)
\]  

(35)
\[ I(dB) = 10 \log_{10} \left( \frac{P_{\text{input}}}{P_{\text{coupled}}} \right) \] (36)

Normally, the isolation port is loaded with a 50 ohm load. In this case, the isolation port was used as input port of the noise signal, while the input port of the coupler was connected with the NVNA port 1. In this way, the output of the coupler now contains both signal excited from the NVNA and the noise signal.

The insertion loss of the coupler and the noise path loss were measured. The total noise power at the input of the DUT \( P_{IN,\text{Noise}} \) and the total signal power at the input of the DUT \( P_{IN,\text{Signal}} \) can be calculated using the following equations. Assume that the source power from the NVNA is \( P_S \) and the noise source power is \( P_N \). The noise path loss noted as \( L_1 \) is defined as the loss from the isolation port to the output port of the coupler and is 21.20 dB from measurements. The insertion loss noted as \( L_2 \) is defined as the loss from the input port to the output port of the coupler and is 0.45 dB from measurements.

\[
P_{IN,\text{noise}} (dBm) = P_N + \text{Gain(Amp1)} - L_1 - \text{attenuation} \\
= P_N + 30 - 21.20 - 3 = P_N + 5.8 \quad (37)
\]

\[
P_{IN,\text{signal}} (dBm) = P_S - L_2 - \text{attenuation} \\
= P_S - 0.45 - 3 = P_S - 3.45 \quad (38)
\]

The total input power of the DUT \( P_{IN} \) can be calculated in the following equations.

\[
P_{IN,\text{Noise}} (mW) = 10^{0.1 \times P_{IN,\text{Noise}} (dBm)} \quad (39)
\]

\[
P_{IN,\text{Signal}} (mW) = 10^{0.1 \times P_{IN,\text{Signal}} (dBm)} \quad (40)
\]

\[
P_{IN} (mW) = P_{IN,\text{Noise}} (mW) + P_{IN,\text{Signal}} (mW) \quad (41)
\]

The source match error is caused when the reflection signal of the DUT reflects at the signal source and enters the DUT again, and this is improved by the 3 dB attenuator at the input of the DUT.
4 Measurement Results and Discussion

As device under test two amplifiers were chosen. Two different amplifiers were tested, ZHL-2-8 and ZHL-42 both from Mini-Circuits. Since the results for the two amplifiers were similar, the result for only one amplifier (ZHL-2-8) was presented. The single sinewave generated by the PNA-X was the signal source, while the Gaussian noise was generated with the vector signal generator SMU 200A from Rohde & Schwarz. The measured results of the internal source test, the classical measurements and the noise measurements are presented in this section.

4.1 Internal Source Test

The internal source test was done for the PNA-X using the measurement setup in Figure 3.1. For internal source test, the input power (A1 waves) and the output power (B2 waves) of the DUT for the same excited power and frequency were plotted against repeated measurements.

![Image](image.png)

Figure 4.1: Internal source results with the setting source off after measurement.

With the default settings of the NVNA, the source is turned off after each measurement, and the power levels of the source against repeated measurements were shown in Figure 4.1. As shown in Figure 4.1, the decrease of the source power with repeated measurements from measurement number 1 to 43 was obvious. The difference between
measurement number 1 and 43 was about 0.3 dB which was not acceptable. After a 3 minutes pause (after measurement number 43), the source power level took a step up. Note that this means that the source was off during the 3 minutes measurement pause. With a 5 minutes pause between measurement number 61 and 62, the jump in the source power level became bigger than that after the first pause. This behavior of the internal source made the comparison of repeated measurements difficult because it is not possible to know where the current measurement is in the whole loop of decreasing. Furthermore, the wanted source power level was -6 dBm while the actual source power was around -5.6 dBm.

The power level decreasing of the continuous repeated measurements and the jump up after a pause was probably caused by the heating effects of the internal source. When taking the repeated measurements continuously or with short pauses for saving the data, the internal source is heating, and cooling off when turned off during the short pause. The heating of the internal source, could make the source power decreasing and the cooling off during pauses could make the source power increasing. To avoid the changes caused by turning off the source after each measurement, the source was set always on during the whole experiments and the results were shown in Figure 4.2.

![Figure 4.2: Internal source results when source was set always on during the whole experiments.](image)

From Figure 4.2, it is clear that the variations of the source power were much smaller than the ones in Figure 4.1 when the source is turned off after each measurement. The
difference with measurement number 2 and 51 was 0.038 dB. This was significant lower than the noise influence, which was good enough for this project. Also the power level did not have obvious cooling or heating effects as in Figure 4.1. When taking a 2 minutes pause between measurement number 30 and 31, the power level dropped 0.006 dB which is very small and negligible. The first measurement in Figure 4.2, can be considered as an outlier and is probably due to the internal initialization in the instrument. Thus the first measurement will not be used in any other further processing. Several measurements will be performed to get the instrument and the DUT initialized and then the measurements results will be recorded for further processing. However, the actual source power was around -5.88 dBm when the wanted source power was -6 dBm. The additional source calibration was then done to make sure that the power level at the reference plane was accurate.

The additional source calibration was performed when the DUT was connected to the system, and the open loop mode was set off to get accurate source power at the reference plane. The results were shown in Figure 4.3.

From the tested results in Figure 4.3, the power level started at a very accurate power level of -6.001 dBm, when the wanted power level was -6 dBm. During the repeated measurements, the source power went up with certain steps. When taking a short pause of 1 minute between measurements number 40 and 41, the power level dropped 0.005 dB which is considered negligible for this project. However, when the pause between two

Figure 4.3: Internal source results with additional source calibration when the source was set on and the open loop mode was set off.
measurements was 5 hours, as between measurements number 42 and 43, the power level will drop to the initial position. The variations of the source power between the repeated measurements were almost same in Figure 4.2 and 4.3, where a deviation of approximately 0.002 dB was detected between two adjacent measurements. Since the source power was much closer to the wanted power level in Figure 4.3, the settings with source power continuously on and the open loop mode ‘off’ with an additional source calibration were chosen in the following measurements.

4.2 Classical Measurements

The classical measurements were done for amplifier ZHL-2-8 using the measurement setup in Figure 3.2. The datasheet values for the tested amplifier, ZHL-2-8 from Mini-circuits [32] was shown in Appendix. Frequency sweep was set from 50 MHz to 1 GHz with a step of 50 MHz. The source power sweep was set from -6 dBm to 6 dBm with a step of 0.1 dB. Since a source calibration was done, the reference plane was corrected and the estimated input power of the amplifier should be equal to the source power sweep. The measured input power of the amplifier, A1 wave, was plotted versus the theoretical input power for different operating frequencies as shown in Figure 4.4.

Figure 4.4: The difference between measured and wanted input power versus the wanted input power the amplifier for different operating frequencies.

From Figure 4.4, it is seen that the differences between the measured and the wanted input power were about 0.05 dB at most of the operating frequencies, and was larger at
100 MHz. The surface in from 200 MHz to 1000 MHz was almost flat, which indicated the source power was very stable during the changes of amplitude and frequency. The internal source of the NVNA port 1 was very stable for this classical measurement, and would not have much influence on the following measurement results of the DUT.

The time domain and frequency domain signals for the classical measurements with operating frequency 900 MHz were presented in Figure 4.5 and 4.6. Looking at Figure 4.5, the incident wave at the input of the amplifier, $A_1$ wave, was amplified, and results in the reflected wave at the output, $B_2$ wave. The reflected wave at the input, $B_1$ wave and the incident wave at the output , $A_2$ wave were small enough for this project, which indicates that the input mismatch and the output mismatch in the system were acceptable. When applying 6 dBm input power to the tested amplifier, the tested amplifier is operating in the compression region, and the nonlinear effects, the second and the third harmonics were clearly shown as in Figure 4.6.

![Waveforms](image)

Figure 4.5: Time domain signal of the classical measurements with operating frequency 900 MHz. For each waveform a zoom of the wave was shown on the right.
Figure 4.6: Output spectrum of the classical measurements with 6 dBm input power and 900 MHz operating frequency.

In the following measurement results, the basic properties: gain, 1 dB compression point and TOI point of an amplifier were shown for different operating frequencies.

Figure 4.7: Measured amplifier gain versus input power of the amplifier for different operating frequencies. Note that the input power is plotted in reverse in order to make the gain compression more clearly visible.
From Figure 4.7, it is clear that the gain of the amplifier was compressed when the input power was increased for all tested frequencies. The surface also indicates that the compression was different for different operating frequencies, which will result in different 1 dB compression point as shown in Figure 4.8 and 4.9. In Figure 4.7, the measured linear gain of the amplifier was around 29 dB which is 6 dB less than that specified in the datasheet. However, the amplifier is working well and the deviations of the linear gain will not affect the results in this project.

The error bounds with 95% confidence were calculated for the 10 repeated measurements results. The results should follow the Rice distributions. For high SNR, the Rice distributions can be treated as Gaussian distributions. The Rice distributed estimators calculated using methods in [29] gave unfit error bounds, therefore were not used. Due to the difficulties of estimating the Rice distribution parameters correctly, the Gaussian assumption was used. For Gaussian distribution \( X \sim N(\mu, \sigma^2) \), the 95% confidence interval is \([\mu - 2\sigma, \mu + 2\sigma]\).

As shown in Figure 4.8 and 4.9, the 1 dB compression point of the amplifier changes for the different operating frequencies and it is clear that for certain operating frequencies, such as 150MHz and 850MHz, the repeated measured results had larger variations than others. This is a characteristic of the amplifier under test.

![Figure 4.8](image)

Figure 4.8: Measured 1 dB compression point referred to the input power for different operating frequencies. The blue stars represent 10 repeated measurements, and the red lines represent the measured error bound with 95% confidence.
Figure 4.9: Measured 1 dB compression point referred to the output power for different operating frequencies. The blue stars represent 10 repeated measurements, and the red lines represent the measured error bound with 95% confidence.

From Figure 4.10 and 4.11, the 10 repeated measured TOI point refer to the input power and output power are shown. The TOI point of the amplifier is changed with different operating frequency and it is clear from figure that the variations of the repeated measured results were much smaller for certain operating frequencies, such as 150MHz to 350MHz compared with others. Also, it is clear that the TOI point have much larger uncertainty than 1 dB compression point with these 10 repeated measurements. In Figure 4.6, it is clear that the third harmonic has much lower power in comparison to the fundamental whereas the noise is more or less the same. This creates a much lower SNR when the third harmonic is measured. Also in the TOI plot, a small deviation in third harmonic linear regression line will make a big difference in TOI point results.
The normalized standard deviation of the harmonics $\textit{Norm} \_\delta_m$ is defined as the standard deviation of the output waves divided by the measured mean value of the input waves in the 10 repeated measurements as in Equation (42). Three harmonics of the output waves
are measured, the sub index ‘m’ represents the order of the harmonics. $B_m(i)$ represents the output waves, and $A(i)$ represents the amplitude of the input waves.

$$
\delta_m = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (B_m(i) - \mu_B) ((B_m(i) - \mu_B))^2}
$$

(42)

$$
\mu_B = \frac{\sum_{i=1}^{N} B_m(i)}{N}
$$

(43)

$$
P_A = \frac{\sum_{i=1}^{N} A(i)}{N}
$$

(44)

$$
Norm_{\delta_m} = \frac{\delta_m}{P_A}
$$

(45)

Since the source power is not known, the influences caused by the source variations can be reduced by dividing the mean of the source. Thus, the normalized standard deviation in this project is calculated by Equation (45).

The standard deviation plots in Figure 4.12, 4.13 and 4.14 were all normalized by the mean value of the input waves of the amplifier. From these figures, the uncertainties of the measurements for different harmonics and for different operating frequencies were shown. It is obvious that the uncertainties of the measurements were changing from different frequencies, and with different tested power levels. It was much higher in 50 MHz operating frequency than others. Thus a figure with frequency range from 200 MHz to 1000 MHz was plotted next to the classical results.

Figure 4.12: Normalized standard deviation for fundamental harmonic of the received $B_2$ waves with 10 repeated measurements versus input power for different operating frequencies.
Figure 4.13: Normalized standard deviation for 2\textsuperscript{nd} harmonic of the received $B_2$ waves with 10 repeated measurements versus input power for different operating frequencies.

Figure 4.14: Normalized standard deviation for 3\textsuperscript{rd} harmonic of the received $B_2$ waves with 10 repeated measurements versus input power with different operating frequencies.

### 4.3 Noise Measurements

The noise measurements were done for amplifier ZHL-2-8 using the measurement setup in Figure 3.4. Operating frequency was set to 900 MHz. Source power sweep was set from -6 dBm to 6 dBm with a step of 0.1 dB. Since a source calibration was done, the reference plane was corrected and the theoretical input power of the amplifier should be equal to the source power sweep. An additional AWGN noise source was added to the input of the amplifier with a bandwidth of 50 kHz and a total noise power sweep from -40 dBm to 0 dBm with a step of 4 dB.
To get a reference measurement for the noise measurement setup, a 50 ohm load was connected instead of the noise source. The 1 dB compression point and TOI point plots of the reference measurements were shown in Figure 4.15 and 4.16. The input power in the following text, all refers to the total input power of the amplifier which is the sum of signal power and the noise power.

From both Figure 4.15 and 4.16, the compression of the fundamental and third harmonic was very clear, and the slope of the third harmonic was approximately 3 times of the fundamental harmonic, as discussed in the theory section. The measured results using the classical measurement setup in Figure 3.2 are almost the same with the reference measurements results using the noise measurement setup in Figure 3.4.

Also in Figure 4.15, the differences between two reference measurements were shown in the zoom plots. This difference is small and caused by the difference of the measurement setups. Thus to compare the measurement results and to study the noise contribution, the reference measurement using the noise measurement setup is a better choice than the one using classical measurement setup.

Figure 4.15: 1 dB compression point plot of reference measurements. The measured results using the classical measurement setup were in green, while the measured results of reference measurement using the noise measurement setup were in blue. Their linear regression lines are in red, and black respectively. A zoom of the linear region was shown on the left, while a zoom of the nonlinear region was shown on the right to make the difference visible.
The 1 dB compression point plot and TOI point plot with added noise power equal to 0 dBm are shown in Figure 4.17 and 4.18 respectively. The time domain signal and the spectrum with 0 dBm added noise power, and 6 dBm signal power are shown in Figure 4.19 and 4.20 respectively.

When noise power at the input of the amplifier increased to 0 dBm as seen in Figure 4.17, the 1 dB compression point is difficult to measure. The linear regression line in red was calculated using the first 20 measured values, which were meant to be the values in the amplifier’s linear region. In the first 20 measured values, the SNR was the worst case, when the signal power was lowest with fixed noise power. When the noise power at the input of the amplifier was fixed, the signal power was swept from -6 dBm to 6 dBm and then the total input power of the amplifier was swept from about 1 dBm to 7 dBm. Thus the whole measurements are all in the amplifier’s nonlinear region. To find the clear linear region, the total input power of the amplifier should be decreased, which will result in even lower SNR, and noisy measurements. Thus the 1 dB compression point with added noise power equal to 0 dBm was not included in Figure 4.21 and 4.22.

This is also true for the TOI point plot, as seen in Figure 4.18, the TOI point is the interception point of these two linear regression lines, thus, if the linear regression line for the fundamental harmonic was not valid, so did the TOI point. Besides, the third
harmonic has much lower power in comparison to the fundamental whereas the noise is more or less the same as shown in Figure 4.20. This creates a much lower SNR when the third harmonic is estimated.

Figure 4.17: 1 dB compression point plot of noise measurements. The measured results with 0 dBm added noise were in green, while the measured results of reference measurement for the noise measurement setup were in blue. Their linear regression lines are in red, and black respectively.

Figure 4.18: TOI point plot of noise measurements. The measured results with 0 dBm added noise were in green, while the measured results of reference measurement for the noise measurement setup were in blue. Their linear regression lines are in red, and black respectively.
Looking at Figure 4.19, the 10 repeated waveforms of the noise measurements with 0 dBm added noise had larger variations than the classical measurements. It is clear that in Figure 4.20, the third order harmonics for the noise measurements had about 5 dB less power, and more variations than those in the classical measurements. From these plots, one can see the reason why the 1 dB compression and TOI point are hard to measure with 0 dBm added noise from another point of view.

Figure 4.19: $B_2$ waveforms with 6 dBm input power and 900 MHz operating frequency. The 10 repeated measurements for the classical measurements were plotted in blue and the 10 repeated measurements for the noise measurements with 0 dBm added noise were plotted in red. A zoom of the peak was shown on the right for a clear view.

Figure 4.20: Spectrum with 6 dBm input power and 900 MHz operating frequency. The 10 repeated measurements for the classical measurements were plotted in blue and the 10 repeated measurements for the noise measurements with 0 dBm added noise were plotted in red.
From the results in Figure 4.21 and 4.22, one can see that the added noise at the input of the amplifier will influence the 1 dB compression measurement of the amplifier. With higher noise power added in the system, the 1 dB compression point will decrease when referred to the input power. From the plot with different 1 dB compression point referred to the input power versus varying noise power, this trend of decreases with increasing added noise power is clear, except that from -8 dBm to -4 dBm which will be explained later. It is also true if the 1 dB compression point referred to the output is studied. This trend of 1 dB compression point decreasing is caused by the added noise. The contribution of the added noise to the 1 dB compression point of an amplifier was considered to be dual: it changed the linear regression line, and it reduced the useful output power and made the compression start at a lower power level as discussed in section 2.8.

The 95% confident errors bound shown in Figure 4.21 and 4.22 indicates that the results became much noisier when increasing the added noise power. Notice that in Figure 4.21, the error bound for -4 dBm added noise was less than that for -8 dBm. This was a very interesting behavior which will be explained later.

![Figure 4.21: Measured 1 dB compression point referred to the input power with different added noise power. The one with -45 dBm added noise power represents the reference results from the classical measurements. The one with -44 dBm added noise power represents the reference measurement using the noise measurement setup. The blue stars represent 10 repeated measurements, the red stars represent the mean value, and the red lines represent the measured error bound with 95% confidence.](image-url)
Figure 4.22: Measured 1 dB compression point refer to the output power with different added noise power. The one with -45 dBm added noise power represents the reference results from the classical measurements. The one with -44 dBm added noise power represents the reference measurement using the noise measurement setup. The blue stars represent 10 repeated measurements, the red stars represent the mean value, and the red lines represent the measured error bound with 95% confidence.

When studying the 1 dB compression plot with noise, the deviations of the measured output values with the linear regression line are mainly caused by the noise power. Thus the deviations are highly dependent on the noise power. The higher the noise power, the larger the deviations will be. In Figure 4.23, the linear regression lines with different added noise power were plotted together. Assume that the linear regression line with no noise added was the correct one, and it is obvious that the added noise power changed the linear regression line, especially when the noise power was high. Since the 1 dB compression point is the point where for the output power there is 1 dB difference from the linear regression line to the real measured value, the linear regression line is a key role. The linear regression line is calculated based on the first 20 measured values, where the amplifier is working at its linear region. However, these first 20 measured values were measured with lower signal power and fixed noise power, thus lower SNR. So the added noise will cause deviations which will influence the linear regression line, and finally influence the 1 dB compression point.
Besides the influence that caused by the deviations in the linear region of the amplifier, the main influence of added noise was to make the compression start at a lower input power. The higher the added noise power, the lower input power the compression started at. The simulations in section 2.8, shows that for a certain total power, the noisy waves have less useful power as it is with pure sinewave as seen in Figure 2.13. The compression effect was because of the current limitation of the transistor in the amplifier. When the input power level was pushed into the current limitation of the transistor, the output waves were clipped. With noise added to the input power, larger variations will be in the waves, and the total power will be reduced due to hard clipping, which means less useful power.

Furthermore, when increasing the noise power the SNR became very low, and the linear regression line formed by the first 20 measured values was no longer valid and neither the 1 dB compression point as seen in Figure 4.17.

When looking at Figure 4.21, the uncertainties of the 10 repeated measurements results were increasing when the added noise power increased from -40 dBm to -12 dBm. The mean values of the 10 repeated measurements were decreasing when the added noise power increased from -40 dBm to -8 dBm. However, when looking at the input power of these measured 1 dB compression point, the uncertainty with -4 dBm added noise was even smaller than these with -8 dBm or -12 dBm added noise. Besides, the mean values with -8 dBm and -4 dBm added noise were the same. To study this, 30 repeated
measurements were performed for added noise level between -8 dBm to -4 dBm with a step of 1 dB. The results were shown in Figure 4.24 and Figure 4.25.

Figure 4.24: Measured 1 dB compression point referred to the input power for different added noise power. The black stars represent 10 repeated measurements, the red stars represent the mean value, and the red lines represent the measured error bound with 95% confidence.

Figure 4.25: Measured 1 dB compression point refer to the output power with different added noise power. The black stars represent 10 repeated measurements, the red stars represent the mean value, and the red lines represent the measured error bound with 95% confidence.
From the measured results in Figure 4.24 and 4.25, it is clear that when the added noise increases from -8 dBm to -4 dBm, the 1 dB compression point refer to the output power will decrease, but the 1 dB compression point referred to the input power will remain at the same level around 0.5 dBm.

It is important to check the results by studying the 1 dB compression plot and its linear regression line. The measured output power of the amplifier versus total input power with different added noise levels was plotted in Figure 4.26.

From Figure 4.26, it is clear that the measured results were getting noisier with higher added noise power, and for -4 dBm added noise, the linear regression line was still fit. The added noise also changed the slope of the linear regression line as in Figure 4.23 and Figure 4.26. The slope will tilt anticlockwise when the added noise increased. Thus for the same uncertainties of the total input power, the uncertainties of the output power would be very different with different slope, which caused by different noise level as shown in Figure 4.27. This makes the fact that the uncertainty of the total input power is higher with -8 dBm than that with -4 dBm easier to understand. One can see that clearly, the measured results for -4 dBm were actually much noisier than those with -8 dBm. Since the 1 dB compression point referred to the input power is a function of the measured results, the uncertainty of it is depending on both the measurement results and the slope of the linear regression line, as in Figure 4.27.

![Figure 4.26: The measured output power of the amplifier versus corresponding total input power together with their linear regression lines with different added noise power.](image)
As seen from Figure 4.24, 4.25, 4.26, and 4.27, the measured results in Figure 4.21, and 4.22 were correct and reasonable. The 1 dB compression point will decrease when increasing the noise power when referring to the output power. When studying the uncertainties of the repeated measurements of the 1 dB compression point, the slope of the linear regression line should also be considered. The noise contribution was shown in Figure 4.26 as the gaps between those curves. The gaps were narrowing down because of the compression. The measured curves in Figure 4.26 for the three different added noise levels were all tend to achieve a certain clipping level. Due to the power limitation of the amplifier, the maximum input power is set to 6.5 dBm (signal power 6 dBm with -4 dBm added noise).

From Figure 4.28 and 4.29, it is obvious that the TOI point was very unstable with 10 repeated measurements. The deviation for repeated measurements was above 10 dB even for the reference measurements. It is probably because that the third harmonic has much lower power in comparison to the fundamental whereas the noise is more or less the same. The TOI point is very difficult to find with low SNR when measuring the third harmonic as shown in Figure 4.6 and 4.20.
Figure 4.28: Measured TOI point referred to the input power for different added noise power. The one with -44 dBm added noise power represents the reference measurement using the noise measurement setup. The blue stars represent 10 repeated measurements.

Figure 4.29: Measured TOI point referred to the output power for different added noise power. The one with -44 dBm added noise power represents the reference measurement using the noise measurement setup. The blue stars represent 10 repeated measurements.

The normalized standard deviations for fundamental, 2nd and 3rd harmonics were plotted in 3D figures in Figure 4.30, 4.31, and 4.32. From these figures, it is clear that the normalized standard deviation was increasing while the noise input power was increasing. The influence of added noise power at the input of the amplifier to the fundamental, 2nd
and the 3\textsuperscript{rd} harmonic was almost the same from the measurement uncertainty point of view. The ones with -45 dBm noise added input has much larger uncertainty than others. However, since the actual received noise is limited by the narrow band receiver which is only 30 Hz, other reasons such as temperature, humidity could easily dominate the situation and cause the increase of the uncertainty.

Figure 4.30: Normalized standard deviation for fundamental harmonic of the received $B_2$ waves with 10 repeated measurements versus input power with different added noise power.

Figure 4.31: Normalized standard deviation for 2\textsuperscript{nd} harmonic of the received $B_2$ waves with 10 repeated measurements versus input power with different added noise power.
4.4 Methods Discussion

The 1 dB compression point and the third order interception point were measured in both the classical measurements and the noise measurements in this project. The nonlinearity of a system can be considered in two ways: a generation of new spectral components and an amplitude-dependent signal gain. These measurements, the TOI point and the 1 dB compression point represent these two nonlinear effects and from the measurement results, the noise contribution to each measurement can be studied.

To study the noise contribution in the nonlinear measurements, comparisons between classical measurement results and noise measurement results where input noise was added to the system on purpose were done. However, from the measurement results, this comparing method was not as good as expected. The measurement uncertainty was dependent on the measurement setup and thus it is much better to compare the results from same measurement setup with noise source turned off and with varying noise power added. Even though, the classical measurement results were very useful and provided the information about the DUT and made it easier for the noise measurements to build up a suitable setup.

Given the fact that a lot of data sets need to be analyzed and compared with the measurement uncertainties, the standard deviation representation of uncertainty was used for most cases in this thesis. However, the method of expressing the uncertainty using
confidence region was very helpful in this thesis. Not only that the trend of the ellipse changes can be studied, but using this geometric representation, all measured values can be plotted and error measurement can be noticed and eliminated right away.
5 Conclusions and Future Work

5.1 Conclusions

In this thesis, the noise contribution of a nonlinear system when measuring with the PNA-X was studied. The classical measurements and the noise measurements with input noise added to a nonlinear device, were performed and the noise contribution in those measurements were studied by analyzing the 1 dB compression point, TOI point and also the uncertainties of the received waves in different harmonics.

To distinguish the influence caused by the internal source of the PNA-X and the added noise at the input, an internal source test was performed. The best settings to maximize the internal source accuracy were chosen and the tested results were shown in Figure 4.3. The input power of the DUT at reference plane will increase with each repeated measurement with step that is small enough to ignore. However, it can still affect the results.

The contribution of the added noise to the 1 dB compression point of an amplifier is considered dual: it changes the linear regression line as in Figure 4.23, and it reduces the output power as in Figure 2.13. As a result of those two effects, the added noise make the compression start at a lower power level as in Figure 4.21 and 4.22. When the power of the added noise is increased to a certain level, the 1 dB compression point was not very hard to measure as in Figure 4.17. This is due to the noisy linear regression line caused by the extremely low SNR at the linear region.

The contribution of the added noise to the TOI point of an amplifier was not clear. From 10 repeated measurements, the variations of TOI point were big for each noise power level as in Figure 4.28 and 4.29. The TOI point referred to the input had a trend to decrease when increasing the added noise power but the results were too noisy to draw any conclusions.

From the normalized standard deviation plots of the received waves at each harmonics, it was clear that the uncertainty was higher for higher noise power. This showed that the measurement system was working fine, and the variations of internal source power were small enough to ignore.

Noise behavior in a nonlinear system is very different from that in a linear system as discussed in this thesis. Thus when performing nonlinear measurements, the noise effects
should be taken into consideration and further studies are required to get better understanding of the system’s behavior in noisy environment.

### 5.2 Future Work

To further study the noise contribution to the nonlinear system, the noise figure of the nonlinear device can be studied as a function of noise power. The noise figure is defined in Equation (46)

\[ F = \frac{S_i/N_i}{S_o/N_o} \]

(46)

where \( S_i \) is the input signal power; \( N_i \) is the input noise power; \( S_o \) is the output signal power; \( N_o \) is the output noise power.

The accuracy of the noise figure is important. In research, better noise figure accuracy gives better correlation between simulation and measurements, and furthermore better simulation gives possibility for better modeling. Thus, the measurements and analysis on the noise figure as a function of noise power could be very useful on studying the noise contribution. The Y-factor or hot/cold-source technique can be used to measure noise figure [33].

Also, other methods can be used when analyzing the measured results when the added noise power is high, such as 0 dBm as shown in Figure 4.17. A polynomial could be estimated to fit the AM/AM curves of the noisy measurement. In this way, the noise in the linear region of the AM/AM curves could be reduced, and also the linearity of could be defined with certain tolerance of the mismatch.

The classical measurements and the noise measurements in this project could be done for multisine input signal instead of single sinewave excited by the internal source of the PNA-X. When multisine signal applied to the input of the amplifier, things can be very different from a single sinewave. Is there a 1 dB compression point? How to define the nonlinear gain of the amplifier? How to study the contribution of the added noise in Nonlinear Vector Network Analyzer (NVNA) measurements then? These studies will be very useful to learn the noise behavior in nonlinear systems.
Appendix

The datasheet values for the tested amplifier, ZHL-2-8 from Mini-circuits [32] is shown in Table A below.

Table A: The datasheet values for amplifier ZHL-2-8 from Mini-circuits [32].

<table>
<thead>
<tr>
<th>Frequency [MHz]</th>
<th>Gain [dB]</th>
<th>$P_{1dB_OUT}$ [dBm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>35.32</td>
<td>30.21</td>
</tr>
<tr>
<td>100</td>
<td>35.68</td>
<td>30.45</td>
</tr>
<tr>
<td>150</td>
<td>35.70</td>
<td>30.48</td>
</tr>
<tr>
<td>200</td>
<td>35.67</td>
<td>30.45</td>
</tr>
<tr>
<td>250</td>
<td>35.63</td>
<td>30.47</td>
</tr>
<tr>
<td>300</td>
<td>35.52</td>
<td>30.46</td>
</tr>
<tr>
<td>350</td>
<td>35.42</td>
<td>30.55</td>
</tr>
<tr>
<td>400</td>
<td>35.34</td>
<td>30.58</td>
</tr>
<tr>
<td>500</td>
<td>35.20</td>
<td>30.88</td>
</tr>
<tr>
<td>600</td>
<td>35.11</td>
<td>31.07</td>
</tr>
<tr>
<td>650</td>
<td>35.12</td>
<td>31.18</td>
</tr>
<tr>
<td>700</td>
<td>35.15</td>
<td>31.29</td>
</tr>
<tr>
<td>800</td>
<td>35.28</td>
<td>31.35</td>
</tr>
<tr>
<td>900</td>
<td>35.28</td>
<td>31.33</td>
</tr>
<tr>
<td>1000</td>
<td>35.02</td>
<td>31.41</td>
</tr>
</tbody>
</table>
References


