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Dynamic Calibration of Undersampled Pipelined ADCs by Frequency Domain Filtering

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Abstract

Integral nonlinearity (INL) is used for the post-correction of pipeline analog-digital converters (ADC)s. An input-frequency-dependent INL model is developed for the compensation. The model consists of a static term that is dependent on the ADC output code and a dynamic term that has an additional dependence on the input signal frequency. The INL model is subtracted from the digital output for post-correction. The static compensation is implemented with a look-up-table (LUT). The dynamic calibration is performed by a bank of frequency domain filters using an overlap-add structure. Two ADCs of the same type (Analog Devices AD9430) are compensated in the first three Nyquist bands. The performance improvements in terms of spurious-free dynamic range (SFDR) and intermodulation distortion (IMD) are investigated. By the proposed method up to 17 dB improvements are reported in favorable scenarios.

Index Terms

Analog-digital conversion, integral nonlinearity, dynamic modeling, calibration, frequency domain filtering.

I. INTRODUCTION

The analog-digital converter (ADC) is a key element in contemporary digital telecommunication systems. However, ADCs are susceptible to errors, which limit the advantages that the digital technology offers over its analog counterpart. ADC error characterization is vital for differentiating the distortions that are induced by the ADC from those due to other telecommunication devices and the wireless channel. Reliable error characterization forms the basis for ADC calibration, and has been investigated since the advent of digital systems [1]–[3].

Early ADCs typically operated in a narrow frequency band [4] in which the behavior of the ADC barely changed, making static calibration feasible. Static calibration depends on the ADC digital output code k , which spans the interval $[0, 2^N - 1]$ for an N -bit ADC and provides remapping from the digital output $x[k]$ to a new value $s[k]$. Wideband systems that operate at high carrier frequencies are a current trend in wireless communications. Contemporary ADCs operate within a large frequency band, where the ADC behavior can change substantially. Thus, a static calibration will not coherently improve over the band, and, it actually can occasionally deteriorate the performance [5]. Moreover, modern ADCs are used as down converters that operate in the higher Nyquist bands, reaching their analog bandwidth. Thus, dynamic calibration is needed, where one or several characteristics of the ADC stimuli can be used as an additional input (to the code k) to the correction scheme.

ADC post-correction by digital signal processing (DSP) does not require in-chip intervention, which relaxes the performance quality constraints on the ADC design. The previous sample/s of the ADC digital output, the slope and the frequency of the

Fig. 1. ADC measurements in each Nyquist band, where t and n is the continuous and discrete time indices, respectively. f_N is the Nyquist frequency, which is half of the sampling frequency f_s .

ADC input are potential dynamic variables in the calibration process [6]. Earlier work with dynamic post-correction used a look-up-table (LUT) approach [1]–[3], [7], [8]. In this approach, a correction term $\epsilon[m, k]$, where m represents the dynamic characteristics of the ADC, is added to the ADC digital output $x[k]$ to achieve dynamic post-correction, as given by

$$s[k] = x[k] + \epsilon[m, k] \quad (1)$$

The slope of $x[k]$ was used in [1] as a dynamic variable, resulting in a phase-plane correction. The work in [2] used the previous ADC sample and is called state-space correction. In [3], a slightly delayed, adjacent sampled signal (by another ADC of the same type) was collected to improve the estimate (compared to [1]) of the signal slope. This process resulted in better compensation compared to the latter two approaches [1], [2]. However, it is more complex and requires another ADC for the calibration. Pure sine waves are typically used during the calibration process that was restricted to the first Nyquist band of the ADC [1]–[3]. In [7], a state-space method with several delayed samples as the dynamic variables, was used. A bit-reduction scheme was developed to show that dynamic calibration using an error term with a reduced resolution is superior to static correction (with higher resolution). In [9], the concept of slope-dependent error is further utilized to estimate the coefficients of the calibration filter in each of the Nyquist bands. The ADC digital output is interpolated to acquire an adequate estimate of the slope. The output is upsampled by a factor of 100 during the calibration [9]. With this method, the use of an additional adjacent ADC, as in [3], is avoided. Three calibration frequencies in each Nyquist band were necessary to compute a reliable estimate of the filter coefficients [9], which are computed and saved for calibration. In the calibration process, the ADC output $x[k]$ and certain previous and post samples were filtered to obtain the corrected output at a given code k [9]. The compensated output at a given time depends on certain previous and future samples. All of these methods ([1]–[3], [7], [9]) have shown that positive improvements (for example, in terms of the spurious free dynamic range (SFDR)) cannot be obtained over the whole band, unless the ADC is calibrated for frequencies (pure sines) spanning the whole frequency band. The slope information is deemed to contain the frequency information but acquiring the complete spectrum of slopes requires stepping the calibration sequence over the band.

Recent work has attempted to address the calibration process with a frequency dependent error. A 1-bit based frequency

estimator was implemented in [10] to address a LUT with the frequency content of $x[k]$ as the dynamic feature. In [11], an empirical model-based approach was proposed for pipeline ADC post-correction by translating a frequency dependent integral nonlinearity (INL) model into a compensation scheme based on the INL modeling presented in [12]. The methodology was implemented in [13] based on a modified INL model [14]. In [14], the INL data was modeled by two separate terms: one static term (solely dependent on the code k) and one dynamic term (frequency m and code k dependent). The static component of the INL data, also denoted as the high code frequency (HCF), was modeled by a set of linear segments centered around the zero axis, while the dynamic part, also denoted as the low code frequency (LCF) component was modeled by a polynomial with frequency dependent coefficients [14]. The HCF behavior is considered static because it is related to the ADC structure itself and circuitry [15] and it is independent of the input signal characteristics. The LCF is a dynamic component because it is related to the input signal dynamics.

The post-correction methodology in [13] (refer to (1)) with the INL model of [14] can be described as

$$s[m, k] = x[k] - Q i_{m,k} = x[k] - Q (h_k + \ell_{m,k}) \quad (2)$$

where Q is the least significant bit (LSB) of the ADC or the ideal code bin width, which is the full scale range of the ADC divided by the total number of bits, i.e., $(V_{\max} - V_{\min})/2^N$. The quantity $i_{m,k} = h_k + \ell_{m,k}$ is the correction term based on the INL model, which is divided into the static HCF term (h_k) and the dynamic LCF term ($\ell_{m,k}$). We recall that the HCF was modeled by a set of adjacent segments and the LCF by a polynomial with frequency dependent coefficients [13], [14].

In this work, which is a continuation of the work presented in [13], [14], we will loosen the requirements on a model-based HCF [14], and, circumvent the time-domain filtering approach [13] to **improve the performance with a more straightforward design**. In addition, the calibration scheme is extended to the upper Nyquist bands to target undersampled applications, such as the ones in [16].

In this paper, we use a calibration methodology based on a combination of a static LUT correction in parallel with a model-based dynamic correction. The LUT is based on the measured HCF. The LCF model is then used to design a bank of frequency domain filters that reconstruct the frequency dependent model. The approach developed is evaluated using an Analog Devices AD9430 as the device under test, and experiments are performed in the first three Nyquist bands, employing two different samples of this ADC.

This paper is organized as follows: the INL modeling is presented in Sec. II. The dynamic calibration is described in Sec. III. The ADC, test-bed, and the INL model are presented in Sec. IV. Sec. V shows the calibration results, and Sec. VI provides the conclusion drawn from this work.

II. THE DYNAMIC INL MODEL

For a set of INL measurements over a frequency band, the INL sequence for a given frequency that is, a column vector of length $2^N - 1$ denoted by $\mathbf{i}[m]$, where each entry at a given code k is given by $i[m, k]$. In this work, the LCF part $\ell_{m,k}$ of the INL $i[m, k]$ is modeled by an L -order polynomial with frequency dependent coefficients in a manner similar to [13], [14] while the HCF model is not parameterized to allow a better representation of the static error. The INL model with respect to

the data can be written as

$$i[m, k] = \ell_k^T \boldsymbol{\theta}_m + h_k + \text{residual} \quad (3)$$

where T denotes transpose, $\boldsymbol{\theta}_m$ are the LCF ($\ell_{m,k}$) polynomial coefficients for a given frequency m or

$$\boldsymbol{\theta}_m = \begin{pmatrix} \theta_{0,m} \\ \vdots \\ \theta_{L,m} \end{pmatrix} \quad (4)$$

ℓ_k is the regressor containing the normalized code $\bar{k}$ values

$$\ell_k = \begin{pmatrix} 1 \\ \bar{k} \\ \vdots \\ \bar{k}^L \end{pmatrix}. \quad (5)$$

and the ‘residual’ captures noise, model imperfections, etc. The work in [13], [14] used a $\bar{k}$ in $[-1, 1]$, but a normalized output code in $[0, 2]$ gives an enhanced representation of the polynomial feature of the INL sequence compared with a representation using $[-1, 1]$, which is a smaller approximation error also with a lower order L . For example, as used in the present work, a polynomial order $L = 5$ with a normalized code in the interval $[0, 2]$ outperforms the model using $L = 7$ and $[-1, 1]$, which is used in [13], [14].

The LCF model relates to the INL data for a given frequency m as

$$\mathbf{i}[m] = \boldsymbol{\ell} \boldsymbol{\theta}_m \quad (6)$$

and $\boldsymbol{\ell}$ is the Vandermonde matrix that stacks the normalized ℓ_k regressors as

$$\boldsymbol{\ell} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 1 & 2^{\frac{k}{2^N-1}} & \cdots & \left(2^{\frac{k}{2^N-1}}\right)^L \\ \vdots & \vdots & & \vdots \\ 1 & 2 & \cdots & 2^L \end{pmatrix} \quad (7)$$

Thus, for every measured frequency m , the LCF polynomial coefficients can be computed by a linear least-squares LS estimation

$$\hat{\boldsymbol{\theta}}_m = (\boldsymbol{\ell}^T \boldsymbol{\ell})^{-1} \boldsymbol{\ell}^T \mathbf{i}[m] \quad (8)$$

A similar estimation approach has been presented in [22]. The extracted non-parametric HCF component of the INL becomes

$$\hat{\mathbf{h}} = \bar{\mathbf{i}} - \boldsymbol{\ell} \hat{\boldsymbol{\theta}} \quad (9)$$

where $\bar{\mathbf{i}}$ is the average measured INL data over the frequency and $\hat{\boldsymbol{\theta}}$ is the average estimated polynomial coefficients. Thus,

for a measured INL data sequence at a frequency m , the estimated INL model sequence will be

$$\hat{\mathbf{i}}_m = \ell \hat{\boldsymbol{\theta}}_m + \hat{\mathbf{h}} \quad (10)$$

The estimated INL model with frequency stimuli m at a given code k is given by the k :th entry of the vector $\hat{\mathbf{i}}_m$ in (10), that is

$$\hat{i}_{m,k} = \ell_k^T \hat{\boldsymbol{\theta}}_m + \hat{h}_k \quad (11)$$

where $\hat{h}_k$ is the k -th entry of the vector $\hat{\mathbf{h}}$ in (9).

This process is performed only once, and the parameters $\hat{\mathbf{h}}$ and $\hat{\boldsymbol{\theta}}_m$ are saved for use in the calibration process.

When comparing the INL model presented in this paper with the one proposed in [14], we note that: the model in [14] has a few parameters employing a parametric model, while the model in the current work is non-parametric in HCF to fully represent the static part of the INL data. In addition, the current approach to describe the HCF avoids any optimization scheme, which is required to determine the borders of the HCF segments by the technique in [14].

These estimated quantities are implemented in the post-correction that is performed in a manner similar to [13], or

$$s[m, k] = x[k] - Q \hat{i}_{m,k} = x[k] - Q \ell_k^T \hat{\boldsymbol{\theta}}_m - Q \hat{h}_k \quad (12)$$

The last term in (12) is the static HCF compensation, which is implemented using a LUT, as shown in Fig. 2. The dynamic compensation, i.e., $-Q \ell_k^T \hat{\boldsymbol{\theta}}_m$, is performed with an overlap-and-add frequency domain filtering [20], which will be given in detail in the sequel.

III. DYNAMIC CALIBRATION

A. A Filter Implementation

The main idea of the model-based approach is to reformulate the dynamic LCF model such that it can be used in the post-correction of the ADC digital output $x_n[k]$ (n denotes the running time index). The post-correction of the ADC digital output $x_n[k]$ that is subject to an input stimulus at f_m is a function of the LCF model $\ell_{m,k}$, which is

$$\ell_k^T \boldsymbol{\theta}_m = \sum_{l=0}^L \bar{k}^l \theta_{l,m} \quad (13)$$

Accordingly, the correction is a linear combination of a constant term and the powers of the ADC output $\bar{k}$ up to the power L . In the time domain, (13) can be interpreted as L parallel filters, where an input $\bar{k}_n$ (n is the time index) with a frequency content m (f_m) produces the output in filter branch l :

$$(\theta_{l,m} \bar{k}_n) \cdot \bar{k}_n^{l-1}, \quad l = 1, \dots, L \quad (14)$$

The post-correction (13)-(14) is thus interpreted as a filter bank with L filters in parallel, where each filter is a zero-phase filter (or a π -phase filter if the coefficient is negative) and the filter output is multiplied by a power of the filter input. Accordingly,

the implementation of the post-correction can be described as L filter design problems, where the goal is to design zero-phase finite impulse response (FIR) filters with the gain specified at the M grid-points, which correspond to $\{f_1, \dots, f_M\}$.

A direct approach to implement the current structure is frequency domain filtering, where the filtering is performed by a multiplication of the frequency domain representation of the data and the frequency response of the filter, which is the real-valued $\theta_{l,m}$. When employing frequency domain filtering, one has to ensure that the frequency grid of the filter response coincides with the frequency grid of the set of transformed input data, where the frequency grid is typically determined by the resolution of the discrete Fourier transform applied. Block processing using a rectangular window is an alternative, but it is known to have low attenuation in its stop band [21]. The Hanning window, on the other hand, is a good candidate because of its superior decay in the stop band [21]. However, this window induces amplitude attenuation at the borders of the block of data. The method of overlap-and-add consecutive processed blocks is well known from speech processing, for example. In [20], a method of overlapping the consecutive Hanning windows was presented, where the data is blocked into half-overlapped sequences (that is, 50 % overlap) and each is multiplied by a Hanning window [20] - an approach that is adopted in this work. The calibration process or the reconstruction of the compensation term $l_{m,k}$ is shown in Fig. 2.

In summary, in the frequency filtering procedure, let $\bar{k}_n$ denote the normalized ADC digital output code at a time instant n . To employ frequency domain filtering, the ADC output code is first temporarily segmented into segments of a proper length, which is typically a power of two. For a given sequence, the frequency transform is performed with the Fast Fourier Transform (FFT), yielding a complex-value output of the same length as the input block. For the l :th filter in the bank, the real-valued frequency function is formed from $\{\theta_{l,1}, \dots, \theta_{l,M}\}$ by interpolation to the same length as the transformed data. After multiplication, the time-domain data is obtained by applying the inverse FFT (IFFT).

The zero order coefficients ($\theta_{0,m}$ for $m = 1, \dots, M$) of the LCF contribute a static term. Thus, they can be omitted from the calibration process. The correction structure is sketched in Fig. 2, where the parallel blocks $F_l(\nu)$ correspond to the filters.

Despite working in the upper Nyquist bands, the undersampled signals are downconverted by the ADC to the first Nyquist band. The digital output of the ADC is always completely contained in the first Nyquist band. Thus, frequency filtering is always performed in the first Nyquist band, where the sampled frequencies in the odd Nyquist bands are shifted to the first Nyquist band and the frequencies in the even bands are mirrored around the Nyquist frequency. A bank of filters is constructed out of the respective LCF model polynomials for the correction in each Nyquist band.

B. Comparison with previous work

The calibration scheme presented in this paper is considered to have superior performance than the one in [13] because its dynamic compensation is directly implemented in the frequency domain and thus avoids the process of filter design in the time domain. A non-parametric full LUT for the static correction is expected to achieve superior static performance compared with the approach in [13], where the static correction is based on a model-based description of the LUT with reduced accuracy.

Fig. 2. ADC post-correction block with ADC code k_n , as input. δ_1 is a scaling term defined as $\delta_1 = 2/(2^N - 1)$. The ADC output code k is indexed by n to indicate its time dependence. $\hat{i}_n$ is a short notation of $\hat{i}_{m,k}$ at time instant n .

Fig. 3. A schematic of the test-bed used to characterize the ADC at hand [23].

IV. EXPERIMENTAL ADC CHARACTERIZATION

A. ADC test-bed

The test-bed used here is the same as that in [13], [14], and some additional filters were used in the higher Nyquist bands. More information about the test-bed can also be found in [23]. The test-bed is shown in Fig. 3.

A Rohde&Schwartz SMU200A vector signal generator (VSG) was used at low output signals to avoid distortions that can be generated at high output powers, which make ADC characterization unreliable. A specially designed wideband (30-300 MHz) low distortion amplifier with a gain of 14.4 dB is used to amplify the output of the VSG. The low distortions in this amplifier are guaranteed up to 300 MHz, and this limit was the main reason for stopping the ADC characterization at the third Nyquist band (315 MHz).

The spectral purity of the ADC input signals (harmonics and spurs are attenuated to -80 dBc) is achieved with SLP Mini-Circuits filters. The complete set of SLP filters used for the different ADC bands is shown in Table I.

TABLE I
ADC MEASUREMENT SET-UP.

Frequency band [MHz]	SLP- Filter	Filter passband [MHz]
[30 – 55]	50+	48
[60 – 70]	70+	60
[75 – 115]	100+	98
[120 – 165]	150+	140
[170 – 210]	200+	190
[215 – 300]	300+	270
[305 – 315]	415+	400

Fig. 4. ADC1 LCF polynomials plotted in combination with the static HCF.

To secure the experimental measurement campaign, three sets of data were obtained for each input stimulus. The figures of merit were calculated for each of the input conditions, followed by an averaging of the improvements over the three sets.

B. Device under test and data collection

The ADC tested is the pipeline Analog Devices AD9430 with a sampling rate of 210 MSPS. Two samples of the same ADC were available for testing and calibration and are designated as ADC1 and ADC2.

The INL was characterized over three Nyquist bands from a frequency of 30 MHz up to 315 MHz. The INL characterization is based on single frequency inputs with 5 MHz spacing over the band under test (30-315 MHz), providing $M = 58$ measured INL sequences, each of length $2^N - 1 = 4095$. Coherent sampling was achieved with a slight saturation for the INL measurements according to [17]. The INL is estimated using the histogram test [17].

Additional data were collected at the same frequencies at an input power level of -0.5 dB below full scale (dBFS), which is the proper ADC usage power according to the data sheet. Additionally, three sequences were measured at every frequency under test m . Some additional frequencies within the 5 MHz spacing were also collected.

C. Modeling Results

The INL modeling and ADC post-correction (the compensation scheme in Fig. 2) are implemented in software using Matlab. The estimated LCF and HCF for ADC1 and ADC2 are shown in Figs. 4 and 5, respectively. An $L = 5$ -order LCF polynomial is used for both ADCs.

Fig. 5. ADC2 LCF polynomials plotted in combination with the static HCF.

Fig. 6. The RMS of the INL data versus the input frequency (solid line). The dotted line shows the RMS of the INL data of which the LCF is subtracted. The dashed represents the RMS when the HCF is also removed.

The LCFs of the two ADCs are similar in shape, but ADC2 generally has a more pronounced LCF counterpart and INL, as is obvious in the HCF data shown for the respective ADCs.

Fig. 6 shows the capability of the LCF and HCF to represent the INL data over the three Nyquist bands for both ADCs. The root mean square (RMS) of the INL data is compared with the RMS when the LCF is subtracted and also when the HCF is removed. The LCF is able to capture most of the INL features, especially at higher frequencies. The magnitude of the INL data from ADC2 is approximately 0.6 [LSB] (in RMS sense, refer to Fig. 6) larger than that of ADC1, mainly in the second and third Nyquist bands.

D. Calibration Process

The frequency domain filtering should be performed on a segmented sequence with a length such that no aliasing or spectral leakage occurs. According to [17], for an N -bit ADC, a minimal sequence length of $\pi 2^N$ is needed to avoid such phenomena. Thus, a Hanning window is applied to the $2^{N+2} = 16$ kilo samples (kS) prior to the FFT/IFFT process. The frequency response of the filters $F_l(\nu)$ should be the same size as the windowed sequence, i.e., 16 kS. The response of the filters $F_l(\nu)$ is a linearly interpolated version of the sampled frequency response given by the coefficients $(\theta_{l,m}|_{m=m_N}^{M_N})$, where m_N and

Fig. 7. ADC1 SFDR improvements.

M_N represent the edge frequencies for a given Nyquist band. The calibration process in every Nyquist band is performed with $L = 5$ frequency filters.

The method introduces a delay given by the sum of the block length and the processing time for the filtering.

As a benchmark method to evaluate the proposed calibration structure, a non-implementable reference method is used. The reference method uses the estimated INL data $i_{m,k}$ directly to calibrate the ADC excited by a tone stimulus at f_m [Hz]. Accordingly, it uses the prior knowledge of the stimulus frequency, and it is only applicable for single tone ADC inputs. The reference method is implemented as a two-dimensional (k and m) dynamic LUT (DLUT), where the dependency of m is based on prior knowledge of the ADC test stimulus. The term “dynamic” indicates that a specific INL data is stored for a set of different input frequencies m .

V. ADC CALIBRATION RESULTS

For wideband characterization, the most important ADC parameters are: the SFDR, noise, signal-to-noise and distortion ratio (SINAD), and intermodulation distortion (IMD) [17]. The theoretical investigation in [24] and implementation in [13] have shown that improvements in noise and SINAD are compelled by a 0.7 dB upper bound for an $N = 12$ -bit ADC. Thus, these two parameters will not be considered in this work. SFDR improvements are measured over the first three Nyquist bands. The calibration results for frequencies coinciding with the calibration frequencies are presented first (in Fig. 7 and Fig. 8, for ADC1 and ADC2, respectively). The calibration results for the frequencies that are in between the calibration frequencies are presented in Table II, where the performance improvements for ADC inputs below the nominal power level are also included. The calibration of the IMD products is considered in the last subsection.

Fig. 8. ADC2 SFDR improvements.

A. SFDR Improvements for single tone inputs at the nominal ADC input power level

In this section, we show the calibration results when the ADC is operated at its nominal input power level at -0.5 dBFS. Three sequences are measured for each input frequency, and the average SFDR improvement is reported.

a: Fig. 7 and Fig. 8 show the SFDR improvements in ADC1 and ADC2 over the entire band, respectively. The SFDR improvements of ADC1 obtained with the model-based compensation in Fig. 2 are almost identical to those obtained using the DLUT, except in the frequency range between 105 to 115 MHz, where the DLUT outperforms the model compensation by 2-4 dB. The modest results (compared to the DLUT) in this range can be explained by the following: according to Fig. 6, the model is not able to represent the INL data in this range, or at the surrounding frequencies, i.e., some key features of the ADC distortions are not adequately captured. The model representation and the subsequent calibration process in this specific frequency range are improved (identical to DLUT) when the order of the LCF polynomial L is increased to 7. This translates into two additional filters in the calibration block in Fig. 2. However, this extra complexity does not lead to additional improvements over the remaining range of frequencies because the expected calibration results are almost saturated (the model is virtually identical to the DLUT). Thus, the extra improvements in this frequency range (105 to 115 MHz) are sacrificed for a reduced model order of $L = 5$ LCF frequency filters.

b: The model-based improvements of ADC2 outperform the DLUT table for several calibrated frequencies but are being identical to the latter most in the cases. The average SFDR improvement over the whole range for ADC1 is 5 dB when using the model-based calibration and 5.2 dB when using the DLUT. The magnitudes of these improvements were 9.1 dB and 8.9 dB for ADC2, i.e., the model outperforms the DLUT here. The maximum SFDR improvement for ADC1 was 13.2, dB and the minimum was 0 dB. ADC2 had better results, and the improvements ranged between 2.8 and 16.9 dB. No SFDR degradation was encountered during the calibration process.

Compared with the calibration scheme in [13], this method (Fig. 2) achieves an average improvement of 4 dB over the

first Nyquist band, in case of ADC2. This improvement is mainly due to a significantly improved static (HCF) calibration, in addition to a more accurate LCF calibration. The average improvement was 1 dB in case of ADC1 because the latter has a small HCF error; the difference is due to the improved LCF calibration.

B. Comparison of the performances of the two ADCs

The difference between the two ADCs must be further addressed. ADC1 has a better uncompensated SFDR than ADC2 with an average value of 71.4 dB versus 69.9 dB over the three Nyquist bands, resulting in an average difference of 1.5 dB. This fact also translates to a larger INL data magnitude for ADC2 (refer to Fig. 6). Thus, it is expected that there is more room for improvements in ADC2, especially because the static HCF part of the INL is larger for ADC2 (refer to Fig. 6). A larger static error will inevitably translate into a steadier and (relatively) larger compensation, given that the latter is modeled and estimated adequately. The improvements in ADC2 are better by 4 dB on average than the improvements in ADC1. This difference is clear in Fig. 7 and Fig. 8, where the calibrated SFDR of ADC2 clearly has a larger shift over the uncalibrated SFDR than that of ADC1. ADC2 has an average compensated SFDR of 80 dB, while ADC1 has an average SFDR of 76.4 dB. In general, the compensated SFDR of ADC2 is almost uniformly better than that of ADC1. The improvements in ADC2 are steadier over the band of use. The static correction of ADC1 is quite modest, ranging between 0 and 1 dB, because of the reduced magnitude of the HCF counterpart. Thus, for pipeline ADCs with similar HCF magnitudes, the static correction can be omitted; therefore, the calibration scheme parameters are significantly reduced by $2^N - 1 = 4095$.

C. The reference DLUT

Figs. 7 and 8 show that substantial SFDR improvements are obtained for some frequencies, while modest improvements are obtained at others for both ADCs. Fig. 9 shows the calibrated $f_m = 235$ MHz frequency for ADC2 (ADC1 did not have similarly large improvements at that specific frequency), where the harmonics and spurs are totally suppressed or attenuated to the noise level vicinity using the model compensation or the ideal DLUT. In this case, the model approach is better for attenuating the larger harmonics and totally suppressing the smaller spurious signals. However, for the calibrated $f_m = 155$ MHz frequency, the respective harmonics and spurs are not attenuated as desired (for either ADC). Fig. 10 shows the SFDR improvements of ADC1 for the 155 MHz frequency. In this case, two harmonics are totally suppressed, and one is merely reduced. These phenomena are almost identical for both the DLUT and the model-based calibration approaches. Additionally, the DLUT method gives rise to a small spurious signal that is smaller than the large remaining harmonic. This trend questions the reliability of the reference ideal INL DLUT that is used for INL modeling prior to calibration. The INL is defined in [17] as a static measure (solely code k dependent) at low frequencies, where its testing has been well standardized and optimized. However, the INL has been thoroughly used in the literature as a dynamic error sequence to correct ADCs. It is also worth mentioning that, at the lowest operable range of the ADC (approximately 30 MHz), the INL did not achieve complete compensation. These results show that the INL, despite being a good measure for calibrating ADCs, is not an optimal or complete quantity for calibration.

The small additional SFDR improvements that the model-based method has over the DLUT will be addressed in the next section.

Fig. 9. ADC2 235 MHz input frequency post-correction using the model-based approach in Fig. 2 and the reference DLUT. The plot shows the tones downconverted to the first Nyquist band.

Fig. 10. ADC1 155 MHz input frequency post-correction using the model-based approach in Fig. 2 and the reference DLUT. The plot shows the tones downconverted to the first Nyquist band.

D. SFDR Improvements for single tone inputs with reduced power levels and in-between frequencies

It is clear at this stage that the model correction works well with pure sine inputs coinciding with the calibration frequencies. The response of designed $F_l(\nu)$ filters in the in-between frequencies is a linear interpolation of the response values of the calibrated frequencies. This interpolation is based on the assumption that the in-between frequency behavior (INL data and model) cannot be drastically different than the surrounding calibrated frequencies. However, the calibration process must still be tested in this range. The ADC inputs with reduced power levels are another interesting area for calibration; though they are different than the nominal working conditions of the ADC, such signals are interesting to calibrate because they have a less pronounced slope than the full range (or -0.5 dBFS) inputs. This difference can be problematic if one considers that the slope is the determining factor in dynamic calibration. However, for a pipeline ADC with fully differential bottom plate sampling [25], [26], it can be shown that the frequency and the slope information are similar for a known input signal power level. This property should translate into undiminished calibration results for a pipeline ADC operating at reduced input power levels.

a: In Table II, the model based correction is compared to the DLUT for different power levels (given in negative dBFS) and the SFDR improvement results are presented in dB. The original and model-based compensated SFDRs (denoted by (U)SFDR and (C)SFDR, respectively) are also appended to the table. Two sequences are measured at the same input power level for a

TABLE II
CALIBRATION RESULTS ON SELECTED FREQUENCIES AND INPUT POWERS

ADC	Freq [MHz]	Power dBFS	Model dB	DLUT dB	(U)SFDR dB	(C)SFDR dB
ADC1	52	-0.5	8.1	8.8	77.4	85.5
ADC1	152	-0.5	5.6	5.6	72.5	78.1
ADC1	152	-5	10.8	8.1	76.8	87.6
ADC1	152	-10	2.9	1.8	78.4	81.4
ADC1	200	-3	9.5	9.5	73.4	82.9
ADC1	200	-5	0.4	0	81.2	81.6
ADC1	215	-5	0.1	-0.3	74.8	74.9
ADC1	215	-10	-1.9	-2	78.5	76.6
ADC1	255	-10	0.6	0.3	76.5	77.1
ADC1	257	-0.5	4.2	4.2	63.4	67.4
ADC1	257	-5	3.6	3.5	70.7	74.3
ADC1	257	-10	0	0.2	77.2	77.2
ADC2	45	-4	9	8.8	77.2	86.2
ADC2	45	-5	8.1	3.5	77.2	85.3
ADC2	45	-10	7.9	6.5	69.5	77.4
ADC2	47	-5	7	3.2	77	84
ADC2	47	-10	7.2	4.6	71.7	78.9
ADC2	47	-15	3.5	3	67.5	71
ADC2	64	-0.5	7.4	7.3	79	86.4
ADC2	150	-5	3.2	-0.2	76.5	79.7
ADC2	150	-10	13.1	5.5	68.3	81.4
ADC2	167	-0.5	9.2	9.4	69.6	78.8
ADC2	167	-5	5.6	-1.9	76.6	82.2
ADC2	167	-10	8.3	-2.4	69.5	77.8
ADC2	170	-5	4.8	1.5	77.7	82.7
ADC2	170	-10	9.6	5	69.2	78.8
ADC2	240	-5	10.7	10.6	64.8	75.5
ADC2	240	-10	10.7	10.6	64.8	75.5
ADC2	262	-0.5	7.8	7.6	62.9	70.7
ADC2	262	-5	4	-2	69.9	73.9
ADC2	262	-10	6.7	-4	70.3	77

given input frequency, and the average calibration improvement is reported in Table II. The performance difference between the two sequences is typically 0.3 dB or less, indicating the repeatability of the results. There was no performance degradation for the model-based method, except for ADC1 at the 215 MHz frequency measured at the -10 dBFS power level. ADC1 has no improvements at this frequency when operating at -0.5 dBFS (refer to Fig. 7), and a negative result is not surprising at this stage. The model improvements at the -0.5 dBFS power level for the inputs at in-between frequencies are quite identical to the DLUT, which is an important result that demonstrates the validity of this model-based method over the entire range of frequencies. Regarding the inputs with reduced power levels (-3,-5 and -10 dB), the uncompensated sequence SFDR does not always increase as the input power level is decreased. Actually, decreasing the input power led to SFDR attenuation (before the compensation) in several cases. However, the compensated sequence SFDR is similar (for one frequency with different input power levels) when using the model based correction unless the noise level is reached; there is no room for further correction in the case of reduced power levels. The DLUT correction is not as good as the model based approach, which is not a surprising result because the DLUT is derived for a full scale input power level but it is used at quite reduced power

TABLE III
CALIBRATION RESULTS ON TWO TONES INPUT SIGNALS

ADC	Freq [MHz]	$f_2 - f_1$	$2f_1 - f_2$	$2f_2 - f_1$	$f_1 + f_2$	$2f_1$	$2f_2$
ADC1	28.3 & 29.3	3.5	0.7	-	10.5	0.7	1.6
ADC1	60 & 65	6.2	-	-	8	11.3	10
ADC2	28.3 & 29.3	0	2.2	0	1.7	0	0

levels. Thus, the DLUT shows its worst performance (negative or reduced improvements compared to the model approach) at -10 dBFS. According to [27], the quantization error of a sine wave is concentrated around zero, where the slope of the sine has its largest values. Hence, using an error sequence that is optimized at full scale at a severely reduced input power level can lead to a deterioration in performance.

b: For the model based approach, a constant improvement is obtained in the sense that the compensated SFDR is almost independent of the input power level and the value of the uncompensated SFDR at this level. Using the DLUT, regardless of the magnitude of the sine, the same $i[k, m]$ is appended to a given output code k and frequency m , which is not the case in the model approach due to the frequency filtering process. Performing an FFT provides the frequency of the input in addition to its power. Thus, a given input code $\bar{k}_n$ is multiplied, according to its frequency content m , by $\theta_{l,m}$ to reconstruct the LCF model. At a given time instance n , k_n will not be the same for different input power levels, even at the same frequency and phase. Thus, the error term is scaled according to the amplitude of the input sine wave in the model-based calibration. The effect of a different amplitude/slope for the same input frequency is implicitly included in this process. Compared to this process, the DLUT works blindly because it has no information concerning the input signal power (the output code k_n is not sufficient information for the reduced input power level). Thus, the frequency information in the DLUT approach is quite distorted compared to that in the model-based method, and it is no longer information equivalent to the slope information.

These results also explain the slight edge in performance that the model-based method has over the DLUT because the latter is optimized at full scale and the calibration is performed at -0.5 dBFS. For the case of ADC1 with a 152 MHz measured frequency at -5 and -10 dBFS of power, the calibration suppressed the harmonics and spuri to the noise level such that there was no more room for improvement in the case of the -10 dBFS power level.

E. IMD improvements

According to the AD9430 data sheet, the IMD is measured at an input power level of -7 dBFS for each input. Two sets of frequencies were used for both ADCs. $f_1 = 28.3$ MHz and $f_1 = 29.3$ is the first measured set because the data sheet refers to it. The model is not calibrated at these frequencies and thus is extrapolated in this range. However, improvements still occurred here. Two additional frequencies, $f_1 = 60$ MHz and $f_1 = 65$, were also measured. According to the data sheet, for a two-tone measurement, the most important distortions are the third IMD products ($2f_1 - f_2$ and $2f_2 - f_1$) because they lie in the vicinity of the original two tones, making it notably difficult to filter them out. However, the attenuations of all of the IMD products, including the harmonics of every tone, are presented in Table III. Improvements were clearly achieved. Due to the reduced input power levels, some IMD products were below the noise level prior to the compensation (denoted by a slash). ADC2 has all of its IMD products and most of its harmonics below the noise floor in the case of the $f_1 = 60$ MHz

and $f_1 = 65$ MHz tones; hence, it is not included in the table. Thus, this method is well suited for a multitone calibration.

VI. CONCLUSION

The dynamic calibration of pipeline ADCs is presented in this paper. Prior to the compensation, a dynamic INL model (with the frequency as a dynamic variable) is estimated based on the INL data. The model is subsequently used to design a calibration scheme. The latter reconstructs the correction term with a static LUT and a bank of frequency domain filters. Frequency domain filtering is performed with an overlap-and-add structure. The ADC also provides improvements at inputs below the nominal power level. Generally, the calibration is slightly better than that of the (non-implementable) reference dynamic LUT, where the latter causes performance degradation at reduced input power levels. IMD products are also reduced by the calibration scheme presented in this paper.

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