Ring Resonator with single gap for Measurement of Dielectric Constants of Materials

Rustam Mustafa

June 2013

Master’s Thesis in Electronics

Master’s Program in Electronics/Telecommunications
Examiner: Professor Jose Chilo
Supervisor: Professor Kjell Prytz
Preface

Gratitude to Professor Kjell Prytz for giving us this project to execute; we thank Mr. Rickard a lab technician for fabrication of the ring resonator and the MUT into specified geometries.

“I thank every member of my family, so much grateful to all the support you offered during my study”

-Rustam M.
Abstract

The significance of calculating and measuring dielectric constant is important for all the fields of study and research from biological science to electronics and telecommunication. It gives a better characterization of the material and helps to design the accurate system, and aside these two functions, it has numerous uses. Owing to its huge expediency, various measurement methods of dielectric constant of materials have been developed by scientists and engineers over the years. Each method has its limitations which affect the accuracy of the measurement; these limitations range from frequency, temperature, measurement environment to material under test.

In this research work, we measured the resonant frequency of the ring resonator using different materials and then we measured the dielectric constant of different materials by using that resonant frequency. We discussed different common methods of measuring dielectric constant and the most accurate one, the resonant method, was chosen and driven upon. The project was done by making a mathematical analysis for the geometry of the ring resonator, wrote MATLAB script for the measurement of the resonant frequency, later designed and simulated in HFSS to obtain results which would be comparable to ones obtained in laboratory measurements.

The ring was fabricated and taken to the laboratory for measurement of the resonant frequency for measuring dielectric constant of different materials; two monopole antennae were connected to the two ports of a vector network analyzer with one antenna serving as the transmitter and the other serving as the receiver. When we used the MUT in the gap of the ring resonator we saw frequency shift in the resonant frequency and we noted that resonant frequency.

The resonant frequencies obtained were compared with the geometric parameters of the ring resonator and that of the MUT in an equation written into MATLAB script; this equation was used to extract the dielectric constant of the MUT and then we calculated the dielectric constant by using the resonant frequency of the MUT.
# Table of contents

Preface ........................................................................................................................................... i

Abstract .......................................................................................................................................... ii

Table of contents ........................................................................................................................ iv

Chapter 1 ........................................................................................................................................ 1

1.1 Introduction: .......................................................................................................................... 1

1.2 Objective and Implementation: ............................................................................................. 1

1.3 Thesis Layout: ......................................................................................................................... 2

Chapter 2 ........................................................................................................................................ 4

2.1 Literature Review: .................................................................................................................. 4

2.2 State of Knowledge ................................................................................................................ 5

2.2.1 Split Ring Resonators ....................................................................................................... 6

2.2.2 Dielectric constant ............................................................................................................. 6

2.2.3 Resonance ......................................................................................................................... 7

2.2.4 Quality Factor ..................................................................................................................... 7

2.2.5 Antenna Characterization ................................................................................................. 8

2.3 Different Methods for Measurements: .................................................................................. 8

2.3.1 Transmission and Reflection Line method ....................................................................... 9

2.3.2 Open ended coaxial probe method ................................................................................... 10

2.3.3 Free space method ............................................................................................................ 11

2.3.4 Resonant method .............................................................................................................. 12

Chapter 3 ........................................................................................................................................ 13

3.1 Process and Results: .............................................................................................................. 13

3.2 Analysis ................................................................................................................................... 13

Chapter 4 ........................................................................................................................................ 16

4.1 Simulations ............................................................................................................................... 16

4.2 Excitation and Boundaries in HFSS ..................................................................................... 17

4.3 Simulated resonant frequencies without and with MUT in the gap ...................................... 18
4.4 Measurement ........................................................................................................21
Chapter 5 .................................................................................................................. 25
  Discussion & Conclusions...................................................................................... 25
  5.1 Discussions:..................................................................................................... 25
  5.2 Conclusions..................................................................................................... 26
References .................................................................................................................. 27
Appendix .................................................................................................................... A1
Chapter 1

In this chapter we discussed about the introduction of the thesis work and background relating this thesis. We have also discussed about the Aim/Objective of our thesis.

1.1 Introduction:

The dielectric constant is main factor for measurement of the any dielectric material. The dielectric constant is directly proportional to the permittivity of the material. There are many methods for measuring the dielectric constant of the material which can be used as substrate. The Q Factor and resonance frequency is directly proportional to the dielectric material properties. This measurement is very important in now days in different research field’s e.g. material sciences microwave and etc. First the resonance frequencies of different modes measured and then from these frequencies dielectric constant should be measured. Most frequently used method is the measurement of reflection coefficient lor resonant frequencies. Dielectric constant changes with the different frequencies [1].

1.2 Objective and Implementation:

The aim of this thesis is to measure the dielectric constant for different material using the resonant frequency. The objective of this thesis work is to measure the resonant frequency, calculate the geometry for the design work of the ring resonator, design of the ring resonator in HFSS and then measure the resonant frequency with the VNA.

Following are the steps we will take for the implementation of this work.

- Manual Calculation for resonant frequency.
- Matlab Code Script.
- Ring Resonator Design and Simulation
- Materials selection for measurement.
- Antenna.
- Frequency measurement using VNA.
1.3 Thesis Layout:

![Diagram of thesis process]

Below is the structural layout of this thesis report.

- **Chapter 1** is about the introduction of the whole thesis work for the design of the singly split ring resonator. In this chapter also give brief background and objective implementation procedure.

- **Chapter 2** is about the literature review. In this chapter we explain the theatrical about ring resonator, dielectric constant, quality factor, methods for measurement and also characterize the antenna.

- **Chapter 3** is all process, results and equations which we used for the manual calculation of the resonant frequency.
• **Chapter 4** is all about the measurements and simulations. In this chapter we explain the process of the whole thesis work and show the HFSS simulations and VNA testing. We have also discussed about the results of HFSS simulations and using VNA and antennas for the measurement of the resonant frequency and dielectric constant of different materials.

• **Chapter 5** is all about the discussion and conclusion. In this chapter we discussed our results and conclude the whole research work and also give some suggestions for the further enhancement of this thesis work as a future work.
Chapter 2

2.1 Literature Review:

The discovery of media which have a property of negative effective index, $N_{\text{eff}}$, of refraction made an enormous impact in the science world and recently, it gathered a huge amount of interest. This sprang the discovery of various new and interesting features of left-hand electromagnetism. Following Vassalage proposal in 1968[2], a spontaneous step was taken to make a composite medium whose components simultaneously have negative permeability $\mu < 0$ and negative permittivity $\varepsilon < 0$ ranging certain frequencies[3]. For this to happen, the negative effective index of refraction, $N_{\text{eff}}$, would be negative while the wave vector of the propagating EM wave would be real[4]. Thin metallic wires arranged at regular intervals clearly show the plasma frequency $\omega_p$ in the microwave frequencies; below these frequencies, the material is opaque[5]. For the negative $\varepsilon$ component of composite materials, thin wire arrays are suitable materials because the dielectric permittivity turns negative below $\omega_p$.

Later, a proposal of split ring resonator (SRR) structures was made by Pendry et al., they showed that the structures produced negative permeability near magnetic resonant frequency $\omega_n$ in response to an incident magnet field. In support for the reality of negative refractive index, $n_{\text{eff}} < 0$, experimental confirmation of negative refraction was reported for composite materials of split ring resonators and wires[3,4]. The EM wave components in these media formed left-hand coordinate systems and thus the structures derived their name as left-handed meta materials. The magnetic, electric, and wave vector do not obey the right hand rule in these media [5]. In negative refraction, split ring resonators contribute the negative permeability. The possibility of obtaining left-handed materials hinges on the discovery of SRR structures by Pendry. Since little is known about SRR structures and given the fact that they are presently the essential components of left-handed materials, there are ongoing extensive studies concerning the various features of split ring resonator structures. Transmission features of periodic and disordered split ring resonators are studied experimentally in details in [4] and [5] respectively. The simulation of transmission properties of split ring resonators were reported in [2] while the effective parameters were reported in Extensive study of electromagnetic resonance of SRR, numerical cum theoretical were given in. The understanding of the origin of resonances and the effect of various variables or parameters as regards split ring resonator behaviors is credited to analytical models [6]. Due to their ease of fabrication, the studies on split ring resonators and meta materials are mainly carried out at the gigahertz (GHZ) frequency range but there has been a recent possibility to increase resonant frequencies of split ring resonators to terahertz (THZ) frequencies [7].
2.2 State of Knowledge

The split ring is a basic geometry for the design of sub-wavelength magnetic Meta material resonators [7]. At microwave frequencies, double split rings design has gained so much popularity. From intuition [8], analytical expressions for resonant frequencies of double ring constitution were derived while the ones with more rigors [9, 10] were confirmed by experiment [8, 11]. The resonant properties of single rings have gathered little study, analytically. Here, an analytical expression for the resonant frequency of the singly split single ring resonator is derived. The geometry used for this analysis is shown in Fig.2.1. The parameters are: the inner radius of the ring, R, the thickness, w, the height, h, and the gap width, g the sub-wavelength split ring resonator can be characterized by the inductance, L, and the capacitance, C.

![Figure 2.1 Geometries of the ring under analysis.](image)

\[ f_0 = \frac{1}{2\pi\sqrt{LC}} \]  
(2.1)

The inductance can be approximated by that of a closed ring.

\[ L = \mu_0 R_m \left( \ln \frac{8R_m}{h+w} - 0.5 \right) \]  
(2.2)

\( \mu_0 \) is the permeability of free space and \( R_m \) is the mean radius of the ring, \( R_m = R+w/2 \).

Here, there are two capacitances, the gap and the surface capacitances. Charges in the gap bring about the gap capacitance; the gap can be seen as a parallel plate capacitance.

\[ C_{gap} = \varepsilon_0 \left[ \frac{wh}{g} + \frac{2\pi h}{\ln \left( \frac{2Ah}{w} \right)} \right] \]  
(2.3)

The second term in equation (2.3) is a correction due to finite size.
The surface capacitance is contributed by charges on the surface of the ring; the results obtained by Allen and Segre [12] for infinitely long split cylinders with infinitesimal gap is used. For \( \sigma \), the surface charge density, and the voltage \( V \), we have

\[
\sigma = \frac{\varepsilon_0 V_0}{2\pi R} \cot \frac{\theta}{2}; \quad V = \frac{C}{\pi} (\pi - \theta)
\]

(2.4)

\( V_0 \) is the applied voltage to the gap and the \( \theta \) is the angle defined in figure 2.1 above. With the aforementioned the surface capacitance is given by

\[
C_{\text{surf}} = \int_{\theta_g}^{\pi} \frac{\sigma R d\theta}{V} = \varepsilon_0 h \int_{\theta_g}^{\pi} \frac{\cot \left( \frac{\theta}{\pi} \right)}{\pi - \theta} d\theta \approx \frac{2 \varepsilon_0 h}{\pi} \ln \frac{4R}{g}
\]

(2.5)

Where \( \theta_g = \arcsine (g/2R) \).

With the assumption that the gap and surface capacitances are parallel, the total capacitance is given in equation (1.6).

\[
C = C_{\text{gap}} + C_{\text{surf}}
\]

(2.6)

### 2.2.1 Split Ring Resonators

The SRRs are used to overcome the problem of signal propagation in the narrow band near the resonance frequency. This is done by the magnetic excitation on the ring. On the basis of the SRRs method, many scientists have now introduced other methods for narrow band. Through this method negative permeability can be generated by inductor and capacitor [13][14].

### 2.2.2 Dielectric constant

The extent to which the electrostatic lines of flux are concentrated by a material is called the dielectric constant of that material. When voltage is applied to a material, electric energy is stored in that material, thus dielectric constant can further be said to be the ratio of this stored electrical energy in the material to that stored in vacuum. If there are two capacitors with one having a material as its dielectric and the other having air as its dielectric, then the dielectric constant is the ratio of the capacitance of the capacitor having a material as its dielectric to the ratio of the capacitance of the capacitor having air as its dielectric. Dielectrics with low permittivity are very good insulators used for isolating signal-carrying conductors for one another; high permittivity dielectrics are good materials for holding charges and thus are favored dielectrics for capacitors [15].
2.2.3 Resonance

The phenomenon of resonance occurs due to the characteristics of inductance and capacitance. The combination of inductance and capacitance can have only one frequency of operation and this is known as resonant frequency. The daily life application of resonant frequency could be found in TV channel or Radio broadcasting station tuning which is done through the LC tuner which selects the desired frequency and rejects the others. When the resonant frequency is fed to a series or parallel circuit, XL becomes equal to XC, and the circuit is said to be resonant to a specific frequency. The circuit is then known as resonant tuned circuits. The response of the resonant circuit to the resonant frequency is different from its response to other frequencies.

\[ X_c = \frac{1}{2\pi f C} \]  

(2.7)

Where

\( X_c \) = the capacitive reactance in ohms
\( f \) = the frequency in hertz
\( C \) = the capacitance in farads

\[ X_L = 2\pi f L \]  

(2.8)

where

\( X_L \) = the inductive reactance in ohms
\( f \) = the frequency in hertz
\( L \) = the inductance in henries

2.2.4 Quality Factor

The quality factor is very important in every field or in any device. The quality factor of split ring resonator is also very important. It’s a key feature of the ring resonator. The quality is inversely proportional to the loss of the ring resonator i.e. If the quality factor is high the losses will be very low. The quality factor is a ratio [16].

\[ Q = 2\pi \times \frac{\text{Maximum energy stored per cycle}}{\text{average energy dissipated per cycle}} \]  

(2.9)
2.2.5 Antenna Characterization

Monopole antenna is a dipole antenna cut into two i.e. one half of dipole. It’s a radio antenna mounted perpendicularly on the ground plane. It is the same as the dipole antenna only that its impedance is half of the dipole antenna and its gain is double the gain of the dipole antenna. The radiation pattern and physical structure of the monopole antenna is shown in figure 1.2 below [17].

![Monopole Antenna Diagram]

Figure 2.2: Radiation Pattern and physical structure of a Monopole Antenna.

2.3 Different Methods for Measurements:

Various methods abound for the measurement of dielectric properties of materials with each method limited to certain frequencies, applications, materials, etc. In this thesis work, four methods have been chosen; the methods are listed below and further described in details.

I. Transmission and Reflection Line Method.
II. Open ended coaxial probe method.
III. Free Space Method.
IV. Resonant method.

Below is Table 2.1 highlighting the various measurement methods used for extracting the dielectric properties of different materials.

<table>
<thead>
<tr>
<th>Measurement techniques</th>
<th>Materials</th>
<th>S-parameters</th>
<th>Dielectric properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission/Reflection</td>
<td>Coaxial line, waveguides</td>
<td>S₁₁, S₂₁</td>
<td>εᵣ, μᵣ</td>
</tr>
<tr>
<td>Open-ended coaxial probe</td>
<td>Liquids, biological specimen, semi-solids</td>
<td>S₁₁</td>
<td>εᵣ</td>
</tr>
<tr>
<td>Free space</td>
<td>High temperature</td>
<td>S₁₁, S₂₁</td>
<td>εᵣ, μᵣ</td>
</tr>
</tbody>
</table>
Transmission and Reflection Line Method falls within the well-known method called the broadband measurement method where only the fundamental waveguide mode is assumed to propagate. Here, the transverse electromagnetic mode (TEM) is for the coaxial line while the transverse electric mode (TE) is for the waveguide.

Both the transverse electromagnetic mode (TEM) and the transverse electric mode (TE) mode of propagation is assumed for the resonant method and this method gives high accuracies in the measurement of dielectric constant of materials.

### 2.3.1 Transmission and Reflection Line method

In this method, the object is placed in waveguide or coaxial line and measurement is done using the S-parameters through the Vector network analyzer (VNA). As the name implies, it is used for the measurement of transmitted \( S_{21} \) and reflected signals \( S_{11} \). These S-parameters are then used to find the permittivity and permeability of the materials with the help of equations. Figure 2.2 shows the measurement setup of transmission and reflection line method. Before using this method, calibration must be carried out for accurate results and also sometimes when the object doesn’t fit in the waveguide or coaxial line, simple machining is done [18].
Advantages of Transmission and Reflection Line Method

- Samples whose loss range from medium to high can be measured using coaxial line and waveguide.
- Both the permittivity and permeability of material under test (MUT) can be determined using this method.

Disadvantages of Transmission and Reflection Line Method

- Air-gap effect affects measurement accuracy.
- For a sample whose length is a multiple of one-half, there is bound to be low accuracy.

2.3.2 Open ended coaxial probe method

This is a non-destructive measurement method. Here, the probe makes contact with the MUT or the specimen, the reflection coefficient is determined and used to measure the dielectric constant. In case the material whose permittivity to be measured cannot be cut out as a sample, the probe will be touched to the material in order for a contact to be made. Vector Network analyzer is used to measure the reflection coefficient and calibration needs to be done before any measurement is made [18].

![Figure 2.4: Open ended coaxial probe method.](image)

Advantages of Open-ended Coaxial Probe Method

- It does not need any machining of the sample, thus the sample is easy to prepare.
- It allows for dielectric measurement of numerous samples to be achieved in little time after calibration has been done.
- It allows for measurement to be done in a temperature-controlled environment.

Disadvantages of Open-ended Coaxial Probe Method
• It supports only reflection measurement.
• Susceptible to errors caused by air-gap effect.

2.3.3 Free space method

This method of measurement is used in wide frequency band; it is very suitable if the material under test is under high temperature or in hostile environment. The material under test should have large dimension i.e. should be large and flat. The two antennae are arraigned facing each other and each is connected to a port on the Vector Network Analyzer. The Vector Network Analyzer must be calibrated with any of the methods listed below before any measurement is done.

1. Through-Reflect-Match (TRM)
2. Line-Reflect-Line (LRL)
3. Through-Reflect-Line (TRL)

Of all the calibration methods listed above, the Line-reflect-Line(LRL) method is the best because it the tendency of achieving topmost calibration quality with the line standard achieved by separating the focal planes of the two antenna by $\lambda/4$ and the reflection standard achieved by inserting a metal plate in-between the two antennae. Once calibration is done, the S-parameters are first obtained without the MUT in the sample holder. Later, the MUT is put in the sample holder and the S-parameters are re-obtained [18].

![Diagram of Free Space Method](image)

Figure 2.5: Free Space Method.

Advantages of Free Space Method

• Suitable for high frequency measurements.
• Goes well with non-destructive measurement.
- Can measure material under test in hostile conditions.
- Can evaluate both permittivity and permeability.

Disadvantages of Free Space Method

- It cannot be used for small MUT, thus it needs large as well as flat MUTs.
- There is a problem of multiple reflections between antennae and surface of sample.
- Edge of sample contributes diffraction effect.

2.3.4 Resonant method

This method provides parameters with high accuracy for calculating the permittivity and permeability, but because of its limitations with high frequency and non-destructive testing uncertainty, it is not widely used. Re-entrant Cavities, Split Cylinder, and Cavity Resonator methods are some of the various methods for achieving resonant measurement [19].

With resonance characteristics depending on the MUT in a cavity, its quality factor and resonant frequency can be monitored to determine the dielectric parameters. The dielectric properties can be determined by first measuring the resonant frequency and quality factor of an empty cavity. The second step is to repeat the measurement after filling the cavity with the MUT. The permittivity or permeability of the material can then be computed using the frequency, volume, and q-factor. For this method we don’t need to calibrate the vector network analyzer [19].

Advantages of Resonant Method

- Suitable for measuring small material under test.
- It uses approximate expressions for fields in both cavity and sample.

Disadvantages of Resonant Method

- It requires a Vector Network Analyzer (VNA) of high frequency resolution.
- It has a limitation of narrow frequency bands.
Chapter 3

Chapter 3 is all about the measurements and simulations. In this chapter we explain the process of the whole thesis work and show the HFSS simulations and VNA testing. We have also discussed about the results of HFSS simulations and using VNA and antennas for the measurement of the resonant frequency and dielectric constant of different materials.

3.1 Process and Results:

In this chapter, we applied our understanding of the knowledge base in Chapter 2. We used shift in resonant frequency to measure the dielectric constant of any solid material of interest. We achieved this by first measuring the resonant frequency of the ring without the MUT (Material under Test); second we put the MUT in the gap of the ring and re-measured the resonant frequency; this time, the resonant frequency changed to a lesser one. Third, we combined the new resonant frequency with the geometric parameters of the ring and MUT to derive equation which we used to calculate the dielectric constant. $S_{21}$ was used to characterize the resonant frequencies. The chapter is divided into three sections, Analysis, Simulation, and Measurement.

3.2 Analysis

Equation (3.1), which incorporated equations (3.3) to (3.5) and (3.6) could only give the first resonant frequency of the ring resonator without MUT in the gap but to measure the dielectric constant of the material, we need the resonant frequency of the ring when the MUT is placed in the gap; thus an extensive derivation of the new resonant frequency equation is needed to be able to achieve the aim of the project. What follows is the derivation of this formula and the inherent accompany ones.

Our ring resonator could be idealized as a parallel plate capacitor whose capacitance is written as

$$C = \frac{\varepsilon_0 A}{d}$$  \hspace{1cm} (3.1)

By extending this formula to the case of this project, the capacitance of the parallel plate capacitor is likened to the capacitor in the gap of the ring resonator and is given as $C_{\text{gap}}$ in equation (2.3). Putting material in the gap splits this capacitance into two which are $C_{\text{mut}}$ and $C_{\text{air}}$. The $C_{\text{mut}}$ is the capacitance due to the MUT and the $C_{\text{air}}$ is the capacitance due to the air left uncovered by the material. The two capacitances are in series and their relation is given in equation (2.2) below:

$$\frac{1}{C_{\text{gap}}} = \frac{1}{C_{\text{mut}}} + \frac{1}{C_{\text{air}}}$$  \hspace{1cm} (3.2)
From equation (3.3), we know that

\[ C_{\text{gap}} = \varepsilon_0 \left[ \frac{wh}{g} + \frac{2\pi h}{\ln\left(\frac{2.4h}{w}\right)} \right] \quad (3.3) \]

From here, we have

\[ C_{\text{mut}} = \varepsilon_0 \varepsilon_r \left[ \frac{wh}{g_{\text{mut}}} + \frac{2\pi h}{\ln\left(\frac{2.4h}{w}\right)} \right] \quad (3.4) \]

\[ C_{\text{air}} = \varepsilon_0 \left[ \frac{wh}{g_{\text{air}}} + \frac{2\pi h}{\ln\left(\frac{2.4h}{w}\right)} \right] \quad (3.5) \]

\[ C_{\text{gap}} = \left[ \frac{1}{C_{\text{mut}}} + \frac{1}{C_{\text{air}}} \right]^{-1} \quad (3.6) \]

\[ C = C_{\text{gap}} + C_{\text{surf}} \]. From equation (2.6), this means that

\[ C = \left[ \frac{1}{\varepsilon_0 \varepsilon_r \left[ \frac{wh}{g_{\text{mut}}} + \frac{2\pi h}{\ln\left(\frac{2.4h}{w}\right)} \right]} + \frac{1}{\varepsilon_0 \left[ \frac{wh}{g_{\text{air}}} + \frac{2\pi h}{\ln\left(\frac{2.4h}{w}\right)} \right]} \right]^{-1} + \frac{2 \varepsilon_0 h}{\pi} \ln \frac{4R}{g} \quad (3.7) \]

where

\[ C_{\text{surf}} = \frac{2 \varepsilon_0 h}{\pi} \ln \frac{4R}{g} \quad (3.8) \]

Equation (3.6) is the total capacitance when material is inserted in the gap.

From equation (1.1),

\[ f_0 = \frac{1}{2\pi\sqrt{LC}} \]
Customizing this equation to the case of an MUT in the gap, this resonant frequency changes to the one below:

\[
f_{\text{onew}} = \frac{1}{2\pi \sqrt{L(C_{\text{gap}} + C_{\text{surf}})}}
\]

\[
f_{\text{onew}}^2 = \frac{1}{(2\pi)^2 L(C_{\text{gap}} + C_{\text{surf}})}
\]

\[(2\pi f_{\text{onew}})^2 L(C_{\text{gap}} + C_{\text{surf}}) = 1
\]

\[C_{\text{gap}} + C_{\text{surf}} = \frac{1}{(2\pi f_{\text{onew}})^2 L}
\]

\[C_{\text{gap}} = \frac{1}{(2\pi f_{\text{onew}})^2 L} - C_{\text{surf}}
\]

\[C_{\text{gap}} = \frac{1 - (2\pi f_{\text{onew}})^2 L C_{\text{surf}}}{(2\pi f_{\text{onew}})^2 L}
\]

\[
\left[\frac{1}{C_{\text{mut}}} + \frac{1}{C_{\text{air}}}\right]^{-1} = \frac{1 - (2\pi f_{\text{onew}})^2 L C_{\text{surf}}}{(2\pi f_{\text{onew}})^2 L}
\]

\[
\frac{1}{C_{\text{mut}}} = \frac{(2\pi f_{\text{onew}})^2 L}{1 - (2\pi f_{\text{onew}})^2 L C_{\text{surf}}} - \frac{1}{C_{\text{air}}}
\]

\[
\frac{1}{\varepsilon_r \times 1.7134 \times 10^{-13}} = \frac{(2\pi f_{\text{onew}})^2 L}{1 - (2\pi f_{\text{onew}})^2 L C_{\text{surf}}} - \frac{1}{C_{\text{air}}}
\]

\[
\frac{1}{\varepsilon_r} = \left[\frac{(2\pi f_{\text{onew}})^2 L}{1 - (2\pi f_{\text{onew}})^2 L C_{\text{surf}}} - \frac{1}{C_{\text{air}}}\right] \times 1.7134 \times 10^{-13}
\]

Equation (3.7) can be used to predict the new resonant frequency when the MUT is inserted in the gap provided the dielectric constant of the MUT is known. Equation (3.8) is what was used for the dielectric constant extraction after the shift in resonant frequencies was obtained. The MATLAB scripts for these equations and the accompanying ones are found in the appendix.
Chapter 4

4.1 Simulations

Based on the mathematical analysis in Chapter 3, a practicable ring resonator was designed; it could hold the MUT (material under test), whose dielectric constant is to be measured, in its gap. The values of the geometric parameters of the ring were chosen such that the first resonant frequency of the ring fell within the measurable frequency range (1-14GHz) of the VNA available in the laboratory; the first resonant frequency was calculated with the different parameters constituting the geometry of the ring resonator; putting the four parameters: height(h), width(w), radius(R), and gap(g) of the ring resonator in equation (3.1) repeatedly to get the resonant frequency and ensuring the fabrication practicability of the ring is a gritty task without a programmed that could iterate this process; a MATLAB script was written to address this issue. This script allows for iteration to be done with all the geometric parameters in many times over so that a practicable ring of the designer’s choice could be realized; it further gave the resonant frequency of any geometry of the ring. The script is in Appendix A.

After getting a ring resonator whose geometry we were convenient with, we simulated the ring and the measurement environments with HFSS 13.0. The material for the ring resonator was stainless steel and its geometry is given below:

Height (h) = 5mm
Width (w) = 2mm
Gap(g) = 6mm
Internal radius (R) = 13mm

Putting the above geometric parameters in the MATLAB script in Appendix A, the first resonant frequency of this ring is 1.6566GHz. Figure 3.1 below shows the simulated model of the ring resonator, without the MUT, in HFSS.

![Figure 4.1: Ring Resonator Model without MUT.](image)

The ring was enclosed in an air box because in HFSS, the background is a PEC (Perfect Electric Conductor) and since the resonator was not to be put in a PEC during real-time measurement, the air...
box had to surround the structure so that the transmission and reception of the EM (Electromagnetic Waves) could occur in free space. The dimension of the air box is at least $\lambda/4$ of the lowest resonant frequency.

**4.2 Excitation and Boundaries in HFSS**

To simulate the ring resonator in HFSS, boundaries and excitations had to be defined. The top and bottom faces of the box were defined as the Perfect H (this falls on the Z-plane) while the left and right faces were defined as the Perfect E (this falls on the Y-plane); Perfect H and Perfect E are the boundary definitions. Figures 4.2 and 4.3 below show the boundaries in HFSS.

The Perfect H and Perfect E are orthogonal to each other due to the principle of electromagnetic waves; Perfect H means perfect magnetic waves while Perfect E means perfect electric waves.

Excitation was done by defining two wave ports; a port is a point through which a signal or wave enters and leaves a structure. In this case, Port 1 was used as the wave entry point while Port 2 was used as the wave exit point. Figures 4.4 and 4.5 below show the two ports.
In summary, the direction of wave propagation was along the $x$-axis, electric field was along the $y$-axis while the magnetic field was along the $z$-axis. The solution type was set to Driven Model, this solution type is the one appropriate for finding the resonant frequencies of structures in HFSS.

4.3 Simulated resonant frequencies without and with MUT in the gap

After defining the proper boundaries and excitations with the appropriate solution type, a simulation for resonant frequencies from 0.5GHz to 15GHz was run, this yielded a sizeable number of resonant frequencies depicted in figure 3.6 below:

The calculated fundamental resonant frequency from the formula matched the first resonant frequency seen from the simulation result. Here, $S_{21}$ is in blue while $S_{11}$ is in brown. The dips in $S_{11}$ and $S_{21}$ indicate the points of resonant frequencies. Either $S_{21}$ or $S_{11}$ could be used to define the resonant frequency; for $S_{11}$ only one port is needed while two ports are needed for $S_{21}$. In our case, $S_{21}$ was used for defining the resonant frequency and this makes it clear why two ports were defined in the excitation. A clearer and more exact depiction of the first resonant frequency is shown in figure 3.7 below:
From figure 4.7 above, a resonant frequency of 1.64GHz could be read, this shows a clear conformity with the calculated one and thus we had very little error.

When a sample MUT of porcelain was inserted in the gap and a simulation for resonant frequencies was done with the HFSS, there were noticeable shifts in the resonant frequencies compared to the ones obtained in figure 4.6 without material in the gap. The new resonant frequencies of the ring with material in the gap are shown in figure 3.8 below:

From figure 4.8 above, it could be seen that the first resonant frequency shifted from 1.64GHz to less than 1.5GHz and likewise the subsequent resonant frequencies. More accurate plots of the shifts in resonant frequencies will be seen in the subsequent parts of the report when the simulation is confined within a short frequency range of 0.5GHz to 2.5GHz.

The simulations and measurements were done in the frequency range of 0.5GHz to 2.5GHz because from analysis, that is where the first fundamental resonant frequency falls and moreover, despite having a low quality factor, it has the best frequency shift; this fits very well into achieving the aim of this thesis since the resonant frequency shift is what is required for the measurement of dielectric constant.

For a shift in resonant frequency to be achieved, the dimension of the MUT must be very small compared to that of the ring resonator; further, the MUT must fill the gap. Anything short of these two
conditions will achieve no result. The ring resonator with material in the gap is shown in figure 4.9 below:

The MUT is the little black object filling the gap and it can be seen that its dimension is very small compared to that of the ring resonator.

Now, what follow are the simulated resonant frequencies of the ring resonator with MUTs. The one without MUT (figure 4.7) was used as the referencing standard for the ones with MUTs. The simulation was done within a frequency range of 0.5GHz to 2.5GHz with a step size of 0.01GHz and 10 maximum numbers of passes. These settings were chosen so as to get very accurate results from HFSS.

Four different materials were used, they are Porcelain, Quartz glass, Marble and Plexiglas. Each case of material inclusion had different resonant frequency; no two materials had the same resonant frequency because the materials were different from each other.
Further, plots of current distributions in the ring at each resonant frequency from 1.64GHz up to 10.2GHz were made; these plots gave us knowledge of the places of lowest and highest current in the structure at each resonant frequency. The figures that follow depict the current distributions.

4.4 Measurement

After simulating for resonant frequencies without material and with different materials, we needed to execute this project in real life scenario. It is real life measurement that will tell whether the designed and fabricated ring resonator will work or not and this can only be done in the laboratory.

Rhodes and Schwarz ZVB 14 VNA operating at a frequency range of 10MHz to 14GHz with a pair of monopole antennae were used. TOSM (Through-Open-Short-Match) full 2-port was used as the calibration for the VNA; the VNA was calibrated for the entire operating frequency range.

The ring resonator was made of stainless steel; coaxial cables used for the monopole antennae were designed to be $\lambda/2$ [19] of the measuring frequency range; the exposed center conductor, which acted as the transmitter and receiver, was 75mm long, this conformed with $\lambda/2$. The distance between the two monopole antennae was 45mm. The steps followed in the measurement are listed below:
1. One antenna was connected to port 1 of the VNA while the other was connected to port 2. The one at port 1 was used as the transmitter while the one at port 2 was used as the receiver.
2. The ring resonator was placed between the antennae with equal distance from either of them.
3. The VNA was set to measure the resonant frequency of the ring resonator between 0.5GHz and 2.5GHz without material in the gap and the resonant frequency was noted and recorded.
4. The VNA was set to measure the resonant frequency of the ring resonator between 0.5GHz and 2.5GHz with material in the gap and the resonant frequency was noted and recorded. After this step, a different material was put in the gap; the resonant frequency was also noted and recorded.

The materials used were Teflon and wood because these were the only available materials in the laboratory as at the time of making the measurement.

The measured results without and with MUT in the gap are given in the figures that follow.

![Figure 4.12: Measured S21 vs. f0 from 1-2.5GHz for the Ring resonator without the material, the resonant frequency is at 1.65GHz](image)

Comparison of results obtained in figure 4.13 to that of figure 4.7 showed a very good result and conformity between the simulated result and measured result. The simulated result gave a resonant frequency of 1.64GHz without MUT in the gap while the measured result produced a resonant frequency of 1.65GHz without MUT.
Further, with the two different test materials put in the gap, the resonant frequency shifted for the measurement as predicted in the aforementioned analysis and as obtained in the HFSS simulation with different MUTs in the gap. Please note that the MUTs used in the simulation were not the same as the ones used in real life laboratory measurement; thus their resonant frequency plots would not be the same but the similarity in the results is that they all produced resonant shifts.
Above in figure 4.17 are the practical setup of the laboratory measurement, in which we have shown the whole setup for resonant frequency measurement. As shown in the above figure we used two monopole antennas with VNA and measured the ring resonator resonant frequency and also the material under in also put in gap of the ring resonator.
Chapter 5

Discussion & Conclusions

Chapter 5 is about the discussion and conclusion. In this chapter we discussed our results and conclude the whole research work and also give some suggestions for the further enhancement of this thesis work as a future work.

5.1 Discussions:

From the aforementioned, we resonant method for the measurement of dielectric constant of materials because it gives more accurate results. The analytical, simulated, and measured results are given in the tables that follow.

<table>
<thead>
<tr>
<th>Analytical $f_0$ without MUT</th>
<th>Simulated $f_0$ without MUT</th>
<th>Measured $f_0$ without MUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6566GHZ</td>
<td>1.64GHZ</td>
<td>1.65GHZ</td>
</tr>
</tbody>
</table>

Percentage Error

1% 0.39%

Table 5.1: Resonant frequency results obtained from analysis, simulation, and measurement.

<table>
<thead>
<tr>
<th>Material</th>
<th>Simulated $f_0$ with MUT</th>
<th>Dielectric constant, $\varepsilon_r$ of MUT obtained from simulated $f_0$</th>
<th>Standard dielectric constant of MUT</th>
<th>Percentage Error in $\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porcelain</td>
<td>1.39GHZ</td>
<td>5.98</td>
<td>5.7</td>
<td>4.9%</td>
</tr>
<tr>
<td>Glass_PTFE</td>
<td>1.57 GHZ</td>
<td>2.60</td>
<td>2.5</td>
<td>4.0%</td>
</tr>
<tr>
<td>Marble</td>
<td>1.28 GHZ</td>
<td>8.18</td>
<td>8.3</td>
<td>1.45%</td>
</tr>
<tr>
<td>Quartzglass</td>
<td>1.49 GHZ</td>
<td>3.99</td>
<td>3.78</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

Table 5.2: Resonant frequencies and Dielectric constants obtained from simulation with MUT in the gap.

<table>
<thead>
<tr>
<th>Material</th>
<th>Simulated $f_0$ with MUT</th>
<th>Dielectric constant, $\varepsilon_r$ of MUT obtained from measured $f_0$</th>
<th>Standard dielectric constant of MUT</th>
<th>Percentage Error in $\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass_PTFE</td>
<td>1.59GHZ</td>
<td>2.55</td>
<td>2.5</td>
<td>2.0%</td>
</tr>
<tr>
<td>Wood</td>
<td>1.56GHZ</td>
<td>2.78</td>
<td>2.9</td>
<td>4.1%</td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>------</td>
<td>-----</td>
<td>------</td>
</tr>
</tbody>
</table>

Table 5.3: Resonant frequencies and Dielectric constants obtained from measurement with MUT in the gap.

The dielectric constants for each simulated and measured case was calculated with equation (3.8); the equation accepts the inductance, surface capacitance, air capacitance, and the new resonant frequency values when MUT is put in the gap as inputs. The script for the equation is in the Appendix.

From tables 5.2 and 5.3, we could see that the errors in dielectric measurements are very minimal. The possible sources of errors are:

I. The inherent errors in the measuring instruments as there are no 100% perfect such instruments.

II. Measurement error of the sample dimension.

III. Errors from the calibration of the VNA.

IV. Errors from bends in the measuring cables.

V. Errors from exact positioning of the resonator between the two antennae.

VI. Errors from the non-proper alignment of the edges of MUT with the two edges of the ring resonator forming the gap.

On the whole, the results are good, the designed and fabricated resonator is reliable, and the project was well executed.

**5.2 Conclusions**

Dielectric constants of different materials were successfully measured with the resonant method employed in this thesis; it was done with a singly split single ring resonator.

Further work on this project will be in the form of concatenation of ten or more of this same single ring resonator and the arraignment will be used for the measurement of dielectric constant of materials as done in this project.

One major difference will be that two horn antennae will be used instead of monopole antennae used here.
References


Appendix A

Resonant Frequency of Ring without MUT in the Gap

%Script file: resf
%Returns the resonant frequency of a Singly Split Single Ring Resonator
disp('************************************************************************************')
disp('This script Returns The Resonant Frequency of a Singly Split Single Ring Resonator')
disp('******************************************************************************')
R=input('*** Enter the Internal Radius of Ring in (mm):');
w=input('*** Enter the Width of Ring in (mm):');
g=input('*** Enter the Length of Gap in (mm):');
h=input('*** Enter the Height of Ring in (mm):');
Rm=R+w/2;
L=4*pi*10^-7*Rm*(log(8*Rm/(h+w))-0.5)*0.001;
C_gap=8.854*10^-12*((w*h/g)+(2*pi*h/log(2.4*h/w)))*0.001;
C_surf=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001;
C=C_gap+C_surf;
disp('************************************************************************************')
disp('The Resonant Frequency f_o in GHZ is:')
f_0=(1/(2*pi*sqrt(L*C)))*1e-9
disp('************************************************************************************')

Resonant Frequency of Ring with MUT in the Gap

%Script file: nresf
%Returns the new resonant frequency of a Singly Split Single Ring Resonator
%with MUT in the Gap
disp('************************************************************************************')
disp('This script Returns The Resonant Frequency of a Singly Split Single Ring Resonator with MUT in the Gap')
disp('**********************************************************************************')
R=input('*** Enter the Internal Radius of Ring in (mm):');
w=input('*** Enter the Width of Ring in (mm):');
h=input('*** Enter the Height of Ring in (mm):');
g=input('*** Enter the Gap of Ring in (mm):');
E_r=input('*** Enter the Dielectric Constant of the MUT :');
Rm=R+w/2;
L=4*pi*10^-7*Rm*(log(8*Rm/(h+w))-0.5)*0.001;
C_mut=E_r*8.854*10^-12*((w*h/5.5)+(2*pi*h/log(2.4*h/w)))*0.001;
C空气=8.854*10^-12*((w*h/0.5)+(2*pi*h/log(2.4*h/w)))*0.001;
C_surf=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001;
C_gap=((1/C_mut)+(1/C空气))^(-1);
C=C_gap+C_surf;
disp('************************************************************************************')
disp('The Resonant Frequency f_o in GHZ is:')
f_0new=(1/(2*pi*sqrt(L*C)))*1e-9
disp('************************************************************************************')
Inductance of Ring Resonator

%Script file:inductance
%Returns the Inductance of a Singly Split Single Ring Resonator
disp('********************************************************************')
disp('This script returns the inductance of a Singly Split Single Ring Resonator');
disp('********************************************************************')
R=input('*** Enter R in (mm):');
w=input('*** Enter w in (mm):');
h=input('*** Enter h in (mm):');
Rm=R+(w/2);
disp('********************************************************************')
disp('The inductance L in H is :')
L=4*pi*10^-7*Rm*(log(8*Rm/(h+w))-0.5)*0.001
disp('*************************************************************')

Gap Capacitance without MUT

%Script file:cgap
%Returns the Gap Capacitance of a Singly Split Single Ring Resonator without MUT
disp('********************************************************************')
disp('This script returns the gap capacitance of a Singly Split Single Ring Resonator');
disp('********************************************************************')
w=input('*** Enter w in (mm):');
g=input('*** Enter g in (mm):');
h=input('*** Enter h in (mm):');
disp('********************************************************************')
disp('The gap capacitance in F is :')
C_gap=8.854*10^-12*((w*h/g)+(2*pi*h/log(2.4*h/w)))*0.001
disp('******************************************************')

Surface Capacitance

%Script file:csurf
%Returns the Surface Capacitance of a Singly Split Single Ring Resonator
disp('********************************************************************')
disp('This script returns the surface capacitance of a Singly Split Single Ring Resonator');
disp('********************************************************************')
R=input('*** Enter R in (mm):');
g=input('*** Enter g in (mm):');
h=input('*** Enter h in (mm):');
disp('********************************************************************')
disp('The surface capacitance in F is :')
C_surf=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001
disp('********************************************************************')
Total Capacitance a Singly Split Single Ring Resonator without MUT

%Script file:ctot
%Returns the Total Capacitance of a Singly Split Single Ring Resonator
disp('************************************************************
*** This script returns the Total Capacitance of a Singly Split Single Ring Resonator
*** RETURNS THE TOTAL CAPACITANCE OF A SINGLY SPLIT SINGLE RING RESONATOR
**************************************************************
');
R=input('*** Enter R in (mm):');
w=input('*** Enter w in (mm):');
g=input('*** Enter g in (mm):');
h=input('*** Enter h in (mm):');
C_gap=8.854*10^-12*((w*h/g)+(2*pi*h/log(2.4*h/w)))*0.001;
C_surf=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001;
disp('****************************************************************
The total capacitance C in F is :');
C=C_gap+C_surf
disp('****************************************************************

Capacitance contributed by Air in a Singly Split Single Ring Resonator with MUT

%Script file:cair
%Returns the capacitance contributed by air in a Total Capacitance of a Singly Split Single Ring Resonator
disp('****************************************************************
*** This Script Returns the Capacitance contributed by Air in a Singly Split Single Ring Resonator
*********************************************************************
');
w=input('*** Enter w in (mm):');
h=input('*** Enter h in (mm):');
disp('******************************************************
The capacitance in F is :');
C_air=8.854*10^-12*((w*h/0.5)+(2*pi*h/log(2.4*h/w)))*0.001
disp('******************************************************

Capacitance contributed by Material in a Singly Split Single Ring Resonator with MUT

%Script file:cmut
%Returns the capacitance contributed by the MUT.
disp('****************************************************************
The script returns the Capacitance contributed by the MUT in a Singly Split Single Ring Resonator
');
w=input('*** Enter w,the Width of the Ring in (mm):');
h=input('*** Enter h,the Height of the Ring in (mm):');
E_r=input('*** Enter E_r,the Dielectric Constant of MUT:');
disp('***********************************************
The capacitance in F is :');
C_mut=E_r*8.854*10^-12*((w*h/5.5)+(2*pi*h/log(2.4*h/w)))*0.001
disp('***********************************************

A3
The Resultant Gap Capacitance a Singly Split Single Ring Resonator with MUT

%Script file: cgap
%Returns the gap capacitance of a Singly Split Single Ring Resonator
disp('*****************************************************************************

This script returns the gap capacitance of a single split single ring resonator');
disp('***************************************************************

|w|=input('*** Enter w, the width of the ring in (mm):');
|h|=input('*** Enter h, the height of the ring in (mm):');
|E_r|=input('** Enter E_r, the dielectric constant of the MUT:');
|C_mut|=E_r*8.854*10^-12*((w*h/5.5)+(2*pi*h/log(2.4*h/w)))*0.001;
|C_air|=8.854*10^-12*((w*h/0.5)+(2*pi*h/log(2.4*h/w)))*0.001;
disp('The Gap Capacitance, C_gap, in F is:')
|C_gap|=(1/C_mut+1/C_air)^-1
disp('*****************************************************************************

New resonant frequency of a Singly Split Single Ring Resonator with MUT in the Gap

%Script file: nresf
%Returns the new resonant frequency of a Singly Split Single Ring Resonator
%with MUT in the Gap
|disp('*****************************************************************************

|This script Returns The Resonant Frequency of a Singly Split Single Ring Resonator with MUT
|in the Gap');
disp('***************************************************************

|R|=input('*** Enter the Internal Radius of Ring in (mm):');
|w|=input('*** Enter the Width of Ring in (mm):');
|h|=input('*** Enter the Height of Ring in (mm):');
|g|=input('*** Enter the Gap of Ring in (mm):');
|E_r|=input('** Enter the Dielectric Constant of the MUT:');
|Rm|=R+w/2;
|L|=4*pi*10^-7*Rm*(log(8*Rm/(h+w))-0.5)*0.001;
|C_mut|=E_r*8.854*10^-12*((w*h/5.5)+(2*pi*h/log(2.4*h/w)))*0.001;
|C_air|=8.854*10^-12*((w*h/0.5)+(2*pi*h/log(2.4*h/w)))*0.001;
|C_surf|=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001;
|C_gap|=1/(1/C_mut+1/C_air)^(^1;
|C|=C_gap+C_surf;
disp('The Resonant Frequency f_o in GHZ is:')
|f_0new|=(1/(2*pi*sqrt(L*C)))*1e-9
disp('*****************************************************************************

The Dielectric Constant of the MUT in the Gap

%Script file: dielec
%Returns the The Dielectric Constant of the MUT in the Gap
|disp('*****************************************************************************

|This script The Dielectric Constant of the MUT in the Gap');
disp('***************************************************************

|R|=input('*** Enter the Internal Radius of Ring in (mm):');
|w|=input('*** Enter the Width of Ring in (mm):');
h=input('*** Enter the Height of Ring in (mm):');
g=input('*** Enter the Gap of Ring in (mm):');
f_0new=input('*** Enter the new resonant frequency in HZ:');
Rm=R+w/2;
L=4*pi*10^-7*Rm*(log(8*Rm/(h+w))-0.5)*0.001;
C_air=8.854*10^-12*((w*h/0.5)+(2*pi*h/log(2.4*h/w)))*0.001;
C_surf=(2*8.854*10^-12*h/pi)*log(4*R/g)*0.001;
disp('***********************************************************************
The Dielectric Constant of the MUT is :')
E_r=((((2*pi*f_0new)^2*L)/(1-(2*pi*f_0new^2)*L*C_surf)-(1/C_air))*1.714^13)^1*(1e9)
disp('***********************************************************************')