



**UNIVERSITY
OF GÄVLE**

FACULTY OF ENGINEERING AND SUSTAINABLE DEVELOPMENT

A Measurement System for Static Non-linearities

Hanbing Xu & Kailin Hou

September 2012

Bachelor's Thesis in Electronics

Bachelor of Science in Electronics

Examiner: José Chilo

Supervisor: Efrain Zenteno

Acknowledgement

The authors appreciate Efrain Zenteno's work very much for his enthusiastic and professional supervision, and providing effective MATLAB® code for the thesis. Also, sincerely thanks to Niclas Björzell for giving us the general description to the project topic with warm hearted suggestions. Special thanks to master student Hongxu Zhu and Tianyang Feng for giving us useful advices on this paper.

Abstract

Measuring static nonlinearities systems has been a hot topic in recent years. In this thesis, amplifier is a device as a static nonlinearities system. In most cases, amplifiers operate as a linear device under small signal conditions. However it becomes more nonlinear and distorting with increasing drive levels. It is applied in many fields, such as communication, mathematical, biology.

This thesis is aiming to investigate the differences between linear and nonlinear modeling of circuits by modeling of a device and implementing a measurement system for electrical devices in MATLAB[®] as the tool. The approach to fulfill our aim is to find the coefficients of the model and getting the error between models and measured. In this thesis, the error is used to determine the goodness of fit in a specific model.

Through working while following our project design, a comparison is made in the end between the modeling capabilities for two model structures: linear and polynomial, for two operation modes of an amplifier with lower and higher input power.

Table of contents

Acknowledgement.....	i
Abstract	ii
Table of contents	iii
Table of figures	v
1 Introduction	1
1.1 Background	1
As a matter of fact, all the physical systems are non-linear, because there is always a non-linear phenomenal existing such as dead zone, saturation and gap. If we keep constant pressure of low-power and high-bandwidth applications, we will push an increasing number of devices beyond the edge of their linear range into a nonlinear operation region[1]. A static nonlinear system is a system in where there is no change, the output only depends on the present value of input, and the output is not proportional to the input. Nonlinear behavior becomes essential and study of nonlinear systems has become increasingly significant. In natural science and engineering technology, a large amount of phenomena cannot be used in linear model, for instance, Pendulum strongly, Relay diode characteristics, Self-oscillating circuit Mechanism and so on[2].A nonlinear system can describe a physical device or process more clearly than linear system. Under this circumstance, more and more engineers are focusing on measuring and modeling linear devices.....	1
1.2 Goals	2
1.3 Thesis Outline	2
2 Theory	4
2.1 Linearity and Nonlinearity	4
2.1.1 Linearity	4
2.1.2 Nonlinearities	5
2.2 Model structures.....	5
2.2.1 Linear FIR model:	6
2.2.2 Polynomial model:.....	6
2.2.3 Polynomial with memory:	7
2.3 Amplifier.....	7

2.3.1	Gain	8
2.3.2	Noise.....	8
2.3.3	Negative Feedback Amplifier (DUT - Device Under Test)	8
2.4	FFT.....	9
3	Process and results.....	10
3.1	Setup.....	10
3.2	DUT design	11
3.3	Communication with instruments	11
3.4	Excitation signal.....	12
3.5	Measurements	13
3.6	Modeling Comparisons	17
3.6.1	Nonlinear model	17
3.6.2	Linear model.....	19
3.6.3	Linear model and nonlinear model combined	21
3.7	Error	22
3.8	Model evaluation.....	25
3.9	AM curves.....	26
4	Discussion and conclusions.....	28
	References	29
	Appendix A	1

Table of figures

Figure 1.1 the construction of the instruments.....	2
Figure 2.1 linear device.....	4
Figure 2.2 nonlinear device.....	5
Figure 2.3 the basic principle of the model.....	5
Figure 2.4 negative feedback amplifier.....	8
Figure 3.1 instruments connections.....	10
Figure 3.2 the circuit diagram of the DUT negative feedback amplifier.....	11
Figure 3.3 the signal in frequency domain.....	11
Table 3.1 measured signal in frequency domain at different frequency.....	12
Table 3.2 the command of transferring signal to function generator process.....	13
Figure 3.4 measured input and output at low amplitude.....	14
Figure 3.5 measured input and output at high amplitude.....	14
Figure 3.6 measured input and output reduced noise at low amplitude.....	15
Figure 3.7 measured input and output reduced noise at high amplitude.....	15
Figure 3.8 measured input and output fixed delay at low amplitude.....	16
Figure 3.9 measured input and output fixed delay at high amplitude.....	17
Figure 3.10 nonlinear model compare with measured output at low amplitude.....	18
Figure 3.11 nonlinear model compare with measured output at high amplitude.....	19
Figure 3.12 linear model compare with measured output at low amplitude.....	20
Figure 3.13 linear model compare with measured output at high amplitude.....	20
Figure 3.14 combined model compare with measured output at low amplitude.....	21
Figure 3.15 combined model compare with measured output at high amplitude.....	22
Figure 3.16 compare the error of nonlinear model and linear model at low amplitude.....	23
Figure 3.17 compare the error of nonlinear model and linear model at high amplitude.....	23
Figure 3.18 the error of combined model at low amplitude.....	24
Figure 3.19 the error of combined model at high amplitude.....	24
Figure 3.20 measured input and output of new signal with modeling at low amplitude.....	25
Figure 3.21 measured input and output of new signal with modeling at high amplitude.....	26
Figure 3.22 measured ratio of V_{out} and V_{in} at high amplitude.....	26
Figure 3.23 measured the ratio of V_{out} and V_{in} at low amplitude.....	27

1 Introduction

Since the first decade of the 21st century, engineers have reached a high level of the study on the linear system theory. Linear system is a mathematical model of a system based on the use of a linear operator. And we can see the linear framework dominates the engineering applications in a long time period. However, with development of device, linear region is not able to satisfy the needs of people in some certain fields any more, for instance, communication, mathematical, measurement and so on. People are increasingly realized that the linear system is only limited within small-scope of work. To get breakthrough in higher field is in great need. Therefore, it motivates us to choose work on the non-linearity.

1.1 Background

As a matter of fact, all the physical systems are non-linear, because there is always a non-linear phenomenon existing such as dead zone, saturation and gap. If we keep constant pressure of low-power and high-bandwidth applications, we will push an increasing number of devices beyond the edge of their linear range into a nonlinear operation region[1]. A static nonlinear system is a system in which there is no change, the output only depends on the present value of input, and the output is not proportional to the input. Nonlinear behavior becomes essential and study of nonlinear systems has become increasingly significant. In natural science and engineering technology, a large amount of phenomena cannot be used in linear model, for instance, Pendulum strongly, Relay diode characteristics, Self-oscillating circuit Mechanism and so on[2]. A nonlinear system can describe a physical device or process more clearly than linear system. Under this circumstance, more and more engineers are focusing on measuring and modeling linear devices.

In measuring nonlinearities, the driving force is the need for good nonlinear models for some components and devices. In our study, we chose Power Amplifiers. There are different approaches to design a measurement instrument for nonlinear measurements. The aim of our project is to find the coefficients for a polynomial model of a device[3].

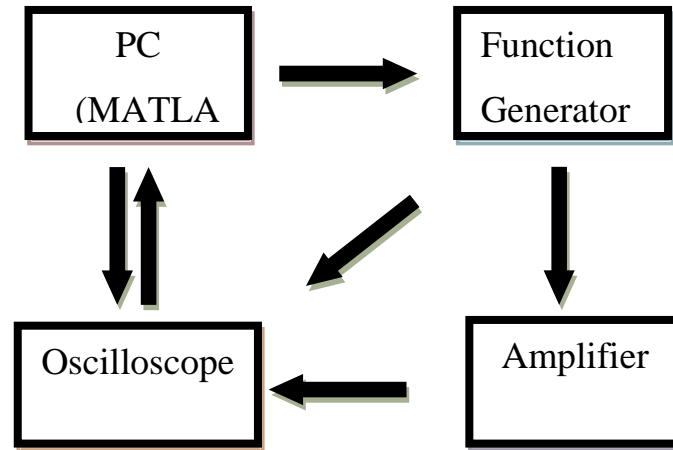


Figure 1.1 the construction of the instruments

As it is shown in Figure 1.1, the project is to build a model by using MATLAB[®], function generator, amplifier and oscilloscope. A signal which is built by MATLAB[®] is sent to a function generator. Later, such signal is used to excite an amplifier. An oscilloscope is used to measure both input and output signals from the amplifier. The oscilloscope is controlled by a PC through GPIB. Finally, MATLAB[®] serves as a tool for analyzing the measure data from the oscilloscope.

1.2 Goals

The goal of this thesis is to find out the differences between linear and nonlinear modeling of circuits through modeling of a device and implementing a measurement system for electrical devices as the tool. To achieve our goal, the approach we chose is to find the coefficients of the model and getting the error between models and measured. The main difficulty of the measurement is to structure a nonlinear model by using MATLAB[®].

1.3 Thesis Outline

The project thesis will be organized as follows:

After the introduction sector, a theory chapter (Chapter 2) will be presented. The definition and theory behind the linearity and nonlinearity concepts would be introduced as well as an overview of the model structures. Besides, theoretical knowledge of the amplifier and the use of FFT will be included. Chapter 3 will be the experiment process and results statement. In this chapter, a detailed information of our project design will be explained and our main findings. Apart from it, modeling comparison is made between linear and nonlinear models. At last, an evaluation of the model is provided.

In the last chapter, main conclusion from our study is made related to our objectives. We discussed the possible differences between the two models together with suggestions for further research.

2 Theory

This chapter would introduce the theoretical knowledge that our study based on, including concepts of linearity and nonlinearity, selecting and contracting fitful models for linear FIR model and polynomial model, basic theory of amplifier are discussed and FFT(Fast Fourier Transform).

2.1 Linearity and Nonlinearity

2.1.1 Linearity

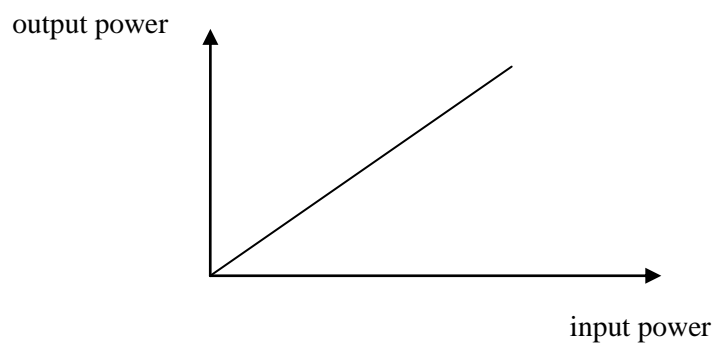


Figure2.1 linear device

A signal is input to linear device. The basic formulation of linear model as below:

$$y = ax \quad (2.1)$$

Where x is input signal, y is output signal, a is arbitrary constant.

As shown in Figure 2.1, the output is proportional to the input. Such device is called linear device. In this thesis, negative feedback inverter amplifier was used as a device. Thus, the output was inverted and amplified by the amplifier. In process part, the device was in linear region with low power, 100mV, while the output was proportional to the input signal.

2.1.2 Nonlinearities

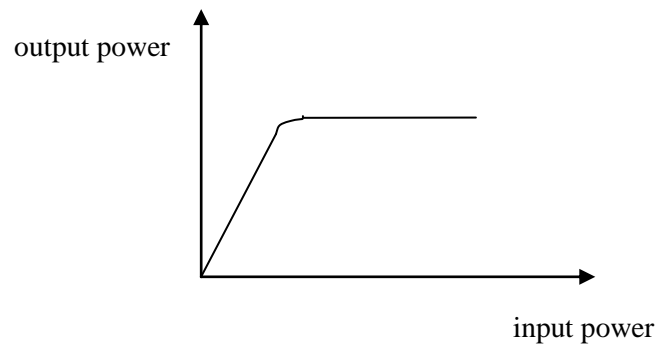


Figure 2.2 nonlinear device

A signal is input to a device. However, the output is not similar to the input like Figure 2.1.

$$y = ax + bx^2 + cx^3 \quad (2.2)$$

where x is input, y is output, a, b, c are arbitrary constants.

As shown in Figure 2.2, it is a curve. It means that the output is not proportional to the input. With the increase of the input power, the device is pushed into compression region. The compression means that with the increase of the input power, the output power reaches maximum point and then the output is compressed. After that, the output power is stable. In process part, the device was in nonlinear region with the increase of the input power. In this case, the input power is 500mV, while the output was pushed into compression region.

The equation(2.2) means that the output is not proportional to the input signal. In this thesis, the amplifier with high power was pushed away from linear region to nonlinear region.

2.2 Model structures

The amplifier was modeled. In this thesis, three models were built: linear FIR model, polynomial model and polynomial model with memory.

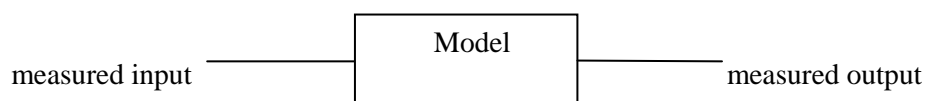


Figure 2.3 the basic principle of the model

As shown in Figure 2.3, the coefficients can be found by the data of measured input. Thus, the model can be built and then compare the model with measured output.

2.2.1 Linear FIR model:

FIR model was used in this thesis as a linear model. It fits the linear device, but it is not suitable for nonlinear device.

The linear FIR model was built as below:

$$y = a_0x(n) + a_1x(n-1) + a_2x(n-2) \quad (2.3)$$

The equation (2.3) is a 2nd memory FIR model. Thus, the nth memory linear FIR model can be written as below:

$$y = \sum_{i=0}^n a_i x(n-i) \quad (2.4)$$

In this thesis, 4th memory FIR linear model was built. It can be clearly seen that the linear model only fits the linear part.

2.2.2 Polynomial model:

Polynomial model as a nonlinear model was used in this thesis. This model can fit both linear and nonlinear device. However, with low input power, FIR model is more suitable than polynomial model in linear region[4]. With high input power, the device was pushed into nonlinear region.

The polynomial model was built in this thesis as below:

$$y = a_0x(n) + a_1x^2(n) + a_2x^3(n) \quad (2.5)$$

The equation (2.5) is a 3rd order polynomial model. Hence, the nth order polynomial model can be written as below:

$$y = \sum_{i=0}^{\infty} a_i x^{i+1}(n) \quad (2.6)$$

In this thesis, 4th order polynomial model was built. In process part, it can be seen that the model is suitable for linear and nonlinear systems.

2.2.3 Polynomial with memory:

Polynomial with memory model is combining the linear FIR model with the nonlinear polynomial model. It can suit linear and nonlinear devices. Through the results of error, this model is better than the other two for most of nonlinear devices. In this thesis, 4th order with 4th memory model was built.

The polynomial with memory model was built as follow:

$$y = a_0 x(n) + a_1 x(n-1) + \dots + a_{n-1} x(1) + b_0 x^2(n) + b_1 x^2(n-1) + \dots + b_{n-1} x^2(1) + \dots + c_0 x^3(n) + c_1 x^3(n-1) + \dots + c_{n-1} x^3(1) + \dots \quad (2.7)$$

In this equation, the memory is $n, n-1, \dots, 1$, which are stored input data in the model.

The coefficients of this model can be found by function. See appendix A.

2.3 Amplifier

An amplifier is a device for increasing the power of a signal by use of an external energy source. Mainly it is used to detect the weak signal of the very low signal-to-noise ratio. The amplifier is referred to any device which is able to use less energy to control a larger energy. In this thesis, a negative feedback amplifier was used to amplify the input signal so that it can be clearly seen the relation between input and output.

Now in daily use, the term often refers to the amplifier circuit. An amplifier with an input-output relationship: often expressed as a function associated with the frequency of input. This relationship is

called the amplifier's transfer function, while the transfer function of the coefficient is defined as the “gain”[5].

2.3.1 Gain

The gain of an amplifier is the ratio of output and input power or amplitude, and it is usually measured in decibels. The gain can be calculated by equation (2.8) and (2.9).

$$G(\text{dB}) = 10 \log(P_{\text{out}}/P_{\text{in}}) \quad (2.8)$$

$$G(\text{dB}) = 20 \log(V_{\text{out}}/V_{\text{in}}) \quad (2.9)$$

In this thesis, the gain is 3.6. It can't be too small. Thus, it can be clearly seen that the output signal is amplified by the device.

2.3.2 Noise

The noise exists in the amplifier and measurement. It makes the data not exactly. So reducing the noise is very necessary. In electrical circuits, there are 5 common noise sources, namely, shot noise, thermal noise, flicker noise, burst noise and avalanche noise. As noise sources have amplitudes vary randomly with time, they can only be specified by a probability density function.

2.3.3 Negative Feedback Amplifier (DUT - Device Under Test)

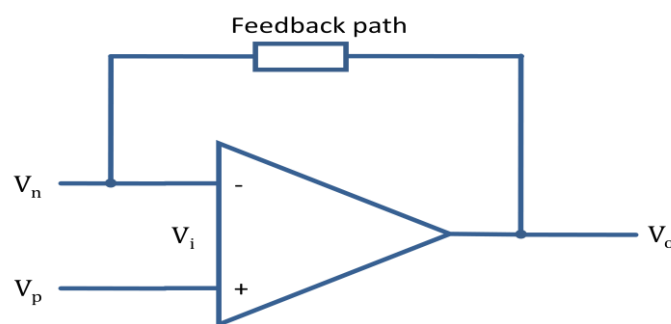


Figure 2.4 negative feedback amplifier

Feedback amplifier is an amplifier which sends the output signal to the input so that amplify the input power. If the feedback signal makes the input increase so that increases the gain, it is positive feedback. Otherwise, it's negative feedback amplifier[6].

2.4 FFT

FFT is the abbreviation of Fast Fourier Transform. It is an efficient algorithm to compute the discrete Fourier transform and its inverse[7].

The functions $Y = \text{fft}(x)$ and $y = \text{ifft}(X)$ implement the transform and inverse transform pair given for vectors of length N by:

$$X(k) = \sum_{j=1}^N x(j)\omega_N^{(j-1)(k-1)}$$

$$x(j) = (1/N) \sum_{k=1}^N X(k)\omega_N^{-j(k-1)}$$

Where $\omega_N = e^{(-2\pi i)/N}$ is an N th root of unity.

In this thesis, it is used to find the sampling frequency and the frequency response of a 3rd order polynomial.

3 Process and results

3.1 Setup

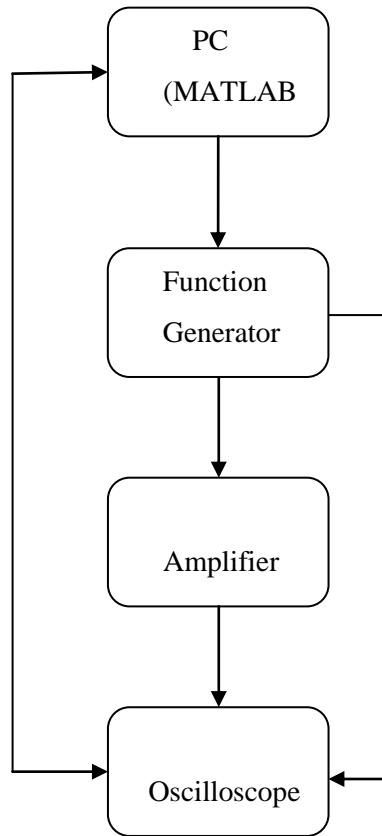


Figure 3.1 instruments connections

Function Generator: HP 33120A 15MHz Function / Arbitrary Waveform Generator

Amplifier: Negative feedback inverter amplifier

Oscilloscope: HP 54600A 100 Hz Oscilloscopes

Power Supply: Power box 3000B

In this thesis, the function generator input connected channel 1 of the oscilloscope and the output of the circuit connected channel 2 of the oscilloscope. And connect SYNC to external trigger through a cable so that to make the signal more stable.

Run MATLAB[®] to communicate the function generator and the oscilloscope using TM tool. A 150 KHz sine signal was transferred to the function generator and the amplitude was 100mV. At 150 KHz frequency and 100mV amplitude, the signal is better to be measured and display on the oscilloscope.

3.2 DUT design

In this thesis, negative feedback inverter amplifier was used. And the frequency was 10 KHz. The higher amplitude was 500mV and the lower amplitude was 100mV. The selected Gain was 3.6. The designed circuit is shown in Figure 3.2.

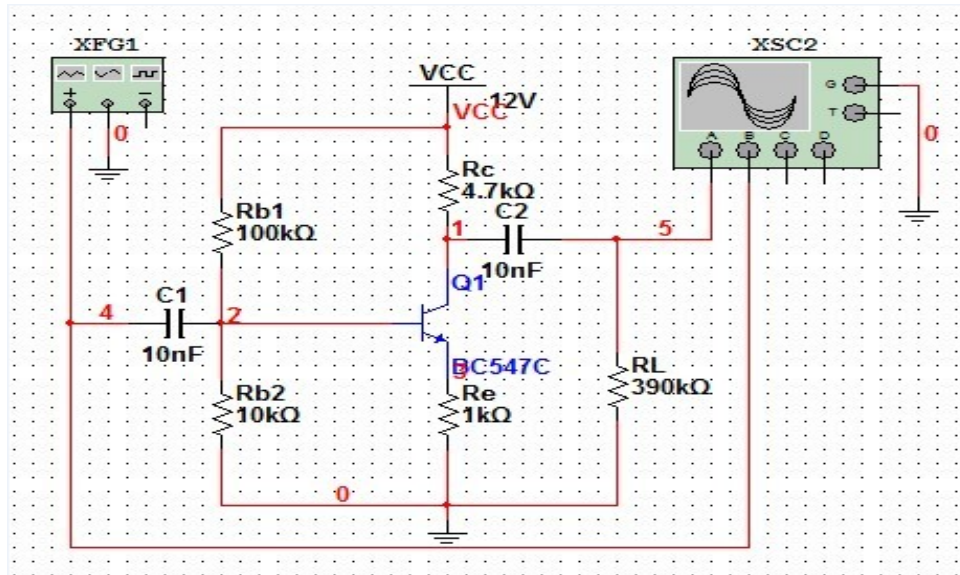


Figure 3.2 the circuit diagram of the DUT negative feedback amplifier

At 10 KHz frequency, the device is pushed into nonlinear region while the amplitude is 500mV; the device stay in linear region while the amplitude is 100mV. And the picture seems good, so these data were used in thesis.

3.3 Communication with instruments

Firstly, the signal in frequency domain was plotted. See Figure 3.3.

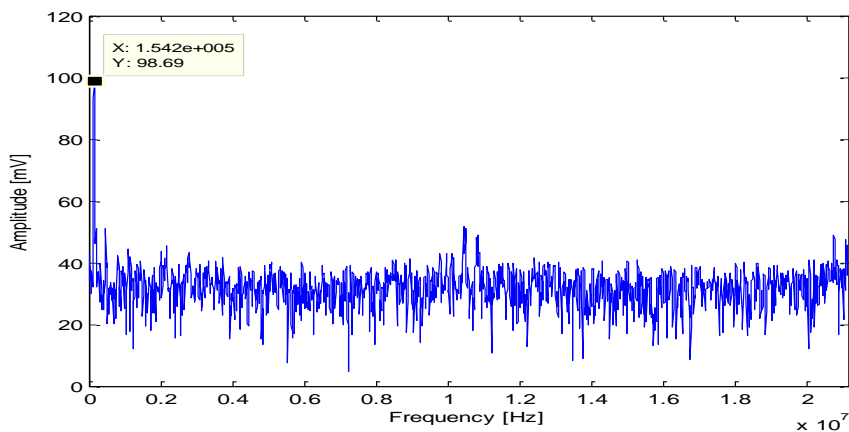


Figure 3.3 the signal in frequency domain

Then two signals were displayed on the oscilloscope. Channel 1 was input signal and channel 2 was output which was amplified and inverted of input signal. Then, the oscilloscope transferred the data to MATLAB[®]. Plot these two signals in time domain and FFT in frequency domain. The sampling frequency was computed and tried many times so that the frequency in FFT was almost same as the input signal frequency. In this case, 42.3e6Hz sampling frequency was used. The frequency in Figure 3.2 was almost same as the input signal frequency, 150 KHz

Input signal frequency (KHz)	Measured in frequency domain (KHz)
225	Error
224	224.0
220	225.2
210	216.0
200	203.5
150	154.2
130	129.5
100	105.0
80	80.2
50	46.2
30	30.8
26	21.5
25	Error

Table 3.1 measured signal in frequency domain at different frequency

As it is stated in Table 3.1, useful band was found. The highest frequency was 224 KHz; the lowest frequency was 26 KHz for the sampling frequency. Three frequencies were used to make a 3rd order polynomial signal, 100 KHz, 150 KHz and 200 KHz. The data included: the sampling number $N = 2950$, the time range in time domain $t = [0: N-1]/f_s$, the time base range was 0.1e-3s.

3.4 Excitation signal

In this thesis, a new signal (3.1) was transferred to the function generator.

$$y = \sin(2*\pi*100e3*t)+\sin(2*\pi*150e3*t)+\sin(2*\pi*200e3*t) \quad (3.1)$$

It is multiple sine waves. The process of transferring the signal to the function generator is as below:

Command	Meaning
fprintf	Transfer the data to instruments
'DATA VOLATILE,y'	Download the data, y
'FUNction: USER VOLATILE'	Turn on output and transfer the data to function generator
'FUNction: SHAPe USER'	Set 'USER' function shape which stored before

Table 3.2 the command of transferring signal to function generator process

10 KHz frequency and 100mV amplitude were used in this project. Then the function generator received the data and transmitted the signal to the oscilloscope, after that, the input signal, channel 1 was displayed on the oscilloscope. At the same time, the function generator input the signal to the negative feedback amplifier, and the output signal, channel 2 was displayed on the oscilloscope. The amplitude of input and output can be measured by the oscilloscope. In this case, the input amplitude was 203.1mV and the output amplitude was 718.7mV. The gain equals to the output amplitude divided by the input amplitude. After that, the oscilloscope transferred the data of these two signals to PC (MATLAB[®]) and measured the two signals using GetOSCDat1 m-file.

3.5 Measurements

By using the m-file GetOSCDat1, y1 and y2 can be measured at the same time, y1 was input signal and y2 was output signal. Then the figure of these two signals can be plotted. At lower amplitude, in this case 100mV, the output of the signal was in linear region. At higher amplitude, 500mV, the output of the signal was pushed into nonlinear region. In high amplitude case, the input amplitude was 1,000V, the output amplitude was 2,688V. 10 KHz was used.

The figure was not exactly. The amplitude in the figure was not same as the amplitude of the measurement. Condition the measured include: set DC to zero and fix scales.

$$y1 = y1 - \text{mean}(y1) \quad (3.2)$$

$$y2 = y2 - \text{mean}(y2) \quad (3.3)$$

These two equations (3.2) and (3.3) were able to set DC to zero, then fix the input and output signal scales by using the amplitude of the measurement divided by the amplitude of the measurement by MATLAB®, then times the measuring signal as below.

$$y1 = 203/(\max(y1) - \min(y1))*y1 \quad (3.4)$$

$$y2 = 719/(\max(y2) - \min(y2))*y2 \quad (3.5)$$

They were used at low amplitude. See Figure 3.4. In the same way, the input and output signal at high amplitude can be plotted. See Figure 3.5.

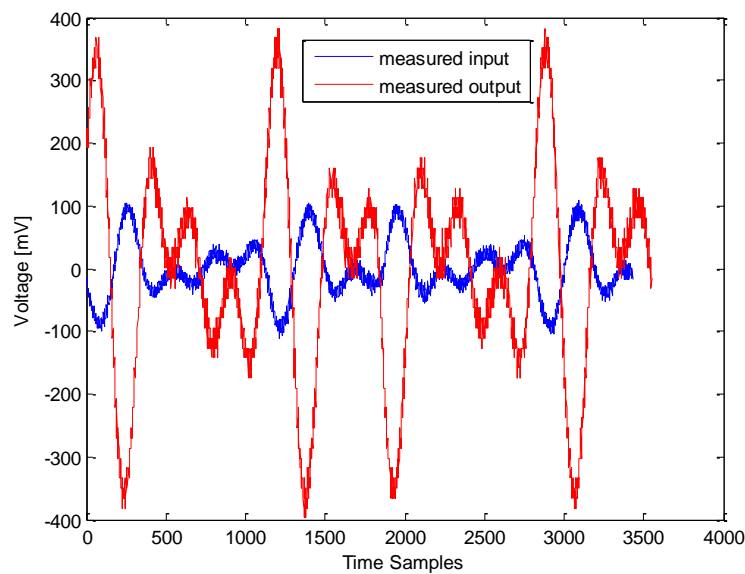


Figure 3.4 measured input and output at low amplitude

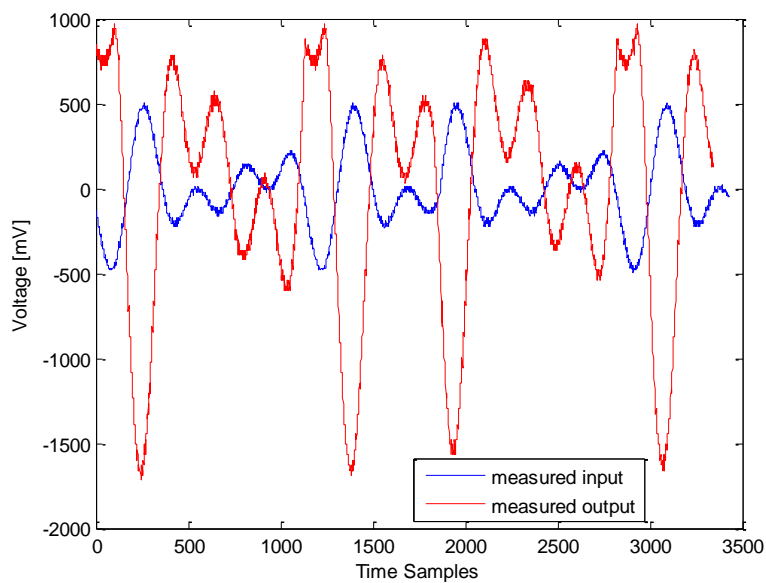


Figure 3.5 measured input and output at high amplitude

As shown in Figure 3.4 and 3.5, the two signals have a lot of noise. Thus, limit the bandwidth to reduce the noise of the input and output signal was used in this thesis. Then the two signals with lower noise were plotted.

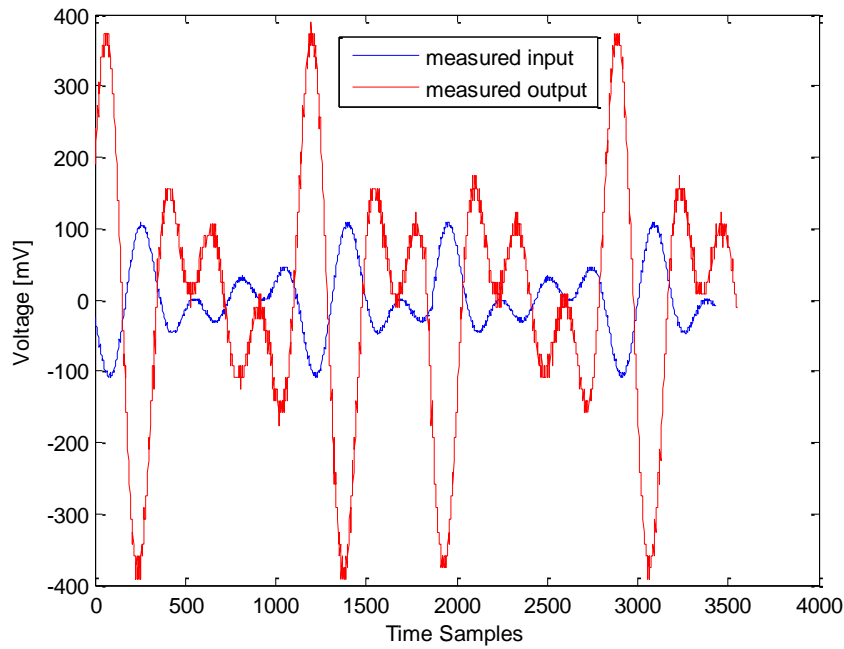


Figure 3.6 measured input and output reduced noise at low amplitude

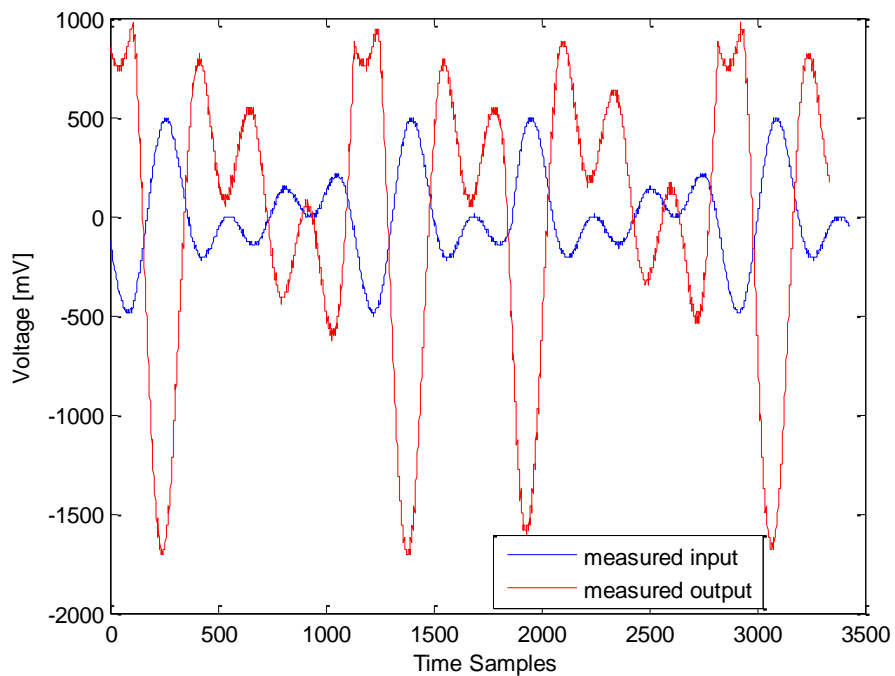


Figure 3.7 measured input and output reduced noise at high amplitude

As shown in Figure 3.6, Figure 3.7, the noise is smaller than before in this method.

Can be clearly seen in Figure 3.6 and Figure 3.7, at peak and trough, the input signal and the output signal have delay between this two signals. Fix delay was necessary. Firstly, make sure device is linear. Secondly, measure the input and output signal. Then remove DC and make scale to zero. After that, plot FFT and compute the angle. Assume X was fast Fourier transform of the input signal. Then plot X . Therefore, the angle can be got through a point in the line. In the same way, the angle of the output signal can be computed either. These two angles should be measured in one period. Then, use the output signal angle minus the input signal angle to compute the value. In this thesis, the value was 25. Command 'circshift (signal, value)' can help to solve the problem. It means that the signal can be shifted left or right by value. In this thesis, shift the output signal right was needed. Hence, use the command circshift to fix the delay. Then plot the input and output signal at low and high amplitude.

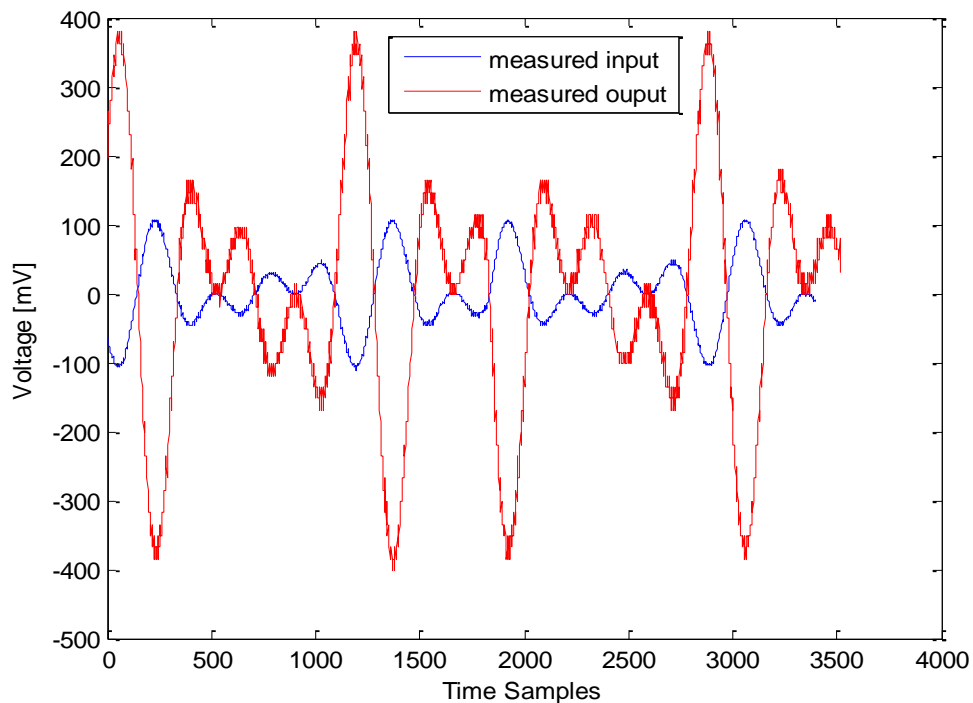


Figure 3.8 measured input and output fixed delay at low amplitude

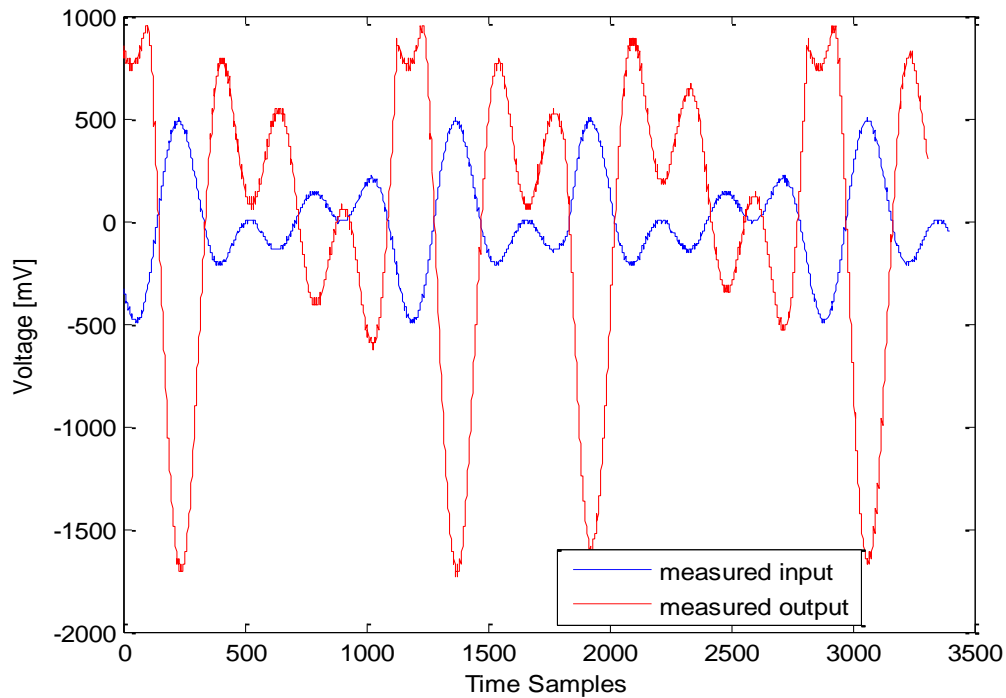


Figure 3.9 measured input and output fixed delay at high amplitude

As shown in Figure 3.8 and Figure 3.9, the problem was solved. The input and output signal were almost no delay there. And then it's better to modeling.

3.6 Modeling Comparisons

In this part, three models were built: nonlinear model, linear model and polynomial model with memory.

3.6.1 Nonlinear model

The input signal and output signal were measured, and then the model of the output should be built. In this thesis, nonlinear 3rd order polynomial model were used at first, $y = ax + bx^2 + cx^3$. Where x was the measured input, y was the measured output. Actually, noise signal was exist but the noise was reduced. It's too small so that it wasn't considered in this thesis. The method of build a model was to find the coefficients, a , b , c . The input signal x and the output signal y were matrix from $x(0)$ to $x(N)$ and $y(0)$ to $y(N)$. Hence, this equation can be written as (3.6).

$$\begin{bmatrix} y(0) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} x(0) & x^2(0) & x^3(0) \\ \vdots & \vdots & \vdots \\ x(N) & x^2(N) & x^3(N) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} n(0) \\ \vdots \\ n(N) \end{bmatrix} \quad (3.6)$$

At lower amplitude, assume A was the matrix of the input signal; p was the matrix of the coefficients. Therefore, the model equation can be written as $y = A * p$. The matrix p was what should be computed. Then $p = (A^T A)^{-1} A^T y$. Then it can be programmed like $p = A \backslash y$. Hence, matrix p can be computed, thus the coefficients a, b, c can be found. Then the modeling at lower amplitude y_m can be plotted on MATLAB[®] see figure 3.10.

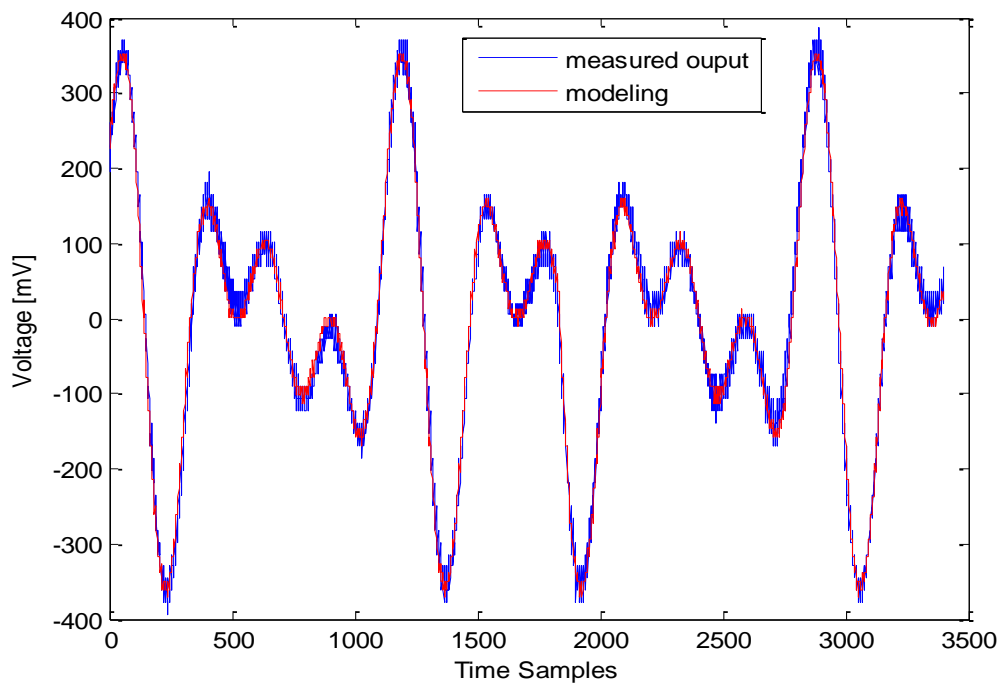


Figure 3.10 nonlinear model compare with measured output at low amplitude

The output signal and the modeling signal were almost superposition. At higher amplitude, the coefficients can also be calculated by following the step like before. The modeling at higher amplitude can be plotted. See figure 3.11.

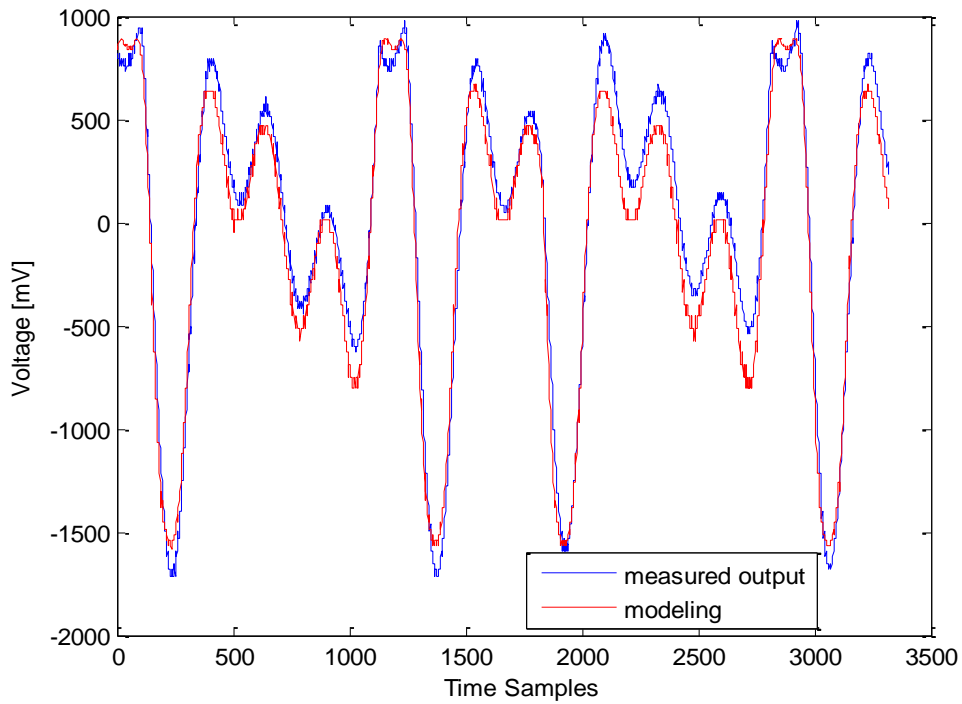


Figure 3.11 nonlinear model compare with measured output at high amplitude

The modeling was not good enough, because of the deviation. Then, the polynomial order was added up to 5, because of rank deficient. The coefficient can be calculated in the same way. Thus, each order of modeling can be plotted.

3.6.2 Linear model

Linear model was used in this thesis so that it can be compared with nonlinear model. The linear FIR model was used, $y[n] = ax[n] + bx[n-1] + cx[n-2]$. As same as before, 3rd memory FIR model was used at first. The model can be written as shown in equation (3.7).

$$\begin{bmatrix} y(0) \\ \vdots \\ y(N) \end{bmatrix} = \begin{bmatrix} x(0) & x(-1) & x(-2) \\ \vdots & \vdots & \vdots \\ x(N) & x(N-1) & x(N-2) \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} n(0) \\ \vdots \\ n(N) \end{bmatrix} \quad (3.7)$$

Assume A was the matrix of the input signal x, p was the matrix of the coefficients. This matrix of the input signal was different from the matrix of nonlinear model. However, the A can be calculated by using a method used before. Command circshift also can solve this problem. As shown in Figure, the first column shifts the last through down by 1 becomes to the second column and the second column shifts the last through down by 1 becomes to the third column. Therefore, it can be solved in easy way. Besides, the terms which were shifted should be zero. As before, the coefficients can be calculated by

$\mathbf{p} = \mathbf{A}\mathbf{y}$. Hence, the linear model y_m at lower and higher amplitude can be plotted on MATLAB[®]. See Figure 3.12 and Figure 3.13:

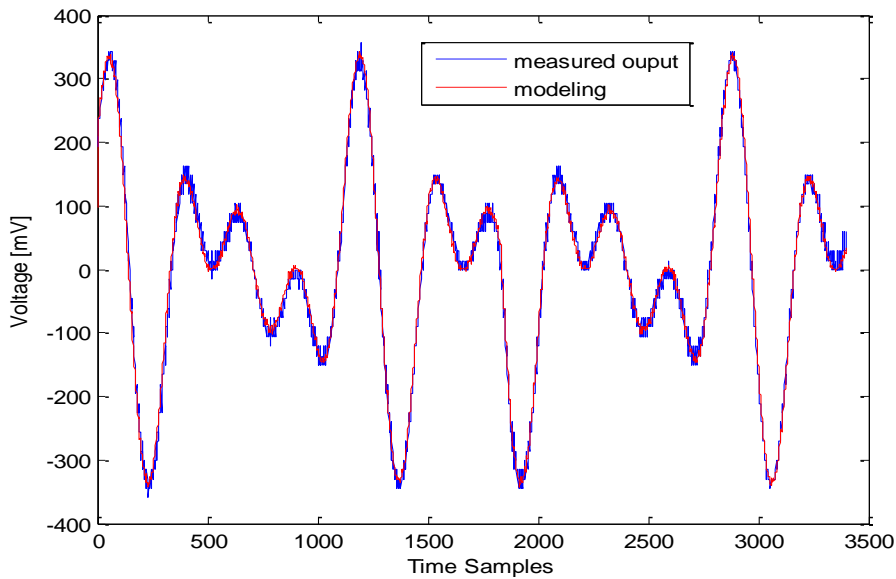


Figure 3.12 linear model compare with measured output at low amplitude

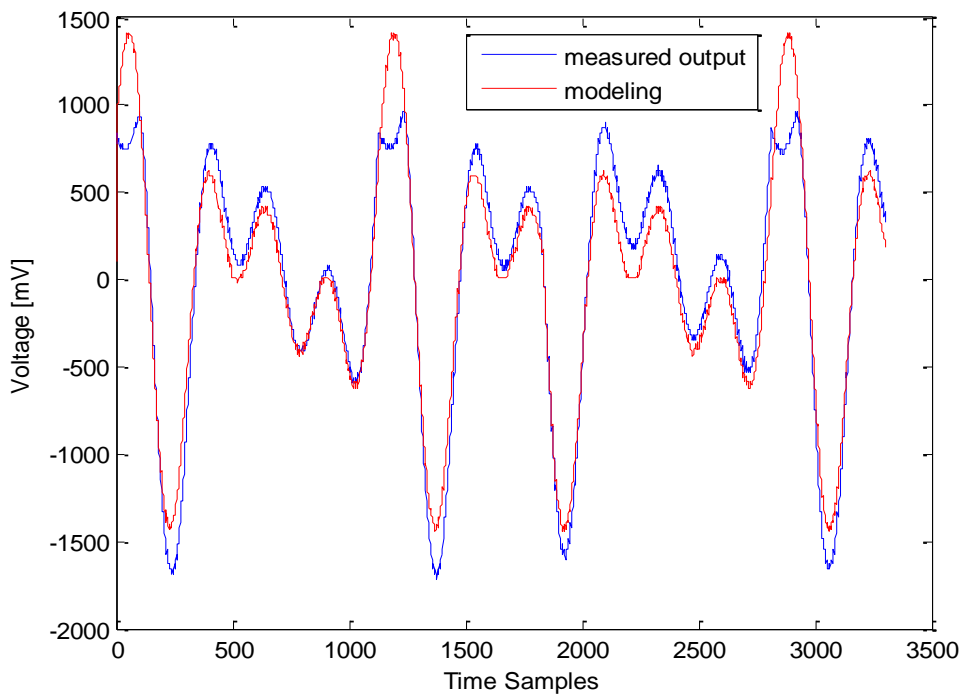


Figure 3.13 linear model compare with measured output at high amplitude

As shown in Figure 3.12 and Figure 3.13, the linear model wasn't suitable at higher amplitude. The measured output is in nonlinear region at the peak. However, the modeling is still in linear region.

3.6.3 Linear model and nonlinear model combined

The linear FIR and the nonlinear polynomial models were used before. What should do next step was to build a combined model: the linear FIR model combined with the nonlinear polynomial model. Using this model, it can be shown clearly that which model was better and compare linear and nonlinear model at lower and higher amplitude respectively. The model was different from before, because of the coefficients. Order and memory were included in this model. In this thesis, 4th order (1~4) and 4th memory (0~3) were used. Because of rank deficient in MATLAB[®] limits the value. Therefore, 4*4 models can be built.

How the matrix A of each model can be achieved in MATLAB[®]? Function was needed here. Two for loops can solve the problem in easy way. Then as before, each p can be calculated by $p = A \setminus y$. Hence, these models can be written and plotted. In this thesis, the modeling of 4th order and 3rd memory was plotted at lower and higher amplitude as shown in Figure 3.14 and Figure 3.15.

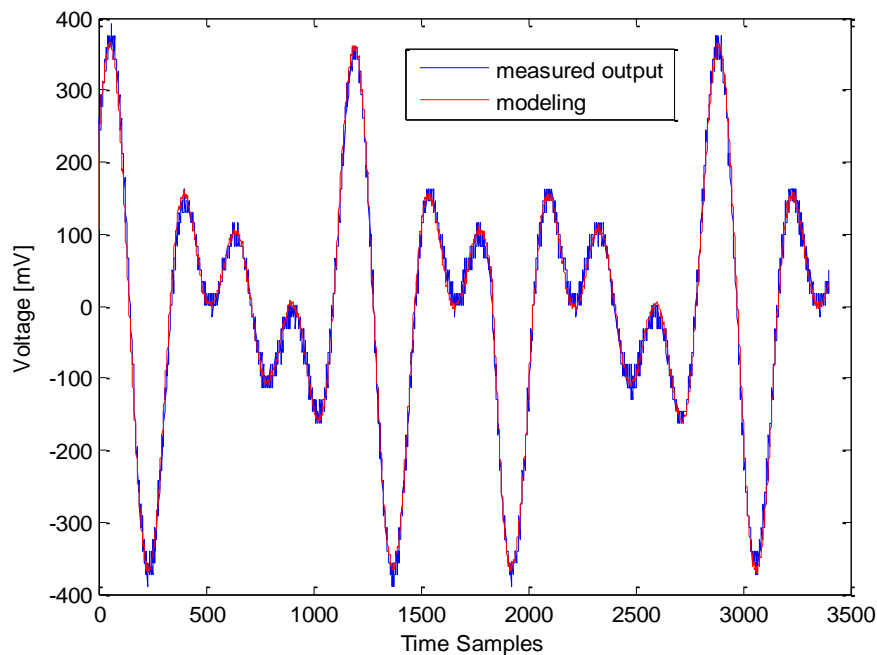


Figure 3.14 combined model compare with measured output at low amplitude

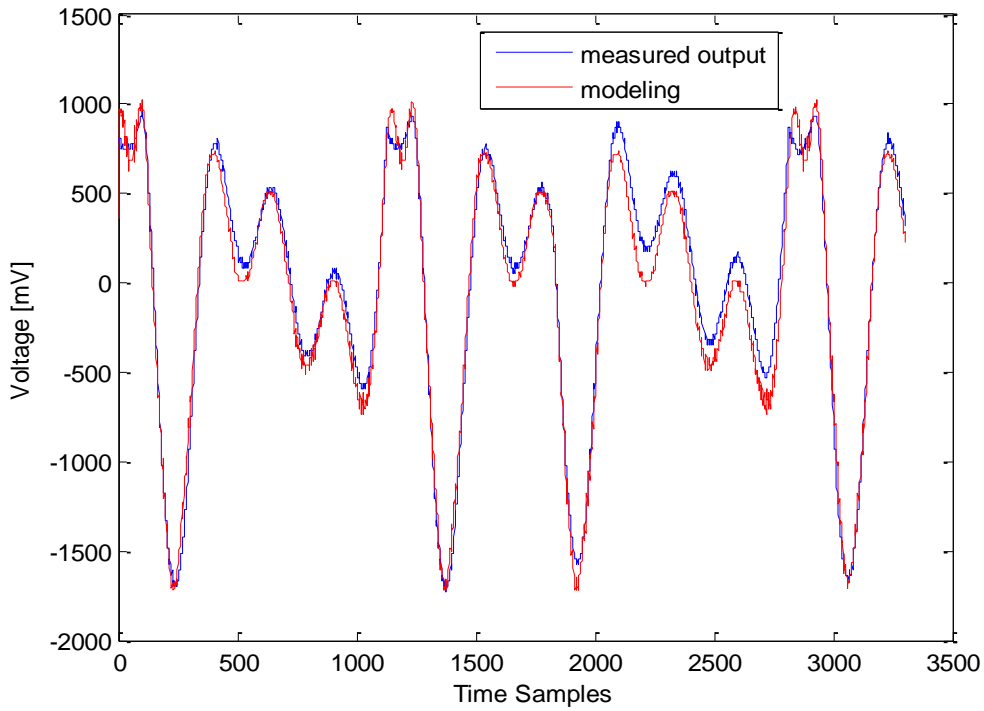


Figure 3.15 combined model compare with measured output at high amplitude

Although it still has large deviation in compression region, it's better than linear and nonlinear model that is hard to modeling in compression region. This model was more suitable than linear and nonlinear model at high amplitude.

3.7 Error

How to determine which models more suitable for the amplifier in this thesis? It can be clearly seen in terms of the error between modeling and the measuring output. The lower error it has, the better model is. In this case, the error can be computed by equation (3.7).

$$e = \sum (y_m - y_k)^2 \quad (3.7)$$

Where y_m was modeling, y_k was measured output.

Thus, the error of the nonlinear polynomial model and the linear FIR model can be plotted at lower amplitude and higher amplitude respectively see Figure 3.16 and Figure 3.17.

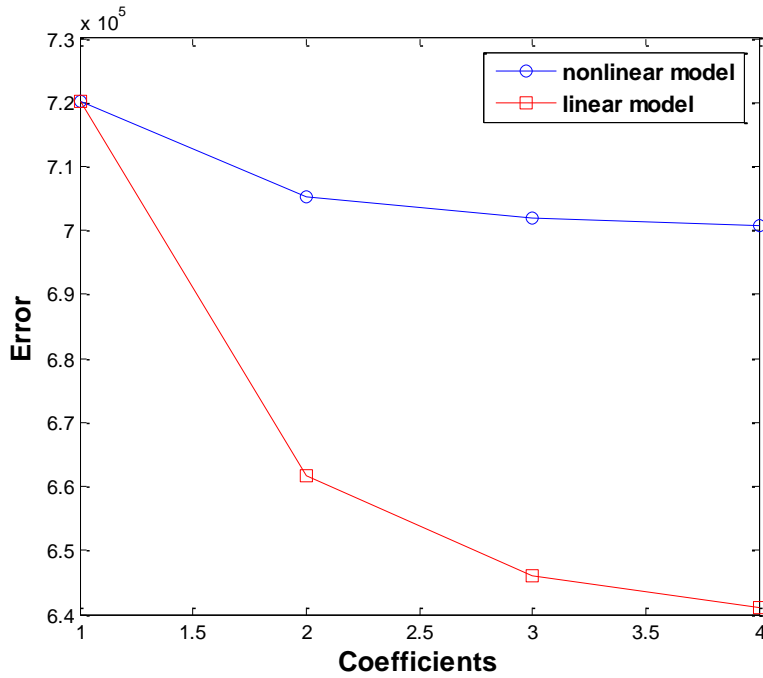


Figure 3.16 compare the error of nonlinear model and linear model at low amplitude

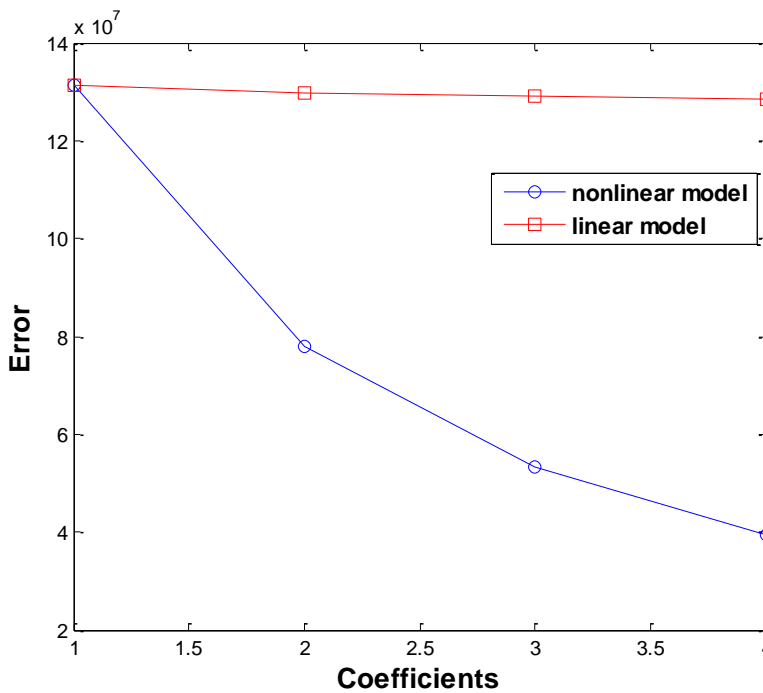


Figure 3.17 compare the error of nonlinear model and linear model at high amplitude

As shown in Figure 3.15 and Figure 3.16, the linear model has lower error than the nonlinear model at lower amplitude and the nonlinear model has lower error than the linear model at higher amplitude. With the increase of the order, the error decreased faster than it decreased with the increase of the memory. Thus, in higher amplitude case nonlinear model is more suitable, in lower amplitude case, linear model is more suitable. Besides, if the order or memory is bigger, the modeling is better.

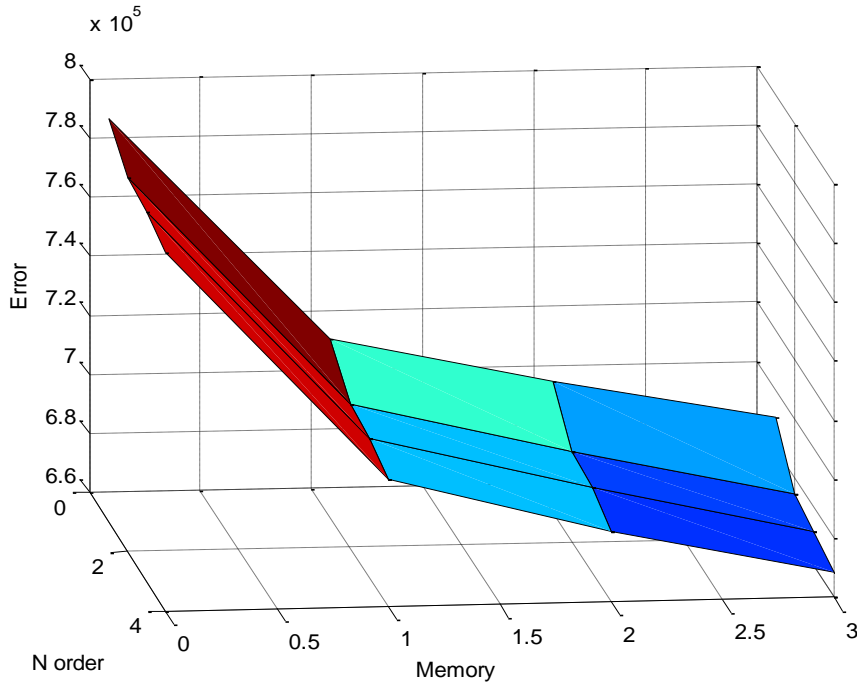


Figure 3.18 the error of combined model at low amplitude

As we can see from Figure 3.18, about the error, order and memory of the combined model at lower amplitude. It shows the error at each order and memory. With the increase of memory, the error reduced quickly. With the increase of order, the error reduced slowly. The relationship between error and coefficients (order and memory) can be shown clearly. In higher amplitude case, order and memory were negative. See Figure 3.19.

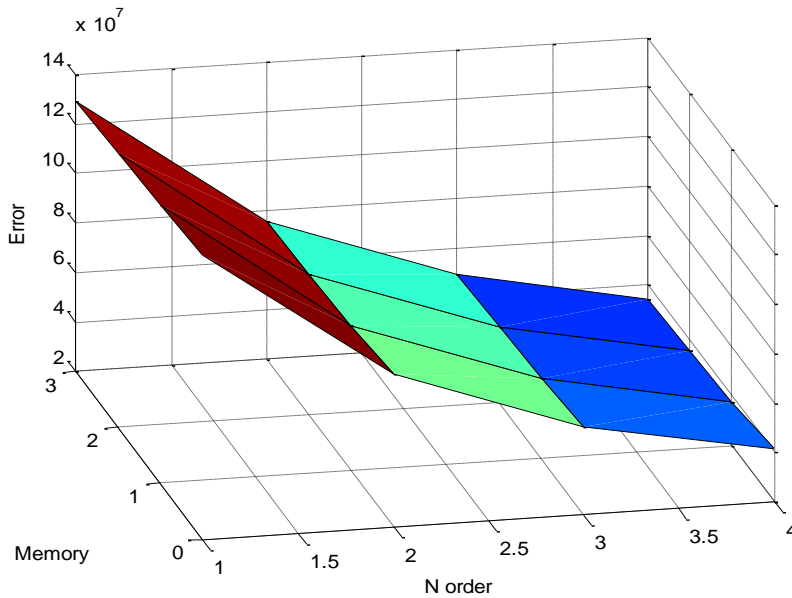


Figure 3.19 the error of combined model at high amplitude

3.8 Model evaluation

At the last part, a new signal was transferred to the function generator to check if the modeling was suitable for the new signal. 10KHz frequency was used and the lower amplitude is 100mV, the higher amplitude is 500mV.

$$y = \sin(2\pi \cdot 100e3 \cdot x) + \cos(2\pi \cdot 150e3 \cdot x) + \sin(2\pi \cdot 200e3 \cdot x + \pi/4) \quad (3.8)$$

In this thesis, a new signal (3.8) was transferred and it was plotted. The combined model was used. Only 4th order and 3rd memory was chosen to compare with the measuring output of the new signal. The coefficients were computed before. Thus, the modeling y_m and the measuring output can be plotted. See Figure 3.20 and Figure 3.21:

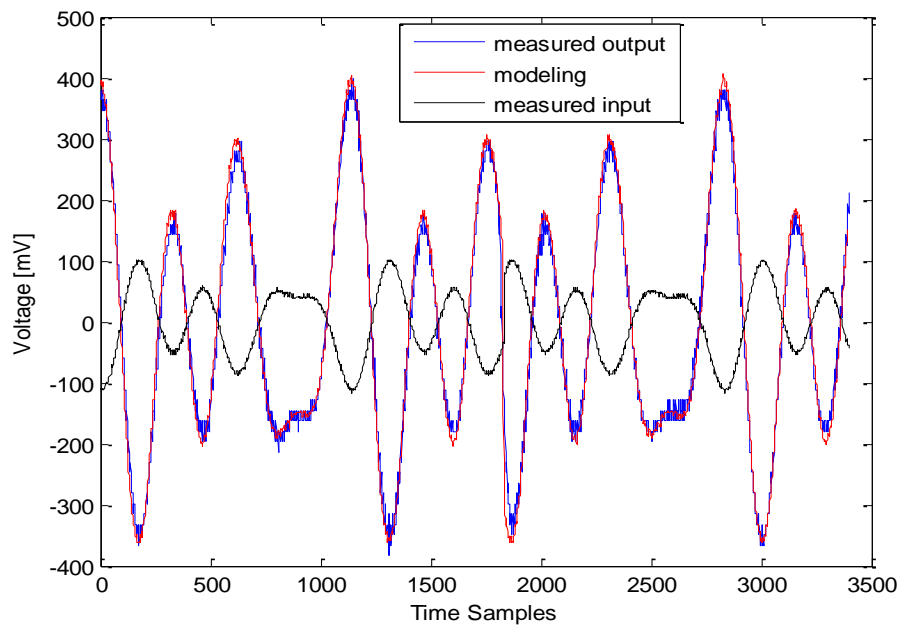


Figure 3.20 measured input and output of new signal with modeling at low amplitude[^]

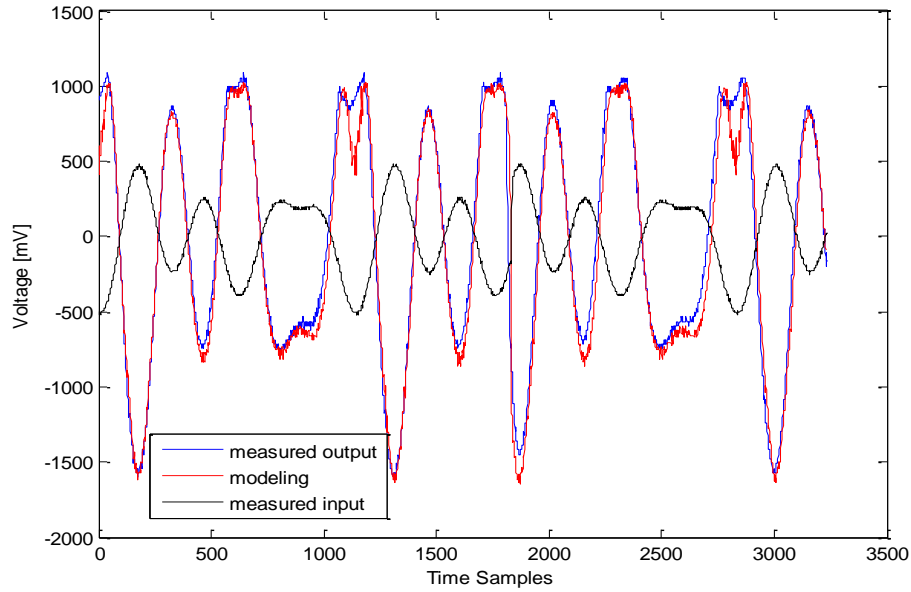


Figure 3.21 measured input and output of new signal with modeling at high amplitude

It can be clearly seen that the modeling was suitable for the new signal. Hence, the modeling was suitable for most of 3rd polynomial system.

3.9 AM curves

AM and AM curves are amplitude. It can be clearly seen the gain and the compression region. The figure about V_{out} and V_{in} was plotted. The input signal was (3.8). V_{in} was the measured input voltage and V_{out} was the measured output of the signal. See Figure 3.22.

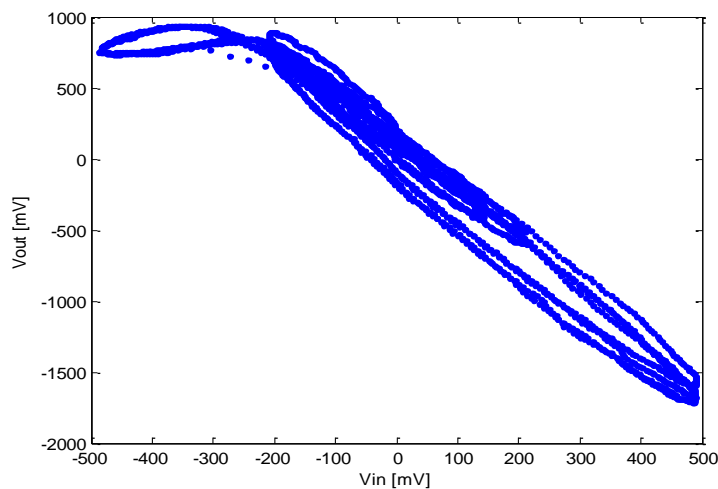


Figure 3.22 measured V_{out} and V_{in} at high amplitude

As shown in Figure 3.22, the phenomenon of clipping appears from -400mV to -500mV. It exists in nonlinear region. In this part, the device was compressed. From -2000mV to 500mV, in this part the device was in linear region with low power. High input power pushed the amplifier's operating point from linear region into compression region.

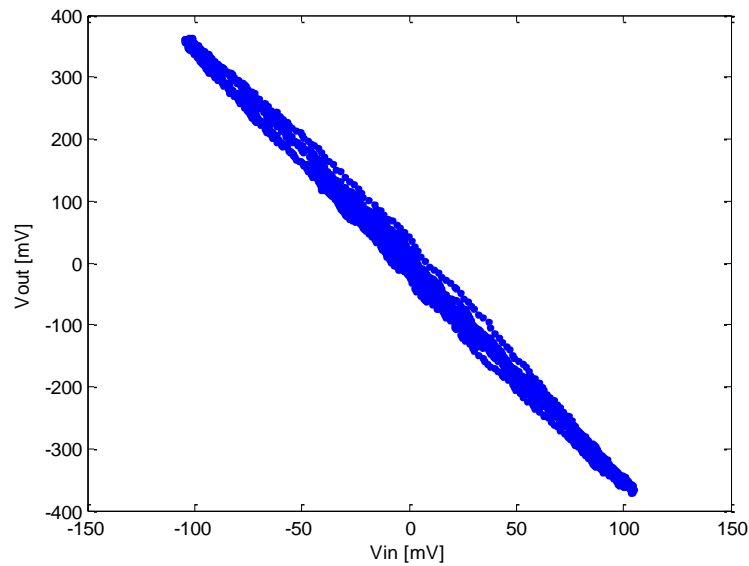


Figure 3.23 measured V_{out} and V_{in} at low amplitude

A negative feedback amplifier was used in this thesis. Thus, $V_{out} = \text{Gain} * V_{in}$. It should be a straight line and the slope is the gain with low amplitude as shown in Figure 3.23. At low input power, the amplifier was in linear region.

4 Discussion and conclusions

According to our main findings, the linear model is only suitable for linear system and the other two models are suitable for linear and nonlinear systems. If increasing the order and memory, more suitable model will be built. In all these figures, it is shown that with the increasing of the memory and order, the error is getting decreased.

The conclusion we draw may not be perfect as there are still errors in it. The problems come from the instruments and measure part probably. The instruments are not very advanced so that the measured data may lack of accuracy due to it. Moreover, MATLAB[®] cannot deal with high values. Temperature is another factor of the problem. It affects the elements of the circuit, so measured data have some errors because of the temperature as well. From the measured results, quantization error is still need to be improved in the future. Some different kinds of amplifier can be used to replace this negative feedback amplifier to check the performance of the models.

From this project, the way of modeling and measure the static nonlinear system is achieved well. In this project, three models were used to measure the nonlinearities of simplified amplifier. Nonlinear model and linear combined with nonlinear model are suited in nonlinear system. In this way, error can be determined. From the results, the error decreases with the increase of the order and memory. Thus, if increasing the order and memory, the small error will be achieved. The error is much smaller than that of linear model. From the results of the error, the combined model is better than the other two models.

To sum up, linear model is not appropriate for nonlinear system. However, linear model is better than the nonlinear and combined models in linear system, because the result of the error is smaller.

References

- [1] W. Van Moer, and Y. Rolain, "A Large-Signal Network Analyzer; Why is it needed," *IEEE Microwave Magazine*, pp.46-62, December. 2006.
- [2] S.A. Maas, "Nonlinear Microwave Circuits," *IEEE New York: Wiley*, pp.3-5, 1988.
- [3] N. Björnsell, "A description to our thesis," *IEEE Högskolan I Gävle*, pp.1, June. 2012.
- [4] Unknown editor, "Modeling with Polynomial Functions," *IEEE McDougal Littell/Houghton Mifflin Company*, pp.137-139, 2007.
- [5] S. Rosenstark, "Feedback Amplifier Principles," *IEEE New York: Macmillan*, pp.57-59, 1986.
- [6] J. Vuolevi, and T. Rahkonen, "Some Circuit Theory and Terminology," in *Distortion in RF Power Amplifiers*, Norwood: Artech House, 2003.
- [7] Robert W. Ramirez, "The FFT fundamentals and concepts," *IEEE Englewood Cliffs, NJ: Prentice-Hall*, pp.178, 1985.

Appendix A

```
function matrix = XHB(y1,P,D);
y1 = y1(:);
A = y1;
for K = 2:P
    A = [A y1.^K];
end
C = A;
for m = 1:D
    temp = circshift(A,m);
    temp(1:m,:) = 0;
    C = [C temp];
end
matrix = C;
```